Incentives and Voluntary Investment in Employer Shares*

Issouf Soumaré†  Ron Giammarino‡

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†Faculty of Commerce & Business Administration, The University of British Columbia, 2053 Main Mall, Vancouver, B.C., Canada V6T 1Z2; Tel: 1-604-822-0184; Email: issouf.soumare@commerce.ubc.ca.

‡Faculty of Commerce & Business Administration, The University of British Columbia, 2053 Main Mall, Vancouver, B.C., Canada V6T 1Z2; Tel: 1-604-822-8357; Email: ron.giammarino@commerce.ubc.ca.
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Abstract

Despite the significant diversification costs, defined contribution plans hold excessive investments in the shares of the employer company. In this paper we present an explanation of why employees would voluntarily purchase large holdings of the shares of their employer. Our explanation builds on the incentive properties of stock ownership and hence our work relates to the standard principal/agent framework that is widely employed to study management compensation. In order to base our analysis on this approach we must first recognize that, in contrast to management contracts where the optimal contract prohibits the purchase, sale and/or short sale of the company’s shares, defined contribution pension plans are invested in assets selected by the employee (the agent). As a result, we add to this literature by deriving an optimal contract when employees are allowed to either buy or sell the shares of the company that employs them. This added feature of the model is accompanied by an added imperfection, that being manager’s type uncertainty, to explain the difference in company shares holdings across firms.

Keywords: Defined contribution pension plans or 401(k) accounts, Hidden information, Incentive, Investment, Moral hazard, Portfolio choice.

JEL Classification: D80, G11, G23, G32, J33

Preliminary and incomplete version.
1 Introduction

Enron’s financial distress cost resulted in both job losses and significant investment losses due to the huge percentage of Enron’s retirement plan that was invested in Enron shares. As of December 31, 2000, Enron shares made up 62% of the assets held in the company’s defined contribution (DC) pension plan, Purcell (2002). An estimated 89% of these shares were purchased by employees while the balance represents the corporation’s matching contribution.

Such a concentration of human and financial capital in one company flies in the face of the main lesson of portfolio theory. Yet, as Table 1 indicates, Enron employees were not alone in holding a large percentage of their pension assets in the shares of the company that employs them.

Although defined benefit pension plans are restricted to holding less than 10% of the plans assets in the stock or real estate of the employer, defined contribution plans (401(k) accounts) are exempted from this rule. In fact, a third of the assets in large retirement savings plans are invested in company stock, and a quarter of workers discretionary contributions are invested in company stock (Benartzi (2001)). Moreover, when employer’s contributions are automatically directed to company stock, employees invest more of their own contributions in company stock.

Table 1: Percentage allocation of retirement plans assets in company stock as of year 2001.

<table>
<thead>
<tr>
<th>Company</th>
<th>% company stock</th>
<th>Company</th>
<th>% company stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procter&amp;Gamble</td>
<td>94.7%</td>
<td>Williams</td>
<td>75.0%</td>
</tr>
<tr>
<td>Sherwin-Williams</td>
<td>91.6%</td>
<td>McDonal’s</td>
<td>74.3%</td>
</tr>
<tr>
<td>Abbott Laboratories</td>
<td>90.2%</td>
<td>Home depot</td>
<td>72.0%</td>
</tr>
<tr>
<td>Pfizer</td>
<td>85.5%</td>
<td>McKesson HBOC</td>
<td>72.0%</td>
</tr>
<tr>
<td>BB&amp;T</td>
<td>81.7%</td>
<td>Marsh&amp;McLennan</td>
<td>72.0%</td>
</tr>
<tr>
<td>Anheuser-Bush</td>
<td>81.6%</td>
<td>Duke Energy</td>
<td>71.3%</td>
</tr>
<tr>
<td>Coca-Cola</td>
<td>81.5%</td>
<td>Textron</td>
<td>70.0%</td>
</tr>
<tr>
<td>General Electric</td>
<td>77.4%</td>
<td>Kroger</td>
<td>65.3%</td>
</tr>
<tr>
<td>Texas Instruments</td>
<td>75.7%</td>
<td>Target</td>
<td>64.0%</td>
</tr>
<tr>
<td>William Wrigley, Jr.</td>
<td>75.6%</td>
<td>Household Int’l.</td>
<td>63.7%</td>
</tr>
</tbody>
</table>

Source: Reproduced from Purcell (2002, Table 1, p.4) (from DC Plan Investing, Institute of Management and Administration, NY).
Some have argued for legislation to cap defined contribution pension investment in company stock. This proposal does not, however, seem to have strong support among plan participants. Erickson (2002) reports: “In a February 2002 poll of 1000 individuals conducted by Putnam Investments, 57 percent of investors who held company stock in their 401(k) accounts were opposed to legislative efforts to restrict the amount of company stock in defined contribution plans; only 20 percent favored such restrictions... Many commentators in the financial press have noted that employees have faith in their employers and do not believe that an Enron-or Adelphia, Qwest or WorldCom-catastrophe could happen to them or to the company stock that seems like, and often has been, such a good investment.” Purcell (2002) notes that, “In most cases, a majority of the company stock in 401(k) plans represents voluntary purchases by employees. It is likely that some workers in these firms would oppose restrictions on the amount of company stock in 401(k).” Sengmuller (2002), “In general it is a bad idea to invest a high proportion of one’s wealth in a single firm. It is even worse when that single firm is one’s employer... But individuals do it, and they like to do it. For instance Motorola eliminated its policy limiting its employees’ investment in Motorola stock to 25% of their contributions after employee complaints.”

In this paper we present an explanation of why employees would voluntarily purchase larger holdings of their employer shares than would be the case based on portfolio considerations alone. Our explanation builds on the incentive properties of stock ownership and hence our work relates to the standard principal/agent framework that is widely employed to study management compensation. In order to base our analysis on this approach we must first recognize that, in contrast to management contracts where the optimal contract prohibits the purchase, sale and/or short sale of the company’s shares, defined contribution pension plans are invested in assets selected by the employee (the agent). As a result, we add to this literature by deriving an optimal contract when employees are allowed to either buy or sell the shares of the company that employs them. This added feature of the model is accompanied by an added imperfection, that being manager’s type uncertainty, to explain the difference of company shares holdings across firms.
Our work builds on the standard principal/agent literature pioneered by Holmstrom (1979, 1982), and Holmstrom & Milgrom (1987), among others. In this literature, the optimal contract imposes suboptimal diversification on workers in order to induce higher levels of effort. This has been applied directly to the case of executive compensation by Garvey & Milbourn (2003), Jin (2002), Oyer & Schaefer (2002) among others.

We depart from this literature by assuming that workers are responsible for deciding how to invest their assets. We argue that, when they have the flexibility to adjust their productivity, workers will tend to invest more in company stock when they believe that the return is higher or the risk is lower. The higher investment level has a second impact through the consequences of higher ownership on firm efforts. In addition, our analysis provides a separate role for managers and other employees. Managers take actions that influence the productivity of the entire work force. Workers on the other hand influence output by adjusting their effort level. The optimal effort level will take into account the productivity decision of management while the productivity decision takes into account workers’ efforts.

The rest of the paper is organized as follows, section 2 provides a survey of the main empirical findings on DC plans asset allocations, section 3 presents the model, we conduct a comparative static of our model with the existing incentive model. Section 4 develops the asymmetry of information with respect to the manager’s type. We conclude with section 5. Proofs are provided in the appendix.

2 Empirical evidence on DC pension plans

Over the last 20 years, DC pension plans have gradually replaced DB pension plans as shown in Table 2. In 2001, there were 700,000 corporate DC plans covering 56 million workers and managing over $2 trillion in assets compared to only 56,000 DB retirement plans covering 23 million active participants (Mitchell & Utkus, 2002). DB plans continue to decline over time.¹

¹While the Employee Retirement Income Security Act of 1974 (ERISA) restricts the investment of DB plans in stock or real estate of the employer to 10% of total assets; DC plans, however, are exempted from this rule. Most DB plans are guaranteed by the Pension Benefit Guaranty Corporation (PBGC), DC plans are not.

A. Number of Pension Plans

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>DB Plans</th>
<th>DC Plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>632,135</td>
<td>170,172</td>
<td>461,963</td>
</tr>
<tr>
<td>1990</td>
<td>712,308</td>
<td>113,062</td>
<td>599,245</td>
</tr>
<tr>
<td>1995</td>
<td>693,404</td>
<td>69,492</td>
<td>623,912</td>
</tr>
<tr>
<td>1998</td>
<td>730,031</td>
<td>56,405</td>
<td>673,626</td>
</tr>
</tbody>
</table>

B. Pension Plans Assets ($ millions)

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>DB Plans</th>
<th>DC Plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>1,252,739</td>
<td>826,117</td>
<td>426,622</td>
</tr>
<tr>
<td>1990</td>
<td>1,674,139</td>
<td>961,904</td>
<td>712,236</td>
</tr>
<tr>
<td>1995</td>
<td>2,723,735</td>
<td>1,402,079</td>
<td>1,321,657</td>
</tr>
<tr>
<td>1998</td>
<td>4,021,849</td>
<td>1,936,600</td>
<td>2,085,250</td>
</tr>
</tbody>
</table>

Source: Reproduced from Mitchell & Utkus (2002, Table 1, p.36)

This importance of DC plans has been the catalyst of recent attention on DC plans investment behavior by the academic community, especially the empirical studies.

Benartzi (2001) found that, a third of the assets in large retirement savings plans are invested in company stock, and a quarter of workers discretionary contributions are invested in company stock. Employees excessively extrapolate past performance of company stock, and then employees of firms that experienced the worst stock performance over the last 10 years allocate 10.37 percent of their discretionary contributions to company stock, whereas employees whose firms experienced the best stock performance allocate 39.70 percent. Allocation to company stock, however do not predict future performance. When employer’s contributions are automatically directed to company stock, employees invest more of their own contributions in company stock because they interpret the allocation of the employer’s contributions as implicit investment advice or “endorsement effect”. In a survey conducted, he found a positive response for the extrapolation of past return, and few respondents rate the firm’s stock as riskier than the market. Roughly a third of the plans buy the shares on the open market, and the remaining two-thirds issue shares. Based on the findings, he argues that an information-based explanation for company stock holdings seems unlikely to hold. People tends however, to be optimistic and overconfident about the future prospect of company stock. Benartzi & Thaler (2001) showed that some investors follow the “1/n naive diversification strategy”: they divide their contributions evenly across the funds offered in the plan. Benartzi & Thaler (2002) found that a majority of their survey participants prefer the median portfolio to the one they pick for themselves. They explain that by the fact that investors do not have well defined preferences.
Sengmuller (2002) found that past performance of the company is the most salient piece of information employees rely on when they make changes to their portfolio (past stock returns, return volatility, and business performance). Investors in 401(k) accounts listen to wrong signals and tend to increase their exposure to company stock if returns have been high and their employer is doing well. They do not react to negative returns, however.

In a survey conducted, Choi et al. (2001) concluded that “... employees often follow ‘the path of least resistance’. For better or for worse, plan administrators can manipulate the path of least resistance to powerfully influence the savings and investment choices of their employees.” Liang & Weisbenner (2001) with their data found that the plan design (number of investment alternatives, employer match in company stock, past firm performance, and firm financial characteristics such as market-to-book ratio, price volatility, etc.) is very important in determining the share of 401(k) assets in company stock. They found that, in 1998, employees allocated 20 percent of their own contributions to company stock purchases. And among plans that offer company stock as option, between 30 and 40% are held in company stock. They can not reject that employees follow a naive $1/n$ diversification rule, investing $1/n$ of their contributions in company stock. Mitchell & Utkus (2002) found that company stock held in DC plans is a large firm phenomenon, and also asset allocation levels to company stock are function of plan size.

Meulbroek (2002) measures the cost of non-diversification to employees when they overload their DC pension plans in company stock. She concludes that the value an employee sacrifices relative to a well-diversified equity portfolio of the same risk average 42% of the market value of the firm’s stock under reasonable assumptions. Despite this high cost of non-diversification, employees still hold their company stock even when they are not required to do so. She argues that, the encouragement by firm to hold company stock may be explicit, by prohibiting employees from selling company stock in their retirement plans, or implicit, by characterizing the firm’s stock as a “good investment” or by suggesting that it provides tangible evidence of employees’ loyalty.
We consider a one period model (two dates: 0 and 1). There are three securities available in the economy: the risk-free security paying a constant return $r_f$, the firm's stock indexed by 1, and the rest of the market portfolio indexed by 2. We normalize the supply of the two risky securities to be 1 at time 0. We assume no taxes and no transaction costs.

The firm hires workers in a competitive labor market at a competitive wage $w_0$. Workers allocate their pension wealth among the three securities: they invest $A_1$ amount in stock 1 (the firm's stock), $A_2$ amount in stock 2, and invest the rest in bonds. Unlike previous incentive models, here workers (the agent) choose their holding percentage of company stock not the principal. In our framework, workers decide about their fund allocation, they then decide voluntarily to invest in company stock or not. When workers invest $A_1$ amount in company stock at the beginning of the period ($t = 0$), they put an effort $l$, which increases the stock's return. We assume the possibility of free-riding among workers, only $q$ portion of workers will participate in the effort production, the other $1 - q$ percent free-ride. The percentage $q$ is exogenously given in this model.

The firm's stock return is a function of workers’ effort $l$. We assume a linear functional form for the stocks returns with respect to the effort $l$

$$\tilde{r}_1 = \mu_1 + ql + \epsilon_1,$$

$$\tilde{r}_2 = \mu_2 + \epsilon_2,$$

where $\epsilon_i$ is the risk source of stock $i$, and is normally distributed with mean 0 and variance $\sigma_i^2$. The correlation between the two risk sources, $\epsilon_1$ and $\epsilon_2$, is $\rho$.

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2The rest of the market portfolio (stock 2) doesn’t include the firm’s stock.
3One implicit assumption we are making here, is that when workers receive company shares as matching contributions from the firm, they can sell the shares on the open market or keep them without restrictions. However it is worth noticing that, some companies impose restrictions on workers from selling the matched contributions in company stock.
3.1 Workers portfolio choice and effort level

Workers’ future wealth is

\[ W_W = A_1(\tilde{r}_1 - r_f) + A_2(\tilde{r}_2 - r_f) + w_0(1 + r_f), \]

where \( w_0 \) is workers total wealth at the beginning of the period, it includes their salary and the matching contributions from the firm. Workers future wealth contains three components: the first is the excess return earned on stock 1, the second term is the excess return from stock 2, and the last term is the value of the initial wealth if it was invested at the risk-free rate.

The production of the effort \( l \) incurs a cost \( C_W(l) \) to workers. The cost function \( C_W(l) \) is an increasing convex function of \( l \): \( C_W'(l) \geq 0 \) and \( C_W''(l) \geq 0, \forall l \geq 0 \). We assume a quadratic cost function

\[ C_W(l) = k \frac{l^2}{2}, \]

where \( k \) is the cost coefficient, and is positive constant.

We assume workers to be risk averse agents with constant absolute risk aversion (CARA) coefficient \( \gamma_w \). Workers make their portfolio choice decision and choose their effort level so as to maximize their expected end-of-period welfare,

\[ \max_{A_1,A_2,l} E[U_W] = E[W_W] - \frac{\gamma_w}{2} Var(W_W) - q C_W(l). \]

The welfare expression has two disutility parts: the disutility for bearing risks and the disutility for putting the effort \( l \). Solving this optimization yields the following portfolio allocation strategies.

**Proposition 3.1.** Workers’ optimal allocation in company stock is

\[ A_1 = \frac{(\mu_1 - r_f) - \frac{\rho_{12}^2}{\sigma_2^2}(\mu_2 - r_f)}{\gamma_w(1 - \rho^2)\sigma_1^2 - \frac{q}{k}}, \]

their holdings amount in stock 2 is

\[ A_2 = \frac{(\mu_2 - r_f)\left(1 - \frac{q}{k\gamma_w \sigma_1^2}\right) - \frac{\rho_{12}^2}{\sigma_1^2}(\mu_1 - r_f)}{\gamma_w(1 - \rho^2)\sigma_2^2 - \frac{\sigma_2^2}{k\sigma_1^2}}, \]
And their effort level is:

\[ l = \frac{A_1}{k}. \] (8)

From these equations, if the firm’s risk, \( \sigma_1 \), is lower or if \( \mu_1 - r_f \) is higher, the investment in company stock (stock 1) will be higher. This is intuitive, since from the mean-variance portfolio framework, the investment in risky stock increases with the stock Sharpe ratio. But the expressions of \( A_i \) we derived are slightly different from the mean-variance ones. \( A_1 \) is decreasing with the cost coefficient \( k \). Indeed, if the production of the effort \( l \) is costly, workers less able to influence the firm’s value, decrease their investment in company stock. \( A_1 \) is increasing with \( q \). Less is the probability of free-riding among workers, \( 1 - q \), and more their investment in company stock will be. Less they free-ride, more they can take advantage of the stock value increase. The effort level \( l \) is an increasing function of workers’ holdings of company stock. The more stakes they have in the company, more involved they are.

In figure 1, workers excess investment in company stock increases with \( q \) and decreases with \( k \). Their effort level decreases with \( k \).

**Figure 1:** Workers' excess investment in company stock and effort level.

These graphs show workers’ excess investment in company stock, and effort level as function of \( q \) and \( k \). LHS: workers’ excess holdings of company stock. RHS: Workers’ effort level. The baseline parameter values are \( \mu_1 - r_f = 0.07, \mu_2 - r_f = 0.07, \sigma_1 = 0.40, \sigma_2 = 0.25, \rho = 0, \gamma_w = 4. \)
3.2 Comparison with the Standard Incentive Model

The aim of this section is to establish the comparative static between workers' investment in company stock obtained in proposition (3.1) and previous incentive works result. We consider a standard incentive model similar to Holmstrom (1979, 1982) and Holmstrom & Milgrom (1987). We assume that, the firm at the beginning of the period decides how much stock to grant to workers. The firm then decides the compensation package share/wage at the beginning of the period. It allocates $A_1^o$ amount of company stock to workers and pay them $w^o$ as fixed salary. Given $A_1^o$ and $w^o$, workers choose their percentage holdings of stock 2, $A_2^o$, and their effort level $l^o$. The firm then solve a single maximization problem with constraints:

$$\max_{A_1, w} (1 - A_1)(1 + \mu_1 + q l) - w, \quad (9)$$

under workers' participation constraint

$$E[U_W(A_1, w)] \geq U_W \quad \text{(Workers PC)},$$

and incentive constraint

$$\{A_2, l\} = \arg\max E[U_W(A_2, l)] \quad \text{(Workers IC)}.$$

Solving this optimization, we get the following proposition.

**Proposition 3.2.** Using the Standard Incentive Model, workers’ holdings of company stock is

$$A_1^o = \frac{q}{k} - \rho \frac{\alpha_2}{\sigma_2}(\mu_2 - r_f) \frac{\gamma_w(1 - \rho^2)\sigma_1^2 + \frac{q}{k}}{\gamma_w(1 - \rho^2)\sigma_1^2 + \frac{q}{k}}, \quad (10)$$

their investment in stock 2 is

$$A_2^o = \frac{(\mu_2 - r_f)\left(1 + \frac{q}{k\gamma_w\sigma_1^2}\right) - \rho \frac{\alpha_2}{\sigma_2}\frac{q}{k}}{\gamma_w(1 - \rho^2)\sigma_2^2 + \frac{q}{k}\sigma_1^2}. \quad (11)$$

---

4 Holmstrom & Milgrom (1987) show that if the manager can influence the firm’s return and if the returns are normally distributed, then the dynamic problem can be reduced to a single period model with a linear compensation scheme. The same framework has been recently used by Jin (2002) and Garvey & Milbourn (2003) to study CEOs compensations.
And their effort level is

\[ l^o = \frac{A_1^o}{k}. \]  

(12)

Their fixed salary is

\[ w^o = \bar{U}\bar{W} - A_1^o(1 + \mu_1 + ql^0) - A_2^o(\mu_2 - r_f) + \frac{\gamma_w}{2} \text{Var}(W^o) + qk\frac{(l^o)^2}{2}. \]  

(13)

The number of shares granted to workers by the firm is increasing with \( q \) and decreasing with \( k \). When more workers free ride (\( q \) low), workers effort is not enough to compensate shareholders for their share dilution in case of new shares issue. When workers effort coefficient is high, the production of an additional effort is too costly; then the number of shares granted to them is lower, because the marginal gains for shareholders aren’t enough to compensate them from giving away part of their shares.

The firm grants \( A_1^o \) portion of company stock to workers. The risk premium of the company stock doesn’t appear on the expression of \( A_1^o \), only the stock volatility is relevant here. Workers will be granted company stock only if they can put an effort to increase the stock value.

**Corollary 3.3.** The difference between workers own picked percentage of company shares, \( A_1 \) [eq. (6)], and the proportion of shares granted to them by the firm, \( A_1^o \) [eq. (10)], is

\[ A_1 - A_1^o = \left[ \frac{\mu_1 - r_f - \rho \sigma_1 \sigma_2 (\mu_2 - r_f)}{\gamma_w (1 - \rho^2) \sigma_1^2 - \frac{q}{k}} + \frac{\mu_1 - r_f - \rho \sigma_1 \sigma_2 (\mu_2 - r_f)}{\gamma_w (1 - \rho^2) \sigma_1^2 + \frac{q}{k}} \right]. \]  

(14)

Assuming the expression of \( A_1^o \) in equation (10) is positive, i.e., \( \frac{q}{k} \geq \rho \sigma_1 \sigma_2 (\mu_2 - r_f) \), and the denominator in equation (6) is positive, \( \gamma_w (1 - \rho^2) \sigma_1^2 \geq \frac{q}{k} \), then workers own picked percentage of company shares, \( A_1 \), will always be higher than the percentage shares granted to them by the firm, \( A_1^o \), if and only if \( \mu_1 - r_f \geq \frac{q}{k} \).

In figure 2, workers’ shares holdings obtained with our model is always above the optimal mean-variance allocation. When \( k = 5 \), the standard incentive model shares allocation is higher than our model generated shares holdings for \( q > 0.5 \). When \( k = 20 \), the standard incentive model fails to generate shares holdings above the mean-variance allocation, while our model does.
These graphs show workers’ excess holdings of company shares as function of $q$ for $k = 5$ (LHS), and $k = 20$ (RHS). Dashed line: Standard Incentive Model, Solid line: our Model. The baseline parameter values are $\mu_1 - r_f = 0.07$, $\mu_2 - r_f = 0.07$, $\sigma_1 = 0.40$, $\sigma_2 = 0.25$, $\rho = 0$, $\gamma_w = 4$.

4 Manager’s type uncertainty

In the previous section, we have assumed $\mu_1$ and $\sigma_1$ exogenously given. We now consider the case where the firm’s manager can choose the level of $\mu_1$ and influence the value of $\sigma_1$. We assume three classes of agents interacting at the firm level: workers, the manager and outside shareholders. To deal with that, we introduce the manager’s type uncertainty in the following time line. Shareholders hire a manager and set her compensation package $(b, s)$, where $b$ is the pay-for-performance sensitivity (PPS) and $s$ the fixed salary. Shareholders expect the manager to put an effort $\mu_1$ which will increase the firm’s expected return. Once the manager is hired, the firm hires workers in a competitive labor market. From workers problem in the previous section, the manager’s effort $\mu_1$, influences workers investment, and de facto their effort level.

4.1 Manager’s compensation and effort level

Similar to Holmstrom & Milgrom (1987), we use a linear compensation scheme. The manager’s end-of-period wealth is

$$W_M = b(1 + \tilde{r}_1) + s,$$  \hspace{1cm} (15)
where \( b \) is the pay-for-performance sensitivity (PPS) and \( s \) is the fixed salary. Since shareholders can’t observe the manager’s effort (Holmstrom (1979)), they determine the manager’s compensation based on their estimation of \( \mu_1 \) and \( l \). From equations (6) and (8), workers’ effort \( l \) is a function of the manager’s choice of \( \mu_1 \). Manager bears a disutility for putting the effort level \( \mu_1 \), represented by the cost function \( C_M(\mu_1) \). The cost function \( C_M(\mu_1) \) is an increasing and convex function of \( \mu_1 \): \( C'_M(\mu_1) \geq 0 \) and \( C''_M(\mu_1) > 0, \forall \mu_1 \geq 0 \). We assume the manager to be risk averse with constant absolute risk aversion coefficient \( \gamma_m \). And hence, the manager’s total welfare is given by:

\[
E[U_M] = E[W_M] - \frac{\gamma_m}{2} Var(W_M) - C_M(\mu_1).
\] (16)

At the beginning of the period, shareholders set the manager’s compensation package by maximizing their expected end-of-period wealth with respect to the PPS, \( b \), and the fixed salary, \( s \), under manager’s participation constraint or reservation utility and incentive constraint, and workers incentive constraint. Outside shareholders who hold well-diversified portfolios are assumed to be risk-neutral. Shareholders maximization program is then formulated as follows:

\[
\max_{b,s} E[(1 - b)(1 + \tilde{r}_1) - s],
\] (17)

under manager’s participation constraint

\[
E[U_M] \geq \overline{U}_M,
\] (18)

and incentive constraint

\[
\mu_1 = \arg \max E[U_M(\mu_1)],
\] (19)

and workers’ incentive constraint

\[
l(\mu_1) = \arg \max E[U_W(l)].
\] (20)

Solving this optimization problem, we get the following proposition.

**Proposition 4.1.** The manager’s PPS, \( b \), is

\[
b = \frac{1}{1 + \gamma_m \sigma_1^2 C'_M(\mu_1) \left[ 1 - \frac{q}{k \gamma_m (1 - \rho^2) \sigma_1^2} \right]^2}.
\] (21)
her effort, \( \mu_1 \), is
\[
\mu_1 = C_M^{\prime\prime(-1)} \left( \frac{b}{1 - \frac{q}{k\gamma(1-\rho^2)\sigma_1^2}} \right),
\] (22)

and her fixed salary, \( s \), is
\[
s = U_M - b(1 + \mu_1 + ql(\mu_1)) + \frac{\gamma_m}{2} q^2 b^2 \sigma_1^2 + C_M(\mu_1).
\] (23)

The PPS, \( b \), is decreasing with the firm risk level, \( \sigma_1 \), the manager’s risk aversion, \( \gamma_m \), and the 2nd derivative of the cost function, \( C_M'' \). It increases with \( q \), and decreases with workers cost coefficient, \( k \). Manager’s effort \( \mu_1 \) increases with the PPS \( b \).

We now turn to the manager’s type uncertainty. When hiring the manager at time 0, the market nor the manager know the manager’s true type. The manager is assumed to have no private information at the time of contracting. The market participants only have the distribution of the manager’s type. Based on that information, shareholders draw randomly a manager and offer her an non-renegotiable binding contract. Once hired, the manager hires workers, and workers and manager observe the manager’s true type. Knowing her own type, manager chooses her effort level, and workers decide their holdings of company shares and effort level to put in.

From manager’s welfare function (equation (16)), we have two disutility functions, the disutility with respect to the firm’s risk, and the disutility with respect to the effort cost. We consider separately two types of uncertainties; first, with respect to the manager’s effort cost; and second, with respect to the risk technology picked by the manager. In the first case, the two types are the low cost and high cost efforts managers. In the second case, one manager when hired chooses a low risk technology for the firm, inducing then the firm risk level to be lower, while the other manager chooses a high risk technology, and then the firm risk is higher under that manager.

4.2 Effort cost uncertainty

There are two types of managers. One with a lower cost of effort and the other one has a higher cost of effort. Manager’s type will be indexed by H and L, with H representing the one with the lower cost of effort (high effort) and L the manager with the higher cost.
of effort (low effort). Let’s define by $C_{M,I}$ the cost function of manager $I$, $I \in \{H, L\}$. By assumption,

$$0 \leq C_{M,H}(\mu_1) < C_{M,L}(\mu_1), \quad \forall \mu_1 > 0,$$

and $C'_{M,I}(\mu_1) \geq 0$, $C''_{M,I}(\mu_1) > 0$, $\forall \mu_1 \geq 0$. The probability of drawing a type $H$ manager is $p$, and the probability of drawing a type $L$ manager is $1 - p$. We assume the managers’ reservation utility, $\overline{U}_M$, to be the same across types.

Shareholders then maximize their expected terminal wealth (equation (17)), under the ‘average’ manager’s participation constraint and incentive constraint (equations (18) and (19)), and workers incentive constraint (equation (20)), with $C_M(\mu_1) = pC_{M,H}(\mu_1) + (1-p)C_{M,L}(\mu_1)$. Shareholders offer to the selected manager, the PPS

$$b = \frac{1}{1 + \gamma_m \sigma_1^2 \left[pC''_{M,H}(\mu_1) + (1-p)C''_{M,L}(\mu_1)\right]} \left(1 - \frac{q}{k \gamma_w (1-\rho^2) \sigma_1^2}\right)^2,$$

and fixed salary

$$s = \overline{U}_M - b(1 + \mu_1 + q l(\mu_1)) + \frac{\gamma_m}{2} b^2 \sigma_1^2 + \left[pC_{M,H}(\mu_1) + (1-p)C_{M,L}(\mu_1)\right],$$

where

$$\mu_1 = \left[pC'_{M,H} + (1-p)C'_{M,L}\right]^{-1} \left(\frac{b}{1 - \frac{q}{k \gamma_w (1-\rho^2) \sigma_1^2}}\right).$$

Figure 3 plots the manager’s PPS and fixed salary as function of $p$ and $q$. $p$ is the portion of low effort cost managers. The PPS increases with $p$ and $q$, and symmetrically the fixed salary decreases as the same time. As the proportion of type $H$ managers increases in the pool, the number of shares granted to manager increases to obtain a high effort level. Manager’s PPS is also increasing with $q$.

Given $b$ and $s$, assuming manager $I$ is picked, she chooses the effort, $\mu_{1,I}$, which maximizes her welfare

$$\mu_{1,I} = C_{M,I}^{(-1)} \left(\frac{b}{1 - \frac{q}{k \gamma_w (1-\rho^2) \sigma_1^2}}\right), \quad I \in \{H, L\}.$$

Workers’ optimal allocation in company stock is

$$A_1(\mu_{1,I}) = \frac{(\mu_{1,I} - r_f) - \rho \frac{\sigma_1}{\sigma_2} (\mu_2 - r_f)}{\gamma w (1-\rho^2) \sigma_1^2 - \frac{q}{k}},$$

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These graphs show the manager’s PPS and fixed salary as function of $q$ and $p$. LHS: Manager’s PPS. RHS: Manager’s fixed salary. The baseline parameter values are $r_f = 0$, $\mu_2 - r_f = 0.07$, $\sigma_1 = 0.40$, $\sigma_2 = 0.25$, $\rho = 0$, $k = 5$, $\gamma_m = 4$, $\gamma_w = 4$, $\overline{U}_M = 1$. We use a quadratic cost function for the manager: $C_{M,1}(\mu_1) = h_1 \frac{\mu_1^2}{2}$, with $h_H = 10$ and $h_L = 30$.

and their effort level is:

$$l(\mu_{1,1}) = \frac{A_1(\mu_{1,1})}{k}.$$

The type affects $A_1$ through manager’s effort level $\mu_{1,1}$. Figure 4 plots the company shares holdings by workers for the two types of managers as function of $p$ and $q$. Workers allocate more of their wealth to company stock when the portion of low effort cost managers is high. Also the holding of company shares with the type H manager is bigger than that with the type L manager.

Workers holdings of company stock increases with the manager’s effort, which increases workers’ effort as well. The effort level $l$ is an increasing function of workers’ holdings of company stock. The more stakes they have in the company, more involved they are. Manager affects the productivity of the entire workforce by putting an effort $\mu_1$.

### 4.3 Risk technology uncertainty

We assume here that there are two types of managers indexed by 0 and P. Manager 0 picks a low risk technology for the firm, and then the firm’s risk under this manager is
These graphs show workers’ holdings of company stock as function of $q$ and $p$. LHS: workers’ holdings of company stock with type L manager. RHS: workers’ holdings of company stock with type H manager. The baseline parameter values are $r_f = 0$, $\mu_2 - r_f = 0.07$, $\sigma_1 = 0.40$, $\sigma_2 = 0.25$, $\rho = 0$, $k = 5$, $\gamma_m = 4$, $\gamma_w = 4$, $\bar{U}_M = 1$. We use a quadratic cost function for the manager: $C_{M,1}(\mu_1) = h_1 \mu_1^2$, with $h_H = 10$ and $h_L = 30$.

$\sigma_{1,0}$. Manager P picks a high risk technology for the firm, and then the firm’s risk under her control is $\sigma_{1,P}$. Hence by assumption

$$0 < \sigma_{1,0} < \sigma_{1,P}. \quad (25)$$

We assume the correlation $\rho$ between the two stocks’ returns to be invariant. The probability of drawing a type O manager is $p$, and the probability of drawing a type P manager is $1 - p$. We assume the reservation utility, $\bar{U}_M$, and cost function, $C_M(\cdot)$, to be the same across types.

As previously, shareholders maximize their terminal wealth (equation (17)), under the manager’s participation constraint and incentive constraint (equations (18) and (19)), and workers incentive constraint (equation (20)), with $\sigma_1^2 = \rho \sigma_{1,0}^2 + (1 - p) \sigma_{1,P}^2$. Shareholders offer to the selected manager, the PPS

$$b = \frac{1}{1 + \gamma_m \sigma_1^2 C_M''(\mu_1) \left[1 - \frac{q}{k\gamma_w(1-\rho^2)\sigma_1^2}\right]}$$

and fixed salary

$$s = \bar{U}_M - b(1 + \mu_1 + q l(\mu_1)) + \frac{\gamma_m b^2 \sigma_1^2}{2} + C_M(\mu_1),$$

16
where

$$\mu_1 = C_M^{(1)} \left( \frac{b}{1 - \frac{q}{k\gamma w(1-\rho^2)\sigma_1^2}} \right).$$

Figure 5 plots the manager’s PPS and fixed salary as function of $p$ and $q$. The PPS increases with the portion of type 0 managers, and the fixed salary decreases with it.

Figure 5: Manager’s compensations.

These graphs show the manager’s PPS and fixed salary as function of $q$ and $p$. LHS: Manager’s PPS. RHS: Manager’s fixed salary. The baseline parameter values are $r_f = 0$, $\mu_2 - r_f = 0.07$, $\sigma_{1,0} = 0.35$, $\sigma_{1,p} = 0.45$, $\sigma_2 = 0.25$, $\rho = 0$, $k = 5$, $\gamma_m = 4$, $\gamma_w = 4$, $U_M = 1$. We use a quadratic cost function for the manager: $C_M(\mu_1) = h \frac{\mu_1^2}{2}$, with $h = 20$.

Given $b$ and $s$, assuming manager $J$ is picked, she chooses her effort, $\mu_{1,J}$, which maximizes her welfare

$$\mu_{1,J} = C_M^{(1)} \left( \frac{b}{1 - \frac{q}{k\gamma w(1-\rho^2)\sigma_1^2,J}} \right), \quad J \in \{O, P\}.$$ 

workers’ optimal allocation in company stock is

$$A_1(\sigma_{1,J}) = \frac{(\mu_{1,J} - r_f) - \rho \sigma_{1,J}^2(\mu_2 - r_f)}{\gamma_w(1-\rho^2)\sigma_1^2,J - \frac{q}{k}};$$

and their effort level is:

$$l(\sigma_{1,J}) = \frac{A_1(\sigma_{1,J})}{k}.$$

The manager’s type affects workers’ holdings of company stock, $A_1$, through two channels: direct impact of $\sigma_{1,J}$ and impact of $\mu_{1,J}$. Figure 6 plots workers’ holdings of company stock for the two types of managers as function of $p$ and $q$. Workers’ allocation
in company stock increases with \( p \) and \( q \). For a 10% risk difference between the two types chosen firm’s risk levels, workers’ allocation in company stock more than double with type 0 manager compared to type P manager.

**Figure 6:** Workers’ holdings of company stock.

These graphs show workers’ holdings of company stock as function of \( q \) and \( p \). LHS: workers’ holdings of company stock with type P manager. RHS: workers’ holdings of company stock with type 0 manager. The baseline parameter values are \( r_f = 0, \mu_2 - r_f = 0.07, \sigma_{1,0} = 0.35, \sigma_{1,p} = 0.45, \sigma_2 = 0.25, \rho = 0, k = 5, \gamma_m = 4, \gamma_w = 4, \overline{U}_M = 1 \). We use a quadratic cost function for the manager: \( C_M(\mu_1) = h \frac{\mu_1^2}{2} \), with \( h = 20 \).

## 5 Conclusion

We provide a new incentive-investment framework to explain workers investment in company stock. The new feature of our model is that, the agent (workers) chooses the number of shares they desire to hold in their DC pension plan. Using this framework, we are able to provide a rationale explanation for workers behavior in overloading in company stock. When workers have the flexibility in their productivity choice, they will hold more company stock, and as a consequence, they will put an additional effort to increase the stock value. When we compare the results of our framework to the standard incentive model results, we observe that our framework is more closed to what is actually observed in 401(k) allocation decision, and is consistent with ERISA of 1974 rules.

We introduce the manager’s type uncertainty to explain the cross sectional company stock holdings differences. Workers in firms with lower managerial effort cost will invest
more in company stock. Also when the manager is more likely to pick a lower risk technology, workers invest more in company stock. When the company risk level changes from 0.45 to 0.35, workers holding of company stock almost triple.
Appendix

Proof of proposition 3.1

Workers maximize their welfare function:

$$\max_{A_1, A_2, l} E[U_W] = E[W_W] - \frac{\gamma_w}{2} Var(W_W) - qC_W(l).$$

The FOCs of this optimization are

$$\frac{\partial E[U_W]}{\partial l} = A_1 q - qC'_W(l) = 0,$$

$$\frac{\partial E[U_W]}{\partial A_1} = (\mu_1 + ql - r_f) - \frac{\gamma_w}{2} (2A_1 \sigma_1^2 + 2A_2 \rho \sigma_1 \sigma_2) = 0,$$

$$\frac{\partial E[U_W]}{\partial A_2} = (\mu_2 - r_f) - \frac{\gamma_w}{2} (2A_2 \sigma_2^2 + 2A_1 \rho \sigma_1 \sigma_2) = 0.$$

Equation (26) implies $l = \frac{A_1}{k}$. Equation (28) implies $A_2 = \frac{(\mu_2 - r_f) - \rho \sigma_1 \sigma_2 \gamma_w A_1}{\gamma_w \sigma_2^2} - A_1 \rho \sigma_1 \sigma_2$, substituting $A_2$ into equation (27), we get

$$A_1 = \frac{(\mu_1 - r_f) - \rho \sigma_1 \sigma_2 (\mu_2 - r_f)}{\gamma_w (1 - \rho^2) \sigma_1^2 - \frac{q}{k}}.$$

And then

$$A_2 = \frac{(\mu_2 - r_f) \left(1 - \frac{q}{k \gamma_w \sigma_1^2}\right) - \rho \sigma_1 \sigma_2 (\mu_1 - r_f)}{\gamma_w (1 - \rho^2) \sigma_2^2 - \frac{\sigma_1^2}{k \sigma_1^2}}.$$

Proof of proposition 3.2

Shareholders maximization problem is

$$\max_{A_1, A_2, l} (1 - A_1)(1 + \mu_1 + ql) - w,$$

under

(Workers PC) $E[U_W(A_1, w)] \geq \overline{U}_W$, & (Workers IC) $\{A_2, l\} = \arg\max E[U_W(A_2, l)]$.

Workers wealth expression changes slightly to

$$W_W = A_1 (1 + \tilde{r}_1) + A_2 (\tilde{r}_2 - r_f) + w.$$

Workers IC: $\max_{A_2, l} E[U_W(A_2, l)] \implies l = \frac{A_1}{k}$, and $A_2 = \frac{\mu_2 - r_f}{\gamma_w \sigma_2^2} - A_1 \rho \frac{\sigma_1}{\sigma_2}$. (30)
Shareholders maximization problem reduces to
\[
\max_{A_1} \left[ (1 + \mu_1 + q \frac{A_1}{k}) + A_2(\mu_2 - r) - \frac{\gamma w}{2} \left( A_1^2 \sigma_1^2 + A_2^2 \sigma_2^2 + 2A_1A_2\rho\sigma_1\sigma_2 \right) - q \frac{A_1^2}{2k} \right], \tag{31}
\]
with \(l\) and \(A_2\) given by equation (30). The FOC of the maximization in equation (31) is
\[
\left( \frac{\partial}{\partial A_1} \right) : \quad q \left( \frac{k}{k} + \left( \mu_2 - r_f \right) \right) \frac{\partial A_2}{\partial A_1} - q \frac{A_1}{k} - \gamma_w \left( A_1 \sigma_1^2 + A_2 \sigma_2^2 \frac{\partial A_2}{\partial A_1} + \rho A_2 \sigma_1 \sigma_2 + A_1 \rho \sigma_1 \sigma_2 \frac{\partial A_2}{\partial A_1} \right) = 0. \tag{32}
\]
From equation (30), we have \(\frac{\partial A_2}{\partial A_1} = -\rho \frac{A_1}{\sigma_2}.\) Substituting this partial derivative into the FOC (equation (32)), gives
\[
A_1 = \frac{q}{k} - \rho \frac{A_1}{\sigma_2} \left( \mu_2 - r_f \right) \gamma_w \left( 1 - \rho^2 \sigma_1^2 + \frac{q}{k} \right). \tag{33}
\]
Substituting \(A_1\) in the expression of \(A_2\) given in equation (30), we get
\[
A_2 = \frac{(\mu_2 - r_f) \left( 1 + \frac{q}{k \gamma_w \sigma_1} \right) - \rho \frac{A_1}{\sigma_1} \frac{q}{k}}{\gamma_w (1 - \rho^2 \sigma_1^2 + \frac{\sigma_2^2}{k \sigma_1^2})}. \tag{34}
\]
The fixed salary \(w\) is such that the participation constraint binds, i.e., \(E[U_W] = \overline{U}_W\), and is
\[
w = \overline{U}_W - A_1(1 + \mu_1 + q \frac{A_1}{k}) - A_2(\mu_2 - r_f) + \frac{\gamma w}{2} \text{Var}(W) + q \frac{A_1^2}{2k}. \tag{35}
\]
**Proof of proposition 4.1**

Shareholders maximize their expected utility
\[
\max_{b,s} (1 - b)(1 + \mu_1 + ql(\mu_1)) - s,
\]
under
Manager PC & IC: \( E[U_M] \geq \overline{U}_M, \quad \mu_1 = \arg \max E[U_M(\mu_1)], \)
Workers IC: \( l(\mu_1) = \arg \max E[U_W(l)]. \)
\[
\text{Manager IC} \implies b(1 + ql'(\mu_1)) = C'_M(\mu_1). \tag{36}
\]
From workers problem (proposition (3.1)), $l'(\mu_1) = \frac{1}{k\gamma w(1-\rho^2)\sigma_1^2-q}$, substituting back into equation (33), implies:

$$
\mu_1 = C_M^{(-1)} \left( \frac{kb\gamma w(1-\rho^2)\sigma_1^2}{k\gamma w(1-\rho^2)\sigma_1^2-q} \right). 
$$

Shareholders problem reduces to

$$
\max_b (1 + \mu_1 +ql(\mu_1)) - \frac{\gamma_m}{2} b^2 \sigma_1^2 - C_M(\mu_1),
$$

with $e$ given by equation (34) and $l$ by equation (8).

The FOC of this maximization is

$$
(1 + ql'(\mu_1) - C'_M(\mu_1)) \frac{\partial \mu_1}{\partial b} - \gamma_m b \sigma_1^2 = 0.
$$

From equation (34), $\frac{\partial \mu_1}{\partial b} = \frac{1}{C_M(\mu_1)} \frac{k\gamma w(1-\rho^2)\sigma_1^2}{k\gamma w(1-\rho^2)\sigma_1^2-q}$, substituting back into equation (35), gives

$$
b = \frac{1}{1 + \gamma_m \sigma_1^2 C_M''(\mu_1) \left[ 1 - \frac{q}{k\gamma w(1-\rho^2)\sigma_1^2} \right]^2}.
$$

The fixed salary $s$ is such that the Manager’s PC binds, i.e.,

$$
b(1 + \mu_1 +ql(\mu_1)) + s - \frac{\gamma_m}{2} b^2 \sigma_1^2 - C_M(\mu_1) = \bar{U}_M.
$$

If the cost function of the manager, $C_M$, is quadratic, i.e., $C_M(\mu_1) = h \frac{\mu_1^2}{2}$, where $h$ is the manager’s cost coefficient, then

$$
b = \frac{1}{1 + h\gamma_m \sigma_1^2 \left[ 1 - \frac{q}{k\gamma w(1-\rho^2)\sigma_1^2} \right]^2}, \quad \text{and} \quad \mu_1 = \frac{b}{h \left[ 1 - \frac{q}{k\gamma w(1-\rho^2)\sigma_1^2} \right]}. 
$$
References


