The Pricing Kernel and Time-Series Characteristics of Asset Returns†

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First draft: January 2002
This version: January 2003

† I am indebted to Günter Franke for invaluable support. I am also grateful for helpful discussions with Axel Adam-Müller, Jan Beran, Ingolf Dittmann, Bertram Dürring, Jens Jackwerth, Jürgen Eichberger, Yuanhua Feng, Bernhard Peisl, Michael Schröder, Susanne Warning, seminar participants at Laval University, at the Centre for European Economic Research, Mannheim, at the Center of Finance and Econometrics, Konstanz and participants of the Annual Meeting of the GEABA in Berlin, and the Symposium on Finance, Banking and Insurance in Karlsruhe. All remaining errors are my sole responsibility. Financial support by the CoFE, Konstanz, the ZEW, Mannheim, and a grant by the Deutsche Bundesbank are gratefully acknowledged.

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Abstract

Starting from an information process governed by a geometric Brownian motion we derive the properties of asset returns when the elasticity of the pricing kernel is not constant. We show that in this case asset returns are predictable. It is also shown that asset prices are governed by a time-homogeneous stochastic differential equation only if the elasticity of the pricing kernel is constant. Moreover, we propose a general characterization of pricing kernels which generates analytical asset price processes. The numerical simulations show that declining elasticity of the pricing kernel may explain short-term momentum and long-term reversals in asset returns.

JEL classification: G12

Keywords: Pricing kernel; Diffusion processes; Stationarity; Predictability of asset returns; Autocorrelation
In spite of the vast literature on deviations from the random walk hypothesis, there is still controversy whether these deviations are attributable to inefficient markets.¹ Even those who favor the efficient market hypothesis do not agree whether these financial market phenomena point to a new model of investor behavior or whether modelers should remain within the traditional framework of rational expectations and von Neumann-Morgenstern utility functions. There is no doubt that the behavioral finance literature has contributed much to our understanding of asset prices. However, it is highly controversial to what extent new behavioral postulates should be adopted.²

Surprisingly, although many models have been proposed for the explanation of asset returns, still little is known on how return characteristics change when the elasticity of the pricing kernel is not constant. Since throughout the paper we assume monotonic elasticity of the pricing kernel, i.e. either monotonic increasing or declining elasticity, our approach is consistent with the traditional framework of rational expectations and von Neumann-Morgenstern utility functions.

On the basis of a parsimonious model for asset price processes this paper analyzes whether predictability can be explained by nonconstant elasticity of the pricing kernel. The results show that predictability may indeed be in-
duced by nonconstant elasticity of the pricing kernel. Since this paper shows what kind of return characteristics can be explained in the traditional framework, the paper provides a benchmark for the necessity of new behavioral postulates. Moreover, we propose a class of pricing kernels which yields analytical asset price processes. This class of pricing kernels is flexible enough to generate many different kinds of pricing kernels, including nonmonotonic ones as those found in recent empirical studies (see for example Jackwerth, 2000). Thus, we provide a powerful method to analyze the implications of different pricing kernels for asset returns. This method simplifies significantly the numerical simulation of asset returns since analytical functions of asset returns can be derived. We also run numerical simulations. They illustrate our theoretical results. Moreover, they show that under pricing kernels with declining elasticity short-term returns tend to be positively correlated but exhibit significant long-term reversals. This provides a new explanation for short-term momentum and long-term reversals which has been documented in several studies.

The economy considered in this paper is very simple. Following the approach of Franke, Stapleton and Subrahmanyam (1999) we start from an exogenously given information process which characterizes investors’ expecta-
tions about the asset price at some terminal date $T$. The information process is assumed to be governed by a geometric Brownian motion with constant instantaneous volatility and no drift. Hence, the information process is consistent with rational expectations. Furthermore, we assume that the pricing kernel is a deterministic function of the asset price which is consistent with a representative investor economy (e.g., Decamps and Lazrak, 2000). Hence, our model is consistent with the model of Black and Scholes (1973) except that we do not assume constant elasticity of the pricing kernel. The simplicity of the framework allows to show more clearly the effect of alternative assumptions on the pricing kernel.

Moreover, the proposed class of pricing kernels yields a new class of analytical asset price processes. These processes are derived from assumptions on the pricing kernel and thus do not lack an economic foundation. In addition, they can capture many stylized facts. Therefore, they may help to understand empirically observed asset return processes.

Our analysis is related to the literature on the viability of asset price processes in a representative investor economy. This paper is closely related to Stapleton and Subrahmanyam (1990) and Franke, Stapleton and Subrahmanyam (1999). In these papers a similar economy is considered.
Our analysis builds on these results and extends them. Our paper is also related to Johnson (2002) who proposes a rational explanation for momentum on the firm level. While we focus on the elasticity of the pricing kernel as an explanation for positive and negative serial correlation of asset returns, Johnson (2002) unveils a stochastic expected growth rate of dividends as an explanation for momentum.

The organization of the paper is as follows. Section I presents the model. In Section II the stochastic differential equation governing the equilibrium asset price process is analyzed for declining and increasing elasticity of the pricing kernel. Section III analyzes the implications for discretely sampled asset returns. In Section IV we present a new pricing kernel specification and we derive analytical asset price processes. We also provide some numerical simulations. Section V concludes.

I. The model

In this paper we consider a market with a given time horizon $T > 0$ and a one-dimensional standard Brownian motion $W$ on a given filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ where $(\mathcal{F}_t)_{t \in [0,T]}$ is the filtration generated by $W$ augmented by all the $\mathcal{F}$-null sets, with $\mathcal{F} = \mathcal{F}_T$. It is assumed that at least one risky asset is traded and hence the market is complete. Moreover, we
assume that the asset does not pay any dividends until terminal date $T$. The distribution of the terminal value of the asset is exogenously given. Following Franke, Stapleton an Subrahmanyam (1999) we define the distribution of the terminal value implicitly by postulating some information process. Such an information process can be interpreted as the representative investor’s conditional expectation about the terminal value of the stock. The information process $I$ is defined by

$$I_t = E(X_T | \mathcal{F}_t), \quad 0 \leq t \leq T,$$

where $X_T$ is the random payoff of the stock at the terminal date $T$. Moreover, to keep things simple and to focus on the pricing kernel, we assume that this information process $I$ is characterized by a geometric Brownian motion with constant instantaneous volatility $\sigma$ and no drift, i.e.

$$dI_t = \sigma I_t dW_t, \quad 0 \leq t \leq T,$$

$$I_0 > 0. \quad (1)$$

Note that with this assumption $I_T$ is lognormally distributed with

$$Var (\ln I_T | \mathcal{F}_t) = \sigma^2 (T - t), \quad 0 \leq t \leq T,$$

and

$$E (I_T | \mathcal{F}_t) = I_t, \quad 0 \leq t \leq T.$$
Thus, the information process is a martingale as required by rational expectations. Note also that at terminal date $T$ the expected value of the asset $I_T$ and the price of the asset $F_T$ are equal, i.e. $I_T = F_T = X_T$. To focus on risk preferences we always consider forward prices instead of spot prices.

After the description of the information structure in the economy we characterize the pricing kernel. It is well known that in an arbitrage-free market an equivalent martingale measure exists. Moreover, in complete markets the equivalent martingale measure $\tilde{P}$ is unique. The transformation from $P$ to $\tilde{P}$ is given by the pricing kernel $\Phi_{t,T} = \frac{\Phi_{0,T}}{\Phi_{0,t}}$ where $\Phi_{0,t} = E(\Phi_{0,T}|\mathcal{F}_t)$, $0 \leq t \leq T$. Thus, the forward price $F_t$ is given by

\[
F_t = E^\tilde{P}(F_T|\mathcal{F}_t) = E(F_T\Phi_{t,T}|\mathcal{F}_t)
\]

\[
= E(I_T\Phi_{t,T}|\mathcal{F}_t), \quad 0 \leq t \leq T.
\]

In general, the pricing kernel is characterized by the Girsanov-functional and thus it is not necessarily a deterministic function of $I_t$ or $F_t$. However, in a representative investor economy with a state-independent von Neumann-Morgenstern utility function over the terminal asset price $F_T$ the pricing kernel is characterized by a deterministic function of time $t$ and either $I_t$ or $F_t$. 

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$F_t$. This follows from the equilibrium condition

$$\Phi_{0,T} = a \frac{\partial}{\partial F_T} U (F_T),$$

(3)

with $a$ some positive scalar and $U$ the state-independent utility function of the representative investor. The pricing kernel considered in this article is path-independent and therefore consistent with a representative investor economy. Moreover, the elasticity of the pricing kernel

$$\eta_t \equiv \frac{\partial \Phi_t}{\partial F_T} \frac{F_T}{\Phi_t},$$

can then be interpreted as the relative risk aversion of a representative investor. Given the information process of equation (1) with $I_T = F_T$, the forward price $F_t$ can then be characterized by the following backward stochastic differential equation

$$dF_t = \left\{ \frac{\partial}{\partial t} v(t, I_t) + \frac{1}{2} \frac{\partial^2}{\partial I_t^2} v(t, I_t) (\sigma I_t)^2 \right\} dt + \frac{\partial}{\partial I_t} v(t, I_t) \sigma I_t dW_t,$$

$$0 \leq t \leq T,$$

$$v(T, I_T) = I_T,$$

(4)

with $v(t, I_t) = F_t$ and the instantaneous drift $\mu_t$ and the instantaneous volatility $\Sigma_t$ being deterministic functions of time $t$ and $F_t$. Note that we can derive a similar stochastic differential equation for any state variable $X_t$, given that $F_T$ is a deterministic function of $X_T$. 
II. Asset returns in continuous-time

In this section we analyze the properties of the stochastic differential equation (4) governing the forward price process. For an information process governed by equation (1) and a pricing kernel with constant elasticity it is known that the forward price is governed by a geometric Brownian motion. In this case the forward price is given by

\[ F_t^C = v^C(t, I_t) = A(t) I_t, \quad 0 \leq t \leq T, \quad (5) \]

where \( A(t) \) is a deterministic function of time \( t \). Thus, in this case expected returns \( \mathbb{E}_t (r_{t, \tau}) = \mathbb{E}_t (\ln F_\tau - \ln F_t), \quad 0 \leq t < \tau \leq T \) do not depend on the level of \( I_t \) nor \( F_t \). Moreover, the elasticity of the forward price with respect to \( I_t \) \( \eta^{F,I}_t \equiv \frac{\partial v(t, I_t)}{\partial I_t} \frac{I_t}{v(t, I_t)} \) is equal to 1 and the instantaneous volatility \( \Sigma_t \) of the forward price process \( F \) is equal to the instantaneous volatility \( \sigma \) of the information process \( I \).

However, the following Lemma states that the elasticity of the forward price with respect to \( I_t \) is higher [lower] than 1 for declining [increasing] elasticity of the pricing kernel. This result is closely related to Theorem 3 of Franke, Stapleton and Subrahmanyam (1999) who show that the ratio between the forward price under declining elasticity of the pricing kernel and the forward price under constant elasticity of the pricing kernel increases
with the level of the latter one.

**Lemma 1** Assume that the information process is governed by a geometric Brownian motion with constant instantaneous volatility and no drift. Then

\[ \eta^{F,I}_t \geq \leq \geq 1, \forall I_t \iff \frac{\partial \eta^{F,I}_t}{\partial F_T} < \geq \geq 0, \forall F_T. \]

**Proof** See Appendix A

\[ \eta^{F,I}_t \geq \leq 1 \] implies that a 1% change in \( I_t \) leads to a higher [less] than 1% change in \( F_t \). Hence, Lemma 1 establishes that the forward price overreacts [underreacts] compared to the case of constant elasticity of the pricing kernel if the elasticity of the pricing kernel is declining [increasing]. To get the intuition for the overreaction [underreaction] think about the elasticity of the pricing kernel in terms of relative risk aversion of the representative investor. A representative investor with decreasing [increasing] relative risk aversion requires a lower [higher] return for the same risk the wealthier he is. Compared to an investor with constant relative risk aversion, his required relative risk premium \( \left( \frac{I_t - F_t}{F_t} \right) \) decreases [increases] the wealthier he is. Hence, the price he is willing to pay for the asset increases more [less] with increasing expected terminal wealth. Thus, with nonconstant relative risk aversion a change in the expected terminal value \( I_t \) also induces a change of the required
risk premium. This change of the risk premium reinforces [diminishes] the purely 'information based' change of the asset price. Thus, since the required risk premium decreases [increases] with the level of $I_t$ for declining [increasing] elasticity of the pricing kernel, the forward price overreacts [underreacts].

The following proposition establishes that predictability of asset returns and nonconstant elasticity of the pricing kernel are closely related.

**Proposition 1** Assume that the information process is governed by a geometric Brownian motion with constant instantaneous volatility and no drift. Then

$$\frac{\partial \eta_{F,F}}{\partial F_T} < [=] > 0, \forall F_T \iff \frac{\partial}{\partial F_t} E (\ln F_T - \ln F_t | \mathcal{F}_t) < [=] > 0, \forall F_t,$$

and we have the following relation between the instantaneous volatility of the forward price $\Sigma_t$ and the instantaneous volatility of the information process $\sigma$

$$\frac{\partial \eta_{F,F}}{\partial F_T} < [=] > 0, \forall F_T \iff \Sigma_t > [=] < \sigma.$$

**Proof** $E (\ln F_T - \ln F_t | \mathcal{F}_t) = \ln I_t - \frac{1}{2} \sigma^2 (T - t) - \ln v(t, I_t)$ with $F_t = v(t, I_t)$. Since $\frac{\partial v(t, I_t)}{\partial I_t} > 0$ the first assertion of Proposition 1 follows from Lemma 1. That the instantaneous volatility $\Sigma_t$ of the forward price process $F$ is higher [lower] under declining [increasing] elasticity of the pricing kernel
follows from Lemma 1 and the definition of the instantaneous volatility of the forward price process

\[ \Sigma_t = \frac{\partial}{\partial I_t} v(t, I_t) I_t \sigma, \quad 0 \leq t \leq T. \] (6)

What do we learn from Proposition 1 and what is the economic mechanism which drives the results? First, expected returns depend negatively [positively] on the level of the forward price if the elasticity of the pricing kernel is declining [increasing]. Since a high respectively low forward price \( F_t \) implies that past returns have been relatively high respectively relatively low, Proposition 1 implies a certain mean reversion [mean aversion] in returns. This effect is also due to the changing risk premium. The higher \( I_t \) the lower [higher] will be the risk premium under declining [increasing] elasticity. Therefore the expected return decreases [increases] with the level of \( I_t \). This mean reversion [mean aversion] can also be related to the overreaction [underreaction] effect. Note first that the distribution of the terminal asset price is independent of the pricing kernel and equal to the distribution of \( I_T \). However, under declining [increasing] elasticity of the pricing kernel the forward price overreacts [underreacts]. Hence, since \( I_T = F_T \) this overreaction
[underreaction] has to be compensated and thus asset returns exhibit mean reversion [mean aversion]. Second, the higher [lower] instantaneous volatility is related to the overreaction [underreaction] effect. The instantaneous volatility of the forward price $\Sigma_t$ measures the instantaneous reaction of the forward price to innovations of the Brownian motion $W$. This Brownian motion drives both processes $I$ and $F$. Since $F$ overreacts [underreacts] relatively to $I$, $\Sigma_t$ must be higher [lower] than $\sigma$.

The overreaction effect also provides an intuition for the mispricing of the Black-Scholes model when the elasticity of the pricing kernel is not constant. This effect has been analyzed in detail by Franke, Stapleton and Subrahmanyan (1999). They show that options are more expensive with declining elasticity than with constant elasticity of the pricing kernel. In the light of our results this overpricing compared to Black-Scholes follows because of the higher instantaneous volatility of the forward price process. It is well known that the amount of money needed for a self-financing strategy which hedges the option (hedging costs) is increasing with the instantaneous volatility of the underlying. Thus, although the terminal value is distributed as in the constant elasticity case, under declining elasticity hedging costs are higher. Higher hedging costs, however, imply a higher price. Hence, option prices
exceed Black-Scholes prices when the pricing kernel has declining elasticity.

Let us now consider the characteristics of the instantaneous volatility in more detail. The following corollary provides a new explanation for the empirically well documented asymmetric volatility phenomenon.

**Corollary 1** Assume that the information process is governed by a geometric Brownian motion with constant instantaneous volatility and no drift. Then,

\[ \Sigma_t \xrightarrow{\mathcal{F}_t} \left[ \begin{array}{c} \sigma \\ \{ \right] \sigma \text{ for } F_t \rightarrow \infty \]

if the elasticity of the pricing kernel is declining [increasing].

**Proof** See Appendix B

Combining this result with Proposition 1 gives a clear characteristic of the instantaneous volatility in the case of declining [increasing] elasticity of the pricing kernel. Note, however, that the relationship established in Corollary 1 is not necessarily monotonic. Let us now discuss this relationship for the case of declining elasticity: Why is the instantaneous volatility \( \Sigma_{t^*}^{\nu^*} \) of the forward price process smaller when forward prices are high? This effect comes from the fact that we assume risk aversion over \( \mathbb{R}^+ \). Thus, for \( I_t \rightarrow \infty \) the representative investor is risk neutral and therefore also constant relative risk averse.
This explanation for asymmetric volatility, i.e. high volatility in bear markets and low volatility in bull markets differs from the two previous approaches, i.e. the leverage effect (see for example Black, 1976, and Christie, 1982) and the volatility feedback effect (see for example Campbell and Hentschel, 1992, and Lüders and Peisl, 2001). Hence, Corollary 1 presents a third mechanism which might contribute to the observed asymmetric volatility. Moreover, recent empirical results show that the negative correlation between volatility and returns is more pronounced for market returns than for individual stock returns. This, however, suggests that the leverage effect is less important than the preference based arguments, i.e. volatility feedback and declining elasticity.

III. Implications for asset returns in discrete-time

A. Time-homogeneity

The purpose of this section is to analyze the characteristics of the stochastic processes in terms of moments of discretely sampled asset returns as for example the autocorrelation and standard deviation of such returns. The following analysis, however, shows that usually asset returns are not stationary. Stationarity requires that the forward price process (the return process) is governed by a time-homogeneous stochastic differential equation, i.e. $\mu_t$
and $\Sigma_t$ of the stochastic differential equation (4) for the price $F_t$ may depend on $F_t$, but they may not depend on time $t$.\(^7\)

**Proposition 2** Assume that the information process is governed by a geometric Brownian motion with constant instantaneous volatility and no drift. Then the forward price process is governed by a time-homogeneous stochastic differential equation if and only if the elasticity of the pricing kernel is constant.

**Proof** First note that because \( \frac{\partial v(T,I_T)}{\partial I_T} \frac{f_T}{v(T,I_T)} = 1 \) and $\Sigma_t = \frac{\partial v(t,I_t)}{\partial I_t} \frac{f_t}{v(t,I_t)} \sigma$, \( 0 \leq t \leq T \) we know that the instantaneous volatility $\Sigma_t$ is constant in $T$ with $\Sigma_T = \sigma$. Moreover, the instantaneous volatility $\Sigma_t$ can be characterized by a function $\tilde{\Sigma}(t,F_t) = \Sigma_t$. Hence, we have $\tilde{\Sigma}(T,F_T) = \text{constant}$ for all $F_T$. If the elasticity of the pricing kernel is declining [increasing] then

$$\tilde{\Sigma}(t,F_t) > [<] \sigma \quad 0 \leq t < T,$$

for all finite values of $F_t$. Since $\tilde{\Sigma}(T,F_T) = \sigma$, the instantaneous volatility of the forward price depends on time $t$. It follows from Proposition 1 that $\tilde{\Sigma}(t,F_t) = \sigma$ if and only if the elasticity of the pricing kernel is constant. \(\blacksquare\)
Proposition 2 states that given our information process only constant elasticity of the pricing kernel yields a time-homogeneous stochastic process for the forward price. Hence, except for constant elasticity of the pricing kernel the transition density will depend on time $t$. This, however, implies nonstationarity of asset returns. Thus, the assumption of a lognormal distribution of the terminal value is not compatible with time independent coefficients and nonconstant elasticity of the pricing kernel. The intuition behind this result is as follows. With, for example, declining elasticity of the pricing kernel the asset price instantaneously overreacts. This overreaction is then compensated by the mean reversion. However, both effects depend on the distance to the terminal date $T$ since at the terminal date the forward price is equal to the lognormally distributed $I_T$. The important point of Proposition 2 is that most estimation techniques rely on the assumption of time-homogeneity. Hence, empirical studies might suffer from the nonstationarity of asset returns. However, it might be questioned whether this nonstationarity is important in reality. Two points might weaken the time-dependence of the transition density. First, in contrast to the asset considered here with just one dividend payment at terminal date $T$, assets usually pay dividends regularly so that the time to maturity effect becomes less severe. Moreover,
the terminal date $T$ is not known in reality. A random terminal date $T$ would also lead to a less pronounced time to maturity effect. Hence, while Proposition 2 implies strong implications for empirical studies these effects might be partly driven by the simplicity of our model. However, researchers should be aware of the fact, that at least weak time dependence could be present in data of asset returns.

**B. Time series properties of asset returns**

We will now deduce some properties of the return process over finite time intervals. It is shown in Proposition 1 that for declining [increasing] elasticity of the pricing kernel the forward price is characterized by a function $v$ satisfying

$$\frac{\partial}{\partial \ln I_t} \ln v^{DE}(t, I_t) > 1, \quad 0 \leq t < T,$$

$$\frac{\partial}{\partial \ln I_t} \ln v^{IE}(t, I_t) < 1, \quad 0 \leq t < T.$$  

It follows immediately that the variance of $\ln v^{DE}(t, I_t)$ is higher and the variance of $\ln v^{IE}(t, I_t)$ is smaller than the variance of $\ln I_t$ for $0 \leq t < T$. For constant elasticity of the pricing kernel both variances are equal since in this case the elasticity of the forward price with respect to $I_t$ equals 1. Note also that this is true for conditional as well as unconditional variances.
More relevant are the properties of the returns over finite time intervals $r_{t,\tau} = \ln F_\tau - \ln F_t$. The following proposition shows that for the case of declining elasticity of the pricing kernel, the conditional variance of returns over finite periods, $\text{Var} (\ln F_\tau - \ln F_t | \mathcal{F}_t)$, and the unconditional variance of returns over finite periods, $\text{Var} (\ln F_\tau - \ln F_t)$, are higher than under constant elasticity of the pricing kernel. It is also shown that the conditional variance of returns is lower if the elasticity of the pricing kernel is increasing. Moreover, it should be noted that the results are not sensitive to whether we consider $\text{Var} (\ln F_\tau - \ln F_t)$ or $\text{Var} (\ln F_\tau - \ln F_t | \mathcal{F}_{t-\theta})$ with $\theta > 0$. Important is only whether $\ln F_t$ is measurable with respect to the filtration on which the variance is conditioned. This means, it is only important whether $\ln F_t$ is known.

**Proposition 3** Suppose that the information process is governed by a geometric Brownian motion with constant instantaneous volatility and no drift. Then

$$\text{Var} (\ln F_\tau - \ln F_t | \mathcal{F}_{t-\theta}) > \text{Var} (\ln I_\tau - \ln I_t | \mathcal{F}_{t-\theta}), \quad 0 \leq t < \tau < T,$$

with $t \geq \theta \geq 0$ if the elasticity of the pricing kernel is declining and

$$\text{Var} (\ln F_\tau - \ln F_t | \mathcal{F}_t) < \text{Var} (\ln I_\tau - \ln I_t | \mathcal{F}_t), \quad 0 \leq t < \tau < T,$$

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if the elasticity is increasing.

**Proof** See Appendix C

In addition, note that the conditional and unconditional variance of returns over finite periods is equal to the corresponding variance of the information process if the elasticity is constant. However, while the conditional variance $\text{Var}(\ln F_\tau - \ln F_t | \mathcal{F}_t)$ is lower for increasing elasticity of the pricing kernel, this is not necessarily true for $\text{Var}(\ln F_\tau - \ln F_t | \mathcal{F}_{t-\theta})$ with $\theta > 0$.

To see this, consider

$$\text{Var}(\ln F_\tau - \ln F_t | \mathcal{F}_{t-\theta}) = \text{Var}(E(\ln F_\tau | \mathcal{F}_t) - \ln F_t | \mathcal{F}_{t-\theta})$$

$$+ E(\text{Var}(\ln F_\tau | \mathcal{F}_t) | \mathcal{F}_{t-\theta})$$

with $E(\text{Var}(\ln F_\tau | \mathcal{F}_t) | \mathcal{F}_{t-\theta})$ being lower under increasing elasticity than under constant elasticity of the pricing kernel. However, while in the case of constant elasticity of the pricing kernel $\text{Var}(E(\ln F_\tau | \mathcal{F}_t) - \ln F_t | \mathcal{F}_{t-\theta}) = 0$, this is positive for nonconstant elasticity. Hence, in contrast to the case of declining elasticity the effect on the unconditional variance is ambiguous under increasing elasticity, since the first term on the right hand side is higher than under constant elasticity but the second term is lower.

The intuition for the higher variance of returns when the pricing ker-
nel has declining elasticity is the same as for the instantaneous volatility. The change in the risk premium increases the reaction to a change in expectations compared to the case of constant elasticity of the pricing kernel. This leads to a higher variance of returns. Moreover, Proposition 3 is closely related to Proposition 2 since it illustrates the nonstationarity of asset returns. For declining elasticity of the pricing kernel the conditional variance of returns over any subperiod \([t, t + \theta]\) with \(0 \leq t < t + \theta < T\), 
\[
\text{Var} \left( \ln F_{t+\theta} - \ln F_t \mid \mathcal{F}_t \right),
\]
is higher than the corresponding variance of the information process \(\text{Var} \left( \ln I_{t+\theta} - \ln I_t \mid \mathcal{F}_t \right)\) except for the returns over the period \([T - \theta, T]\). The conditional variance of terminal returns \(r_{t,T}\) is always equal to the corresponding variance of the information process, i.e.
\[
\text{Var} \left( \ln F_T - \ln F_t \mid \mathcal{F}_t \right) = \text{Var} \left( \ln I_T - \ln I_t \mid \mathcal{F}_t \right), \quad 0 \leq t \leq T. \tag{9}
\]
In contrast, the corresponding conditional variance of the information process is equal for both periods, i.e.
\[
\text{Var} \left( \ln I_{t+\theta} - \ln I_t \mid \mathcal{F}_t \right) = \sigma^2 \theta, \quad 0 \leq t \leq t + \theta \leq T, \tag{10}
\]
which demonstrates that the transition density of asset returns depends on time \(t\). Proposition 3 is also in line with Franke, Stapleton and Subrahmanyan (1999) who have shown that the variance of the forward price is
higher under the declining elasticity pricing kernel.

The previous section showed the fact that returns are predictable under a pricing kernel with nonconstant elasticity. The following proposition shows that the terminal return $r_{\tau,T}$ is conditionally negatively [positively] correlated with the preceding return $r_{t,\tau}$ with $0 \leq t < \tau < T$ if the elasticity of the pricing kernel is declining [increasing].

**Proposition 4** Suppose that the information process is governed by a geometric Brownian motion with constant instantaneous volatility and no drift. Then final period returns ($r_{\tau,T} = \ln F_T - \ln F_\tau$ with $0 < \tau < T$) are conditionally negatively [positively] correlated with preceding returns ($r_{t,\tau} = \ln F_\tau - \ln F_t$ with $0 \leq t < \tau < T$), i.e. $\text{Corr} (r_{\tau,T}, r_{t,\tau} \mid \mathcal{F}_t) < [>] 0$, if the elasticity of the pricing kernel is declining [increasing].

**Proof** Since $\text{sgn Cov} (r_{\tau,T}, r_{t,\tau} \mid \mathcal{F}_t) = \text{sgn Cov} \left( \frac{E(F_T \mid \mathcal{F}_\tau)}{F_\tau}, F_\tau \mid \mathcal{F}_t \right)$ we have to consider

$$\frac{\partial}{\partial F_t} \left( \frac{I(t, F_t)}{F_t} \right)$$

where $I(t, F_t) = E(F_T \mid \mathcal{F}_\tau)$ is the inverse function of $F(t, I_t)$. Note that $F(t, I_t)$ is monotone in $I_t$. Hence, we have negative [positive] conditional
autocorrelation for

\[
\frac{\partial}{\partial F_t} I(t, F_t) < [>] \frac{I(t, F_t)}{F_t}
\]

which is equivalent to

\[
\frac{\partial}{\partial I_t} F(t, I_t) > [>] \frac{F_t}{T(t, F_t)}.
\]

It follows then from Lemma 1 that returns are conditionally negatively [positively] autocorrelated for declining [increasing] elasticity of the pricing kernel. ■

Proposition 4 shows that in contrast to the case of constant elasticity of the pricing kernel, asset returns are negatively [positively] autocorrelated if the elasticity of the pricing kernel is declining [increasing]. Since Proposition 4 and Proposition 1 are more or less two views on the same mechanism, the economic intuition for the serial correlation parallels the explanation of Proposition 1. For declining elasticity, we have seen that asset prices overreact compared to the information process. Hence when, for example, expectations rise the asset price rises more than the change in expectations. Since \( F_T = I_T \), this relatively high first period return has to be compensated by a relatively low return in the following period. In the case of increasing elasticity of the pricing kernel, asset prices underreact and since \( F_T = I_T \) they have to catch up in the second period, which yields positive autocorrelation.
Finally, let us consider the variance ratio

\[
\nu_r = \left( \frac{\tau - t}{T - t} \right) \frac{\text{Var} (\ln F_T - \ln F_t | \mathcal{F}_t)}{\text{Var} (\ln F_\tau - \ln F_t | \mathcal{F}_t)}, \quad 0 < t < \tau < T,
\]

which is widely used in empirical finance to detect serial correlation. Since

\[
\text{Var} (\ln F_T | \mathcal{F}_t) = \text{Var} (\ln I_T | \mathcal{F}_t)
\]

and

\[
\text{Var} (\ln F_\tau - \ln F_t | \mathcal{F}_t) > [\text{Var} (\ln I_\tau - \ln I_t | \mathcal{F}_t)]
\]

for declining [increasing] elasticity of the pricing kernel it is easily seen that the variance ratio is smaller [higher] than 1 for declining [increasing] elasticity of the pricing kernel which is consistent with our earlier finding of mean reversion and negative autocorrelation [mean aversion and positive autocorrelation].

To get a better understanding of the effect of nonconstant elasticity of the pricing kernel on asset returns let us now consider a class of pricing kernels which generates analytical asset prices.

**IV. Analytical asset price processes**

**A. A feasible characterization of asset prices**

To get analytical solutions of the forward price we propose a weighted
sum of power functions as characterization of the pricing kernel, i.e.

\[ \Phi_{t,T}^{\text{general}} = \frac{\sum_{i=1}^{N} \alpha_i I_T^{\delta_i}}{E\left( \sum_{i=1}^{N} \alpha_i I_T^{\delta_i} \mid \mathcal{F}_t \right)}, \quad 0 \leq t \leq T, \quad (12) \]

with \( \alpha_i, \delta_i \in \mathbb{R} \). This specification is rather general so that many different characteristics of the pricing kernel can be matched. For example also the classical pricing kernels derived from a representative investor with exponential utility and from a representative investor with power utility are included by equation (12), i.e.

**exponential utility:**

\[ \Phi_{t,T}^{\text{exponential}} = \frac{\sum_{i=0}^{\infty} \frac{1}{i!} (-\alpha I_T)^i}{E\left( \sum_{i=0}^{\infty} \frac{1}{i!} (-\alpha I_T)^i \mid \mathcal{F}_t \right)}, \quad 0 \leq t \leq T, \]

with \( \alpha \in \mathbb{R}^+ \), which has increasing elasticity and

**power utility:**

\[ \Phi_{t,T}^{\text{power}} = \frac{\alpha I_T^{\delta}}{E\left( \alpha I_T^{\delta} \mid \mathcal{F}_t \right)}, \quad 0 \leq t \leq T, \]

with \( \alpha > 0, \delta < 0 \), which has constant elasticity.

Moreover, the Taylor-series expansion of a function \( f(x) \) around \( x_0 \) can be written as

\[ \sum_{i=0}^{N} \frac{f^{(i)}(x_0)}{i!} (x - x_0)^i = \sum_{i=0}^{N} \frac{f^{(i)}(x_0)}{i!} \left( \sum_{k=0}^{i} \binom{i}{k} x^{i-k} (-x_0)^k \right) \quad (13) \]
where \( f^{(i)} \) is the \( i \)th derivative of \( f \). Hence, our characterization approximates any pricing kernel at least as well as a Taylor-series expansion since the right hand side of equation (13) is a special case of our weighted sum of power functions.

Our proposed class of pricing kernels has the convenient property that the pricing kernel is characterized by a series of noncentral moments of the random variable. Especially with our information process, pricing kernel and asset price are easily computed since the terminal value of the information process is lognormally distributed. The forward price admits the following characterization

\[
F_{t}^{\text{general}} = E \left( \frac{\sum_{i=1}^{N} \alpha_i I_t^{\delta_i+1}}{E \left( \sum_{i=1}^{N} \alpha_i I_t^{\delta_i} \right)} \mid \mathcal{F}_t \right) 
\]

\[
= I_t \sum_{i=1}^{N} \alpha_i I_t^{\delta_i} A_i(t) \exp \left( \sigma^2 (T - t) \delta_i \right) \bigg/ \sum_{i=1}^{N} \alpha_i I_t^{\delta_i} A_i(t), 
\]

\( 0 \leq t \leq T, \)

with \( A_i(t) = \exp \left( \frac{1}{2} (\delta_i - 1) \sigma^2 (T - t) \delta_i \right) \). Thus, equation (14) provides a general analytical characterization of asset prices for a lognormally distributed terminal value. Of course, appropriate parameters have to be chosen, especially to avoid arbitrage possibilities.
B. Example

To illustrate the effect of declining elasticity of the pricing kernel, let us consider the following specification which generates analytical asset price processes for constant and declining elasticity of the pricing kernel

$$\Phi_{t,T}^{l_p} = \frac{1 - q + \beta I_T^\delta}{E\left(\frac{1 - q + \beta I_T^\delta}{F_t}\right)}, \quad 0 \leq t \leq T. \quad (15)$$

This pricing kernel implies a representative investor with a utility function over terminal wealth which is a linear combination of a log-utility and a power-utility function. However, in order to have a well-defined pricing kernel, respectively utility function of the representative investor, we make the additional assumptions:

**no-arbitrage:** to avoid arbitrage possibilities the pricing kernel has to be positive for $I_T > 0$. Therefore we assume $\beta \geq 0$.

**risk aversion:** $\delta \leq 0$ implies a negative slope of the pricing kernel (i.e. marginal utility of the representative investor would be declining).

While the power and the log specification yield constant elasticity of the pricing kernel our extended log-power-utility ($l - p - u$) generates constant ($\beta = 0$ or $\delta = -1$) or declining elasticity ($\beta > 0$ and $\delta \neq -1$) of the pricing
kernel. Moreover, with this pricing kernel we get the following analytical solution for the forward price:

\[
F_t^{l-p-u} = E \left( I_T \Phi_{t,T}^{l-p-u} \right | F_t) \\
= I_t \exp \left( \sigma^2 (t - T) \right) \frac{1 + \beta I_t^{\delta+1} \exp \left( \frac{\delta^2 + \delta - \frac{1}{2} \sigma^2 (T - t)}{} \right)}{1 + \beta I_t^{\delta+1} \exp \left( \frac{\delta^2 - \delta - \frac{3}{2} \sigma^2 (T - t)}{} \right)},
\]

\[0 \leq t \leq T.\]

Equation (16) nests the geometric Brownian motion. This is easily seen since for \( \beta = 0 \) or \( \delta = -1 \) we get the same asset price as under log-utility. Moreover, analyzing equation (16) yields that the forward price in this example is governed by a geometric Brownian motion, \( I_t \exp (\sigma^2 (t - T)) \), multiplied by a random variable.

In each simulation run, we generate 100 observations of the information process with \( \sigma = 0.1 \) and \( I_0 = 100 \). The simulation is repeated 1000 times.\(^9\)

Then, asset returns are calculated for the following specifications:

**Specification 1:** \( \beta = 5, \ \delta = -20 \)

**Specification 2:** \( \beta = 5, \ \delta = -15 \)

**Specification 3:** \( \beta = 5, \ \delta = -10 \)

**Specification 4:** \( \beta = 5, \ \delta = -5 \)

**Specification 5:** \( \beta = 5, \ \delta = -1 \)
Specification 6: $\beta = 5, \ \delta = 0$

Thus, we have for every specification 1000 return-series with 99 observations each. The elasticities of the different pricing kernels are

$$\eta = -\frac{-1 + 5I_T^{\delta+1}\delta}{1 + 5I_T^{\delta+1}}.$$  

Except for $\delta = -1$ (constant elasticity) all analyzed pricing kernels have declining elasticities. Figure 1 displays the standard deviation (Std.) and the autocorrelation of lag 1 (ac(1)) for the returns $(\ln F_t - \ln F_{t-1})$.

Comparing Specification 5 with the other specifications illustrates that the return volatility is higher for declining elasticity of the pricing kernel. Especially for small $\delta$ (Specifications 1, 2 and 3) we find a significantly higher volatility. Specifications 4 and 6, however, yield asset returns which are almost equal to the geometric Brownian motion (Specification 5). For $\delta = -20$ the volatility becomes more than ten times higher than for $\delta = -1$. Moreover, the return volatility is not constant. Figure 1 shows that high volatilities tend to be followed by high volatilities. Thus, the asset returns exhibit some volatility clustering. The pricing kernels with declining elasticity lead also to positive serial correlation of lag 1. Because of the nonstationarity of asset
returns we cannot give a general theoretical result on the sign of the autocorrelation of lag 1 but Figure 1 shows that the autocorrelation of lag 1 becomes positive for the time interval with high volatility. This is consistent with our theoretical result that asset returns overreact under declining elasticity of the pricing kernel. The overreaction leads to short-term momentum (positive autocorrelation of lag 1) and higher volatility of asset returns. Since the terminal distribution is exogenously given, this overreaction has to be compensated. The following Figure 2 shows this long-term reversal. To document the long-term reversal we plot the correlation (ac) between starting period and final period returns. This means we separate the whole period into two subperiods [0, t] and [t, T] and calculate the correlation between the first period return \( r_{0,t} \) and the second period return \( r_{t,T} \). We repeat this procedure for all \( t = 1 \ldots 98 \) to get the time-series of the correlation between starting period and final period returns. We see that this correlation may become significantly negative. For \( \delta = -20 \) we find correlations of up to \(-98\%\). The overreaction is thus compensated by a long-term reversal. Hence, our simulation results indicate that declining elasticity of the pricing kernel generates a behavior of asset returns which is consistent with short-term momentum and long-term reversals. While the autocorrelation of lag 1 of the returns
with higher frequency tends to be positive we find a significant reversal in long-term returns.

- insert Figure 2 here -

Note also the effect of approaching the terminal date $T$. If we get close enough to $T$ the overreaction effect becomes small and thus the volatility converges to the 10% level of the specification with constant elasticity of the pricing kernel. This time-dependence is obvious from the two previous and the following figure. For declining elasticity the asset return volatilities, the autocorrelations and the Sharpe ratios depend also on time $t$. Figure 3 illustrates that the time-variation in asset returns is not completely explained by time-varying standard deviation of returns. The Sharpe ratios $\left(\frac{E(\ln F_t - \ln F_{t-1})}{\text{Std}(\ln F_t - \ln F_{t-1})}\right)$ also exhibit significant changes over time. For $\delta = -20$, the Sharpe ratio varies in a range of more than 200%. Thus, the changing Sharpe ratios (market prices of risk) also contribute to the behavior of asset returns.

- insert Figure 3 here -

Overall we may conclude that the proposed class of pricing kernels and even the special case analyzed here are general enough to generate price
processes which match important stylized facts.

V. Conclusion

Predictability of asset returns is often explained by nonrational expectations or new behavioral postulates which are not consistent with von Neumann-Morgenstern utility functions. However, still little is known on return characteristics in a traditional framework when the elasticity of the pricing kernel is not constant. In this paper we derive the characteristics of asset returns when the elasticity of the pricing kernel is not constant. However, we assume rational expectations and a framework which is consistent with a representative investor with a von Neumann-Morgenstern utility function. Moreover, we derive analytical asset price processes for a rather general class of pricing kernels.

We show that with nonconstant elasticity of the pricing kernel, the forward price $F_t$ is not a linear function of the expected terminal value $I_t = E(F_T|\mathcal{F}_t)$. This nonlinearity induces predictability in asset returns. The predictability is reflected by the fact that expected returns depend on the level of the asset price. Moreover, nonconstant elasticity of the pricing kernel leads to time-varying instantaneous volatility. For declining [increasing] elasticity of the pricing kernel, the instantaneous volatility decreases [increases]
with rising asset prices. Hence, declining elasticity of the pricing kernel leads to asymmetric volatility. In addition, we show that asset returns overreact [underreact] to changes in expectations when the pricing kernel has declining [increasing] elasticity. We also analyze the moments of the discretely sampled return process. The results suggest very careful interpretation of empirical results since the derived return processes are usually not stationary. Our numerical simulations of the analytical asset prices show that declining elasticity of the pricing kernel generates return characteristics which are consistent with short-term momentum and long-term reversals.

Hence, the analysis shows that many observed return characteristics are compatible with a rational expectations model with nonconstant elasticity of the pricing kernel. Moreover, the derived analytical asset price processes are interesting for future theoretical and empirical research. This class of stochastic processes is derived from an economic model and it is consistent with many alternative pricing kernels. Moreover, it should be flexible enough to match many stylized facts. Therefore this class may provide some guidance for future empirical research and it may serve as basis for new derivative pricing models.
Appendix A

The ratio between the forward price under declining [increasing] elasticity of the pricing kernel $F^{DE}_t$ [$F^{IE}_t$] and the forward price under constant elasticity $F^C_t$

$$\frac{F^{DE}_t}{F^C_t}, \text{ respectively } \frac{F^{IE}_t}{F^C_t}$$

increases [decreases] monotonically with increasing $F^C_t$. This follows because

$$\frac{I_t}{F^C_t} \equiv \frac{E(F_T | F_t)}{F^C_t}$$

is independent of the level of $I_t$ since

$$F^C_t = v^C (t, I_t) = A (t) I_t , \quad 0 \leq t \leq T,$$

with $A : [0, T] \to \mathbb{R}^+$, and with declining [increasing] elasticity of the pricing kernel the expected return $\frac{I_t}{F^{DE}_t} \left( \frac{I_t}{F^{IE}_t} \right)$ decreases [increases] with increasing $I_t$. For a more detailed proof see Theorem 3 in Franke, Stapleton and Subrahmanyam (1999). However, this statement in Franke, Stapleton and Subrahmanyam (1999) can be expressed more formally as:

$$\frac{\partial}{\partial I_t} \left( \frac{v^{IE} (t, I_t)}{v^C (t, I_t)} \right) < 0 , \quad 0 \leq t < T,$$

respectively

$$\frac{\partial}{\partial I_t} \left( \frac{v^{DE} (t, I_t)}{v^C (t, I_t)} \right) > 0 , \quad 0 \leq t < T.$$

Simple calculus shows that this is equivalent to

$$\frac{\partial v^{IE} (t, I_t)}{\partial I_t} < \frac{v^{IE} (t, I_t)}{I_t} , \quad 0 \leq t < T , \quad (17)$$

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respectively

\[
\frac{\partial v^{DE}(t, I_t)}{\partial I_t} > \frac{v^{DE}(t, I_t)}{I_t}, \quad 0 \leq t < T. \tag{18}
\]

\[
\text{Appendix B}
\]

We give the proof for declining elasticity, the proof for increasing elasticity is analogous. Since \( \frac{\partial}{\partial I_t} \left( \frac{v^{DE}(t, I_t)}{I_t} \right) \geq 0 \) and \( \frac{v^{DE}(t, I_t)}{I_t} \leq 1 \) for \( 0 \leq t < T \) it follows from the Theorem of Bolzano-Weierstrass that

\[
\lim_{I_t \to \infty} \left( \frac{v^{DE}(t, I_t)}{I_t} \right) = c,
\]

where \( c \) is some positive constant with \( c \leq 1 \) and \( F_t = v^{DE}(t, I_t) \). Since \( \lim_{I_t \to \infty} v^{DE}(t, I_t) = \infty \) it follows from the rule of L'Hopital that

\[
c = \lim_{I_t \to \infty} \left( \frac{v^{DE}(t, I_t)}{I_t} \right) = \lim_{I_t \to \infty} \left( \frac{\partial}{\partial I_t} v^{DE}(t, I_t) \right).
\]

Hence, the elasticity of the forward price with respect to \( I_t \) converges to 1, i.e.

\[
\lim_{I_t \to \infty} \left( \frac{\partial}{\partial I_t} v^{DE}(t, I_t) \frac{I_t}{v^{DE}(t, I_t)} \right) = 1.
\]

For \( I_t < \infty \) we have already seen that the elasticity is higher than 1. Hence,

\[
\lim_{I_t \to \infty} (\Sigma^{DE}_t) = \sigma,
\]

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while \( \Sigma_{t}^{DE} > \sigma \, \forall I_t < \infty \). 

**Appendix C**

Since with declining [increasing] elasticity of the pricing kernel \( \text{Var} \left( \ln F_{\tau} \mid \mathcal{F}_t \right) > [\leq] \text{Var} \left( \ln I_{\tau} \mid \mathcal{F}_t \right) \) for \( 0 \leq t < \tau < T \), it follows immediately that also

\[
\text{Var} \left( \ln F_{\tau} - \ln F_t \mid \mathcal{F}_t \right) > [\leq] \text{Var} \left( \ln I_{\tau} - \ln I_t \mid \mathcal{F}_t \right), \quad 0 \leq t < \tau < T.
\]

Hence, the conditional variance of returns is higher [lower] under declining [increasing] elasticity of the pricing kernel. For the case of declining elasticity of the pricing kernel, consider now the unconditional variance:

\[
\text{Var} \left( \ln F_{\tau} - \ln F_t \right) = \text{Var} \left( E \left( \ln F_{\tau} \mid \mathcal{F}_t \right) - \ln F_t \right) + E \left( \text{Var} \left( \ln F_{\tau} \mid \mathcal{F}_t \right) \right),
\]

\[0 \leq t \leq \tau \leq T,
\]

with

\[
E \left( \ln F_{\tau} \mid \mathcal{F}_t \right) - \ln F_t = E \left( \int_t^\tau \mu - \frac{1}{2} \Sigma^2 ds \bigg| \mathcal{F}_t \right), \quad 0 \leq t < \tau \leq T. \quad (19)
\]

Equation (19) shows that except for constant elasticity of the pricing kernel with nonrandom term \( \mu - \frac{1}{2} \Sigma^2 \)

\[
\text{Var} \left( E \left( \ln F_{\tau} \mid \mathcal{F}_t \right) - \ln F_t \right) > 0.
\]
Moreover,

\[ \text{Var} (\ln I_{\tau} - \ln I_{t}) = \text{Var} (\ln I_{\tau} | \mathcal{F}_{t}), \]

\[ 0 \leq t \leq \tau \leq T, \]

since the information process is governed by a geometric Brownian motion with constant volatility and no drift. Thus, because \( \text{Var} (\ln F_{\tau} | \mathcal{F}_{t}) > \text{Var} (\ln I_{\tau} | \mathcal{F}_{t}) \), we have

\[ \text{Var} (\ln F_{\tau} - \ln F_{t}) > \text{Var} (\ln I_{\tau} - \ln I_{t}). \]
References


Camara, A., 2001, Option Prices Sustained by Risk-Preferences, Working Paper, University of Strathclyde.


Mayhew, S. and C. Stivers, 2001, Stock Return Dynamics, Implied Volatility,
and the Asymmetric Volatility Phenomenon, working paper, University of Georgia.


Notes

1 For an overview on deviations from the random walk hypothesis see for example Cochrane (2001).

2 See for example Brennan (2001) for a discussion of the pros and cons of adopting new behavioral postulates.


4 For a detailed discussion of information processes see Lüders and Peisl (2001).

5 For a more detailed discussion see for example Decamps and Lazrak (2000) or Camara (2001).


7 See also He and Leland (1993) for the analysis of time-homogeneous asset price processes. For the estimation of diffusion models see for example Gourieroux and Jasiak (2001). A recent development on the estimation of diffusion processes is found, for example, in Elerian, Chib, Shephard (2001).

8 For an overview of recent estimation techniques see for example Gourieroux and Jasiak (2001).

9 Random numbers are generated with SAS.
Figure 1. Standard deviation and autocorrelation of asset returns.

Standard deviation (Std.) and autocorrelation of lag 1 (ac(1)) of asset returns 
($ln F_t - ln F_{t-1}$) for $t=1...99$ for the 6 different specifications of the pricing 
kernel.
Figure 2. Long-term Reversals.

Correlation (ac) between starting period returns \( r_{0,t} = \ln F_t - \ln F_0 \) and final period returns \( r_{t,99} = \ln F_{99} - \ln F_t \) for \( t=1...98 \) for the 6 different specifications of the pricing kernel.
Figure 3. Sharpe ratios.

Sharpe ratios (E(ln F_t - ln F_{t-1}) / Std(ln F_t - ln F_{t-1}) over time for the 6 different specifications of the pricing kernel.