

The Generalized Treynor Ratio: A Note

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Abstract

This paper presents a generalization of the Treynor ratio in a multi-index setup. The solution proposed in this paper is the simplest measure that keeps Treynor's original interpretation of the ratio of abnormal excess return (Jensen's alpha) to systematic risk exposure (the beta) and preserves the same key geometric and analytical properties as the original single index measure. Essentially, it must be a monotone function of the portfolio's systematic risk, be comparable to a benchmark, provide a similar ranking to the alpha for portfolios with the same risk, be independent of the scale of the risk premia, and provide performance measures that are independent of the choice of the model. In addition, the adopted measure should be the simplest one respecting these properties. In a linear multi-index, these requirements are fulfilled by normalizing the risk premia using a benchmark portfolio and by rotating the factors to obtain an orthonormed hyperplane for risk dimensions. The resulting Generalized Treynor ratio does not require the identification of the returns generating process. It is defined as the abnormal return of a portfolio per unit of weighted-average systematic risk, the weight of each risk loading being the value of the corresponding risk premium. This performance measure is invariant to the specification of the asset pricing model, the number of factors or the scale of the measure.

The Generalized Treynor Ratio: A Note

After the publication of the Capital Asset Pricing Model developed by Sharpe (1964), Treynor (1965) and Mossin (1966), the issue of the assessment of risk-adjusted performance of portfolios was quickly recognized as a crucial extension of the model. It would provide adequate tools to evaluate the ability of portfolio managers to realize returns in excess of a benchmark portfolio with similar risk. Consequently, four simple measures were proposed and adopted in the financial literature: The Sharpe (1966) ratio and the Treynor and Black (1973) 'appraisal ratio' both use the Capital Market Line as the risk-return referential, using variance or standard deviation of portfolio returns as the measure of risk, whereas the Treynor (1966) ratio and Jensen's (1968) alpha directly relate to the beta of the portfolio using the Security Market Line. The evolution of quantitative performance assessment for single index models bred few new widely accepted measures. Fama (1972) proposed a useful decomposition of performance between timing and selection abilities, while Treynor and Mazuy (1966) and Henriksson and Merton (1981) designed performance measures aiming at measuring market timing abilities. More recently, Modigliani and Modigliani (1997) proposed an alternative measure of risk that also uses the volatility of returns in the context of the CAPM.

There exists some extensions of these performance measures to multi-index model. In the context of Ross's (1976) Arbitrage Pricing Theory, Connor and Korajczyk (1986) develop multi-factor counterparts of the Jensen (1968) and Treynor and Black (1973) measures, while Sharpe (1992, 1994) provides conditions under which the Sharpe ratio can be extended to the presence of several risk premia. Nowadays, most performance studies of multi-index asset pricing models use Jensen's (1968) alpha. Its interpretation as the risk-adjusted abnormal return of a portfolio makes it flexible enough to be used in most asset pricing specifications. Kothari and Warner (2001) only consider this measure for multi-index asset pricing models in their empirical comparison of mutual funds performance measures. Obviously, the lack of a multi-index counterpart of the Treynor ratio represents a gap in the financial literature and in business practice, as it would allow to relate the

level of abnormal returns to the systematic risk taken by the portfolio manager in order to achieve it.

This article aims at filling this gap. It presents a generalization of the Treynor ratio in a multi-index setup. To be a proper generalized measure, it has to conserve the same key economic and mathematical properties as the original single index measure, and also to ease comparison of portfolios across asset pricing models. The solution proposed in this paper is the simplest measure that meets these requirements while still keeping Treynor's original economic interpretation of the ratio of abnormal excess return (Jensen's alpha) to systematic risk exposure (the beta).

The paper is organized as follows. Section I introduces the characteristics of the Treynor ratio. The second section proposes a generalization of this ratio that preserves its key properties. Section III concludes.

I. The classical Treynor performance ratio

The Treynor ratio uses as the Security Market Line, that relates the expected total return of every traded security or portfolio i to the one of the market portfolio m :

$$E_i = R_f + \beta_i [E_m - R_f] \quad (1)$$

where $E_i = E[R_i]$ denotes the unconditional continuous expected return, R_f denotes the continuous return on the risk-free security and $\beta_i = \frac{\text{cov}(R_i, R_m)}{\sigma^2(R_m)}$ is the beta of security i .

This equilibrium relationship corresponds to the market model :

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it} \quad (2)$$

where $r_i = R_i - R_f$ denotes the excess return on security i . If the CAPM holds and if markets are efficient, α_i should not be statistically different from 0.

When considered in the context of portfolio management, the econometric specification of equation (2) translates into an ex-post measure of excess return:

$$\bar{r}_i = \alpha_i + \beta_i \bar{r}_m \quad (3)$$

where $\bar{r}_i = \frac{1}{n} \sum_{t=1}^T r_{it}$ is the average return of the security over the sample period $(0, T)$ and the econometric methodology leading from (2) to (3) ensures that $\bar{\varepsilon}_i = 0$.

Equation (3) constitutes the source of two major performance measures of financial portfolios; Jensen's alpha (1968) and the Treynor ratio (1966).

Jensen's alpha is just α_i in equation (3): it is the percentage excess return earned by the portfolio in addition to the required average return over the period. This is an absolute performance measure – it is measured in the same units as the return itself – after controlling for risk.

The Treynor ratio can either be defined as the *Total Treynor ratio* (TT), as usually treated in the literature, or the *Excess Treynor ratio* (ET) that directly related to abnormal performance. The equations for these two ratios are the following:

$$TT_i = \frac{\bar{r}_i}{\beta_i} \quad (4)$$

$$ET_i = \frac{\alpha_i}{\beta_i} = TT_i - \bar{r}_m \quad (5)$$

These two measures are roughly equivalent. Nevertheless, the link between the Excess Treynor ratio and Jensen's alpha is easier to interpret: the Excess Treynor ratio is just the equal to the alpha per unit of systematic risk of the portfolio. In particular, this formulation corresponds to the original measure developed by Treynor (1966). Thus, this paper will proceed from now on with the Excess Treynor ratio.

It immediately appears that, in this simple classical setup, the Treynor ratio provides additional information with respect to Jensen's alpha: two securities with different risk levels that provide the same excess returns over the same period will have the same alpha but will differ with respect to the Treynor ratio. The improvement comes from the fact that the Treynor ratio provides the performance of the portfolio *per unit of systematic risk*.

From this univariate setup, five key features of the Treynor ratio may be emphasized, provided that systematic risk is positive in all cases:

- *Ratio of distances*: The Treynor ratio is a slope measure, i.e. the ratio of an euclidian

distance in the returns space to an euclidian distance in the risk space: $ET_i = \frac{D(0, \alpha_i)}{D(0, \beta_i)}$, where $D(.,.)$ denotes the euclidian distance operator.

- *Monotonicity with risk:* If a portfolio is riskier than another and both have the same alpha, then the riskier portfolio has the lower Treynor ratio: $\forall i, i'$ such that $\beta_i > (\text{resp. } <, =) \beta_{i'}$ and $\alpha_i = \alpha_{i'} \iff ET_i < (\text{resp. } >, =) ET_{i'}$.
- *Benchmarking:* A portfolio m is taken as a benchmark and the Treynor ratio of every portfolio with the same beta as the benchmark is equal to its alpha: $\forall i$ such that $\beta_i = \beta_m = 1$, $ET_i = \alpha_i$.
- *Cross-sectional independence:* All portfolios whose required return is similar obtain the same ranking as the one given by their alpha: $\forall i, i'$ such that $\beta_i = \beta_{i'}$, $ET_i > (\text{resp. } <, =) ET_{i'} \iff \alpha_i > (\text{resp. } <, =) \alpha_{i'}$.
- *Scale independence:* If the scale of the risk premium is changed from one market, benchmark or period to another, then the Treynor ratio is unchanged: for any two periods where \bar{r}_i is measured at the same time as \bar{r}_m and \bar{r}_i^* is measured at the same time as \bar{r}_m^* such that $\beta_i = \beta_i^*$ and $\bar{r}_m = k\bar{r}_m^*$, $k \in \mathfrak{R}$, then $ET_i > (\text{resp. } <, =) ET_i^* \iff \alpha_i > (\text{resp. } <, =) \alpha_i^*$.

The interpretation of these properties is rather natural. The ratio of distances property comes from the very definition of the Treynor ratio and allows to provide an economic justification. Monotonicity reflects the basic improvement of the Treynor ratio with respect to Jensen's alpha. The other three properties ensure that there is no other dimension than risk impacted by the Treynor ratio; i.e. that this ratio provides similar results to the alpha for other relevant aspects. Benchmarking sets a reference value for the performance measure. Cross-sectional independence entails that portfolios with the same risk and the same abnormal return have the same performance. Scale independence makes sure that the performance measure only depends on the measure of risk and is independent of the scale of the risk premium. This is a crucial property since it guarantees, for instance, that

the Treynor ratio is invariant to a change in currency when a portfolio denominated in a foreign currency is studied in the domestic currency.

II. Generalizing the Treynor ratio

A. Current shortcomings

Since the publication of Ross's (1976) Arbitrage Pricing Theory (APT), it has been widely acknowledged that the use of a single index in a market model is probably not sufficient in order to keep track of the systematic sources of portfolio returns in excess of the risk free rate. Consequently, the developments of unconditional asset pricing models have taken two directions: either they added additional risk premia to the classical CAPM like in Fama and French (1993) and Carhart (1997), or they used a pure multi-factor approach in the spirit of the APT, like in Chen, Roll and Ross (1986). In spite of their differences in conception and statistical assumptions, both approaches share a very general model specification that can be summarized as follows, still considering the ex-post multidimensional equation corresponding to (3):

$$\bar{r}_i = \alpha_i + \sum_{j=1}^K \beta_{ij} \bar{r}_j \quad (6)$$

where $j = 1, \dots, K$ denotes the number of distinct risk premia and $\sum_{j=1}^K \beta_{ij} \bar{r}_j \geq 0$. As it can be immediately noticed, the alpha remains a scalar whereas the systematic risk measures of the portfolio is made of a vector $(\beta_{i1}, \dots, \beta_{iK})$ of loadings to the individual risk premia. This additional source of complexity probably explains why only the Jensen measure has been widely applied in the performance evaluation literature with multi-factor models while the Treynor ratio has been completely ignored.

Yet, it is highly desirable to have a measure of that kind at one's disposal. In the portfolio management area, the single use of Jensen's alpha may provide unfair judgements over the performance of portfolios that invested in very different classes of risks, simply because there presumably exists a positive relationship between the scale of excess returns that can be obtained from an investment strategy and its aggregate level of risk. The

current, general failure to take these relationships into account leads to unduly favoring well-performing risky portfolios (high positive alphas) while condemning bad performers in the same population (low negative alphas).

The most likely explanation for the current shortage of risk-adjusted, relative performance measure like the Treynor measure is the lack of a natural reduction method, going from the K -dimensional risk space to a one-dimensional measure. Obviously, the simplest reduction rules that would account for all sources of risk, like $\frac{\alpha_i}{\sum_{j=1}^K \beta_{ij}}$ or $\frac{\alpha_i}{\sum_{j=1}^K \beta_{ij} \bar{r}_j}$ are not satisfactory, as the first measure does not satisfy the cross-sectional independence requirement while the second one fails to account for scale independence property. Surprisingly, a thorough review of the literature reveals that these basic obstacles were sufficient to prevent theoretical research to go deeper in this area.

B. Desirable Properties

Call the Generalized Treynor ratio (GT) a risk-adjusted performance measure that would fill the same role as the Excess Treynor ratio in a multivariate setup. It has to at least respect the same basic properties as its univariate counterparts. Considering that for every portfolio, the sum of risk premia is positive ($\sum_{j=1}^K \beta_{ij} \bar{r}_j > 0$), one can re-express the previous characteristics in an adapted way:

- (C1) *ratio of euclidian distances*: The Generalized Treynor ratio is a slope measure, i.e. the ratio of an euclidian distance in the returns space to an euclidian distance in the K -dimensional risk hyperspace.
- (C2) *Monotonicity with required return*: If a portfolio has to earn a greater ex-post required return than another and both have the same alpha, then the portfolio with greater required return has the lower Generalized Treynor ratio: $\forall i, i'$ such that $\sum_{j=1}^K \beta_{ij} \bar{r}_j > (\text{resp. } <, =) \sum_{j=1}^K \beta_{i'j} \bar{r}_j$ and $\alpha_i = \alpha_{i'} \iff GT_i < (\text{resp. } >, =) GT_{i'}$.
- (C3) *Benchmarking*: A portfolio m is taken as a benchmark and the Treynor ratio of every portfolio with the same required return as the benchmark is equal to its alpha: $\forall i$ such that $\sum_{j=1}^K \beta_{ij} \bar{r}_j = \sum_{j=1}^K \beta_{mj} \bar{r}_j$, $GT_m = \alpha_m$.

- (C4) *Cross-sectional independence*: All portfolios whose required return is similar obtain the same ranking as the one given by their alpha: $\forall i, i'$ such that $\sum_{j=1}^K \beta_{ij} \bar{r}_j = \sum_{j=1}^K \beta_{i'j} \bar{r}_j$, $GT_i > (\text{resp. } <, =) GT_{i'} \iff \alpha_i > (\text{resp. } <, =) \alpha_{i'}$.
- (C5) *Scale independence*: If the scale of the risk premia is changed from one market, benchmark or period to another, then the Generalized Treynor ratio is unchanged: for any two periods where \bar{r}_i is measured at the same time as \bar{r}_j , $j = 1, \dots, K$ and \bar{r}_i^* is measured at the same time as \bar{r}_j^* , $j = 1, \dots, K$ such that $\forall j, \beta_{ij} = \beta_{ij}^*$ and $\bar{r}_j = k \bar{r}_j^*$, $k \in \mathfrak{R}$, then $GT_i > (\text{resp. } <, =) GT_i^* \iff \alpha_i > (\text{resp. } <, =) \alpha_i^*$.

In addition, the particular issue of a multi-factor model involves that two additional conditions are respected by the Generalized Treynor ratio:

- (C6) *Model independence*: If all portfolios provide the same ex-post required return with two different models for the same period, then the Generalized Treynor ratio is unchanged: for any two models where \bar{r}_i is measured at the same time as \bar{r}_j , $j = 1, \dots, K$ and \bar{r}_j^* , $j = 1, \dots, K^*$ such that $\sum_{j=1}^K \beta_{ij} \bar{r}_j = \sum_{j=1}^{K^*} \beta_{ij} \bar{r}_j^* \forall i$, then $GT_i = GT_i^*$.
- (C7) *Parsimony*: Among all measures that respect conditions C1 to C6, the Generalized Treynor ratio is the one that requires the least number and repetition of parameters.

The fifth condition introduces a consistency of the measure with respect to modeling choices. In particular, for $K = 1$, it requires that $GT_i = ET_i$. The last condition is not properly a technical one, but rather requires that the Generalized Treynor ratio represents the simplest measure respecting the previous technical conditions.

C. The Generalized Treynor ratio

The intuition leading to the Generalized Treynor ratio can be easily presented. Imagine that the asset pricing model involves totally independent sources of risk that correspond to identical risk premia: in this situation, all portfolios whose sum of the

risk loadings (betas) is identical have comparable exposures to risk. Forcing the financial world, where sources of risk are interdependent and of unequal importance, to fit in this framework would leave a very simple problem to solve. Following these lines, the expression for the Generalized Treynor ratio is given by the following Proposition:

Proposition 1: *If the ex-post asset pricing equation of every portfolio i corresponds to equation (6) where $\sum_{j=1}^K \beta_{ij} \bar{r}_j > 0$, whose returns generating process is a K -factor model, then the Generalized Treynor ratio (GT_i) of this security respecting conditions C1 to C7 is given by:*

$$GT_i = \alpha_i \frac{\sum_{j=1}^K \bar{r}_j^*}{\sum_{j=1}^K \beta_{ij}^* \bar{r}_j^*} \quad (7)$$

where $\bar{r}_j^* = \bar{r}_j \beta_{mj}$, $\beta_{ij}^* = \frac{\beta_{ij}}{\beta_{mj}}$, $j = 1, \dots, K$ and β_{mj} is the loading of the benchmark portfolio m of the j 's source of risk.

Proof: From condition C1, the Generalized Treynor ratio can be written

$$GT_i = \frac{f(\alpha_i, \beta_{i1}, \dots, \beta_{iK}, \beta_{m1}, \dots, \beta_{mK}, \bar{r}_1, \dots, \bar{r}_K, K)}{g(\alpha_i, \beta_{i1}, \dots, \beta_{iK}, \beta_{m1}, \dots, \beta_{mK}, \bar{r}_1, \dots, \bar{r}_K, K)}$$

where f and g are measures of euclidian distances. Moreover, from condition C6, it must be that $f(\alpha_i, \beta_{i1}, \dots, \beta_{iK}, \beta_{m1}, \dots, \beta_{mK}, \bar{r}_1, \dots, \bar{r}_K, K) = D(0, \alpha_i) = \alpha_i$. Therefore, g must be a distance measure enabling at the same time the Generalized Treynor ratio to respect conditions C2 to C7.

Consider that the returns generating process that corresponds to the market model for a security i is a factor model, corresponding to the following equation:

$$r_{it} = \sum_{j=1}^K \beta_{ij} r_{jt} + \varepsilon_{it} = \sum_{j=1}^K b_{ij} \rho_{jt} + \eta_{it} \quad (8)$$

where $E(\varepsilon_{it}) = E(\eta_{it}) = 0$ and ρ_{jt} is orthogonal to $\rho_{j't}$ for $j \neq j'$ and to η_{it} . The realized return of security i is given by:

$$\bar{r}_i = \alpha_i + \sum_{j=1}^K b_{ij} \bar{\rho}_j \quad (9)$$

Risk premia are scaled such that $\sum_{j=1}^K \bar{r}_j = \sum_{j=1}^K \bar{\rho}_j$ and $\bar{\rho}_j > 0$.

The fact that the factors are orthogonal ensures the orthogonality of the K -dimensional space of risk loadings. To obtain an orthonormed space, every risk premium should be normalized to a single unit. Define the weight of factor j by:

$$w_j = \frac{\bar{\rho}_j}{\sum_{j=1}^K \bar{\rho}_j} \quad (10)$$

Using (10), Equation (9) can be rewritten as:

$$\begin{aligned} \bar{r}_i &= \alpha_i + \sum_{j=1}^K \frac{b_{ij}}{w_j} \left(\sum_{j=1}^K \bar{\rho}_j \right) \\ &= \alpha_i + B_i \bar{R} \end{aligned} \quad (11)$$

where $\bar{R} = \sum_{j=1}^K \bar{\rho}_j$, $B_i = \sum_{j=1}^K \frac{b_{ij}}{w_j}$.

To respect condition C4, the Generalized Treynor ratio of all portfolios requiring the same return should be equal. In particular, $GT_i = GT_p$ for a portfolio p where $B_p = B_i$ and $b_{p1}^* = b_{p2}^* = \dots = b_{pK}^* = \frac{B_i}{K}$. The euclidian distance between 0 and the coordinates of this portfolio on the K -dimensional orthonormed risk axis is:

$$\begin{aligned} D(0, b_{p1}^*, b_{p2}^*, \dots, b_{pK}^*) &= \sqrt{b_{p1}^{*2} + b_{p2}^{*2} + \dots + b_{pK}^{*2}} \\ &= \sqrt{\left(\frac{B_i}{K}\right)^2 + \left(\frac{B_i}{K}\right)^2 + \dots + \left(\frac{B_i}{K}\right)^2} \\ &= \frac{\sum_{j=1}^K b_{ij} \bar{\rho}_j}{\sqrt{K} \sum_{j=1}^K \bar{\rho}_j} \end{aligned} \quad (12)$$

provided that $B_i > 0$. The last line follows from noticing that $B_i = \frac{\sum_{j=1}^K b_{ij} \bar{\rho}_j}{\sum_{j=1}^K \bar{\rho}_j} > 0$.

By equation (8), $\sum_{j=1}^K b_{ij} \bar{\rho}_j = \sum_{j=1}^K \beta_{ij} \bar{r}_j$ and $\sum_{j=1}^K \bar{\rho}_j = \sum_{j=1}^K \bar{r}_j$. Setting that $g(\alpha_i, \beta_{i1}, \dots, \beta_{iK}, \bar{r}_1, \dots, \bar{r}_K, K) = \sqrt{K} D(0, b_{p1}^*, b_{p2}^*, \dots, b_{pK}^*)$ leads to the *Absolute Generalized Treynor ratio* (AGT_i):

$$AGT_i = \alpha_i \frac{\sum_{j=1}^K \bar{r}_j}{\sum_{j=1}^K \beta_{ij} \bar{r}_j} \quad (13)$$

The AGT_i respects conditions C1 (by construction), C2, C4, C5.(because it is homogenous of degree 0 with respect to the vector of risk premia) and C6.

Since every risk loading only appears once, this expression is the least parsimonious in betas. Furthermore, to respect at the same time conditions C4 and C5, every risk premium must show up at least twice. Hence, this expression is also the least parsimonious in risk premium. Therefore, it respects condition C7. However, it still depends on a scaling factor since $\sum_{j=1}^K \bar{r}_j$ may vary from one rotation of the factors to another.

To respect the benchmarking condition (C3), it suffices to normalize each risk premium by multiplying it with the beta of the benchmark portfolio m :

$$\sum_{j=1}^K \beta_{mj} \bar{r}_j = \sum_{j=1}^K \bar{r}_j^*$$

by defining $\bar{r}_j^* = \bar{r}_j \beta_{mj}$. This rescaling of the risk premia induces a rescaling of each securities' betas: $\beta_{ij}^* = \frac{\beta_{ij}}{\beta_{mj}}$.

the Absolute Generalized Treynor ratio of a security i is normalized by the one of the benchmark portfolio m :

$$GT_i = \alpha_i \frac{\sum_{j=1}^K \bar{r}_j^*}{\sum_{j=1}^K \beta_{ij}^* \bar{r}_j^*} \quad (14)$$

that also respects conditions C1 to C7. \square

This expression for GT_i has an interpretation that directly relates to the original Excess Treynor ratio: it provides the abnormal return of portfolio i per unit of premium-weighted average systematic risk. It reduces to the ET_i for a single index model. Notice that the required excess return is constrained to be positive for both the portfolio under study and the benchmark.

The absence of a benchmark portfolio could still lead to an interpretable measure, but its usefulness could only be sustained within a particular version of the asset pricing model. Then, the dependence of GT_i on the scale of the risk premia forbids any cross-model comparison, while setting the numerator of the Generalized Treynor ratio to a constant for all portfolios would violate condition C5 and make comparisons of securities performance across time impossible.

The key to the geometric argument underlying this Proposition is to normalize the risk referential with orthogonal, normalized axes and to pick a portfolio that has the same

coordinates on every axis. The output of this procedure is such that it allows to switch to the asset pricing model, even if risk premia are not independent.

A crucial property of this model is the perfect independence of the measure with the choice of model specification, provided the compared models are versions each other, i.e. they provide exactly the same required returns for all portfolios. This is verified because the numerator and denominator of the ratio in equation (7) are both independent of the model parametrization provided that it accounts for the same risk dimensions. A test of equalities of the Generalized Treynor ratio for all portfolios could be more relevant to test the consistency of different asset pricing models than a direct test on the Jensen's alphas, obviously less precise.

One may wonder what is the interest of deriving such a simple formula, and especially why the financial literature has never emphasized its usefulness. The reason probably lies in fact that such a formula proposed without proper justification of its applicability would be useless for the financial community. Since no performance measure is acceptable if it is not accepted by the whole funds industry, its isolate adoption would just be taken as another episode in the self-promotion of funds managers.

Through this Proposition, the Generalized Treynor ratio is justified on geometrical as well as analytical grounds. The reduction of a multi-dimensional space into one single distance measure had not yet been performed, probably because it is not natural to consider that portfolios with very different risk profiles but with the same overall exposure to the sources of risk priced by the market could share a single unidimensional risk measure. Such a simplification might be considered as arbitrary, as there are many ways to realize this projection. As a matter of fact, the Generalized Treynor ratio is not the only measure satisfying all desirable technical conditions – probably every distance measure satisfying (11) would lead to a satisfactory solution – but it is the simplest one.

III. Conclusion

This article proposed to generalize the Treynor ratio in a multi-index setup through the recourse to a geometric argument: since the original measure represents a proportion

of distances in the returns and in the systematic risk referentials, a multi-dimensional counterpart can be justified in an orthonormed risk referential. From that starting point, the proposed Generalized Treynor ratio represents the simplest measure that bears this interpretation and, at the same time, manages to conserve the key properties of its one-dimensional counterpart. For a given portfolio, its formula simplifies to a simple ratio of Jensen's alpha over an average of the betas.

Unlike many finance papers, this note has no ambition to be technically involved, but aims at filling a strange gap in the performance evaluation literature. The lack of risk-normalized performance measure for multi-factor models had increasingly become a painful anomaly in the current blossoming of empirical studies focusing on mutual and hedge funds performance. By allowing a safe comparability of performances across risk exposures but also across models and units, the simple formula for the Generalized Treynor ratio has the potential to create more objective classification procedures. We believe that the simple design of these measures, instead of being a hinderance for its adoption, could be the best guarantee of a wide use in the financial community, whether academic or professional.

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