Risk Aversion, Strategic Trading and Mandatory Public Disclosure

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Abstract

This paper studies the optimal dynamic behavior of a risk-averse insider when regulation requires insider to publicly disclose his trades after trades complete. In a fixed time horizon, when trades happen less frequently, mandatory disclosure regulations accelerate price discovery process, and increase market liquidity. If trades happen more frequently, mandatory disclosure requirements slow down price discovery process; and contrary to the case where there is no disclosure requirement, in which the risk-averse agent trade more aggressively at the beginning while less aggressive in the end, when there are disclosure requirements, risk-averse insider trades constantly through time, thus the market is more liquid at the beginning while less liquid near the end than the case without disclosure.

[Keywords] insider trading, risk aversion, securities regulation, price discovery, liquidity.

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1. Introduction

It is generally believed that public disclosure of insider trading reduces trading costs because market makers face less severe adverse selection when insiders disclose their trades and thus set narrower spreads. Carlton and Fischel (1983, p. 868) expresses their belief about public disclosure of insider trading: “The greater the ability of market participants to identify insider trading, the more information such trading will convey”. So US securities laws require insiders associated with a firm to report any equity transactions they make in the stock of that firm to the Securities and Exchange Commission(SEC). The reports are filed after the trade is completed and are immediately available for the public once SEC receives them. However, there are empirical evidences that trading by insiders appears to earn abnormal returns (Jaffe (1974), Seyhum(1986), Meulbroek (1992), Seyhum (1992a)). Stiffer insider trading regulations and enforcement do not seem to curb insiders’ profits (Seyhum (1992b)).

To attempt to explain why even under stiffer regulations insiders continue to earn increasingly profits, a trading model with a risk-averse insider is developed to study the effect of after-trade mandatory disclosure regulations on insider trading. Trades happen as in Kyle (1985) except that the insider has to disclose his trade after his order being executed. We find that when the risk-averse informed trader faces mandatory disclosure regulations, there exists an equilibrium in which the informed trader uses mixed strategies by adding noises to conceal his private information. Following Huddart, Hughes and Levine (2001), we call the behavior of adding noises “dissimulation”. We compare the case where risk-averse insider faces mandatory disclosure requirements to the case in which the risk-averse insider does not need to disclose his trades ex post, which is studied in Holden and Subrahmanyam (1994). We show that the goal of US securities laws may not be achieved: When the number of trading rounds per unit time is small, mandatory disclosure regulations speed up the price discovery process but when the number of trading rounds per unit time is sufficient large, mandatory disclosure requirements slow down the price discovery process. Moreover, when there are no disclosure regulations, the risk-averse insider trades more aggressively in the first several rounds.

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1. Insiders are defined as officers, directors, and beneficial owners of more than 5% of any class of equity securities.
2. Section 16(a) of the US Securities Exchange Act of 1934 imposes a disclosure requirement on insiders that they must report their trades to SEC within ten days following the end of the month in which the trade occurs. Section 13(d) added to the 1934 Act in 1968 by the William Act and amended in 1970 requires any individual with 5% or more of a firm’s stock to report it within ten days. Subsequent material changes to shareholdings must also be disclosed in ten days.
3. Jaffe (1974) and Seyhum (1986) find that insider tend to buy before price increase abnormally and tend to sell before price decline abnormally. Meulbroek (1992) presents evidence that the abnormal return on insider trading is half as large as the price reaction to the public revelation of the information on which the insider trades. Seyhum (1992a) shows that insider trading in their own firms predicts up to 60% of the return variations.
4. Seyhum (1992b) finds that the increased statutory sanctions in the 1980s did not deter insider trading, nor did they decrease the profitability of insider trading.
trading rounds and less aggressively in the last trading rounds, thus the market is much less liquid at the beginning than in the end. On the contrary, when the risk-averse insider are required to disclose his trades ex post, he trades constantly through time, thus the market liquidity remains the same across periods. In addition, when there are fewer trades per unit time, market is more liquid with disclosure regulations; but when there are more trades happening per unit time, market with disclosure requirements is more liquid at the beginning while less liquid near the end than market without disclosure regulations.

The study of insider trading is profoundly deepened by a seminal paper of Kyle (1985). After that, Admati and Pfleiderder (1988), Back (1992), Holden and Subrahmanyam (1992), Foster and Viswanathan (1994), and Rochet and Vila (1994) study insider trading in different situations.\(^5\) More recently, Foster and Viswanathan (1996), Massoud and Bernhardt (1999), Back, Cao and Willard (2000), and Huddart, Hughes and Williams (2000) expand our understanding of insider trading. Since disclosure is believed to reduce trading cost, there are several papers studying voluntary disclosure. Bushman and Indjejikian (1995), and Shin and Singh (1999) conclude that insiders voluntarily disclose their private information to erode other insiders’ superior information. Verrecchia (1983), and Narayanan (2000) show that capitalization of the firm may give insiders incentives to disclose voluntarily. Gregoire (2003) models a multi-asset environment in which insiders voluntarily give up (part of) their information advantage to attract liquidity traders.

There are three recent papers related to our study. Fishman and Hagerty (1995) model a market where the potential insider may or may not be informed and the uninformed insider may have incentives to imitate the informed insider. Market maker is not able to distinguish uninformed insider from informed insider so that combining disclosed information he sets a wider bid-ask spread and thus hurts outsiders. John and Narayanan (1997) reconsider Fishman-Hagerty model for the case where the insider is certainly informed and the probability of good and bad news is different. They show that the mandatory disclosure regulations may create incentives for informed trader to manipulate the market by trading against his information when the market expects good (bad) news and the insider receives good (bad) news. Though Fishman and Hagerty (1995) and John and Narayanan (1997) come to similar conclusions as ours, our paper is different from these two since (a) both papers assume the value of the risky asset takes only two values, while the risky asset in our paper is normally distributed; (b) insider in these two paper can only buy or sell one share, while we allow insider to trade any amount of the risky asset. More importantly, our paper is different from Fishman and Hagerty in that the insider in our model is always informed and he “dissimulates” not because he attempts to pool with anyone but because he wants to preserve his private information. The main difference between our paper and John and Narayanan is that dissimulation always happens in our model no matter what private information he has, while manipulation does not occur when

good and bad news are equally possible or when the insider’s private information is opposite to what
the market expects.

Huddart et al. (2001) introduce the mandatory disclosure requirements to Kyle’s (1985) dynamic
trading game and find that insider uses a mixed strategy to prolong his information advantage.
Our paper is different from Huddart et al. (2001) in that the informed trader in their model is risk
neutral and mandatory disclosure regulations accelerate price discovery process and increase market
liquidity. In our paper, the insider is risk averse and mandatory disclosure rules may slow down
price discovery process and decrease market liquidity.

The remainder of the article is organized as follows. In section 2, we describe the basic structure
of our model. Section 3 first characterizes the linear equilibrium in a special case of two-period model
and then extends to multi-period analysis. In section 4, we compare our model with the case where
the risk-averse insider needs not to report his trades after order executions. Section 5 concludes.

2. Model Structure

There is one risky asset in the market with a liquidation value \( v \), which is normally distributed
with mean \( \mu_0 \) and variance \( \Sigma_0 \). In a time interval from 0 to 1, the risky asset is traded in \( N \) periods,
with \( t_n \) denoting the time at which the \( n \)-th period ends. We assume \( 0 = t_0 < t_1 < \cdots < t_N = 1 \).
There are three types of traders: a risk-averse informed trader, a risk-neutral market maker
\(^6\) and
several “noise” traders. The risk-averse informed trader is endowed with initial wealth \( W_0 \). Let \( W_n \)
denote the wealth at the end of \( n \)-th period. Follow Holden and Subrahmanyam (1994), the informed
trader has negative exponential utility over his terminal wealth, with coefficient of constant absolute
risk aversion (CARA) \( A \), that is,

\[
U(W_N) = -\exp\{-AW_N\}
\]

The informed trader learns \( v \) before first period. Let \( x_n \) denote the aggregate position of the informed
trader after \( n \)-th period, so that \( \Delta x_n \) (defined by \( \Delta x_n = x_n - x_{n-1} \)) denotes the quantity traded
by the informed trader at the \( n \)-th period. The insider chooses his optimal demand for the risky
asset, \( \Delta x_n \), at the start of \( n \)-th period. Besides the informed trader, the noise traders also submit
their orders for exogenous reasons\(^7\) that are not modelled here. The aggregate quantity traded by
noise traders at the \( n \)-th period, denoted \( \Delta u_n \), is normally distributed with mean zero and variance
\( \sigma^2 u \Delta t_n \), where \( \Delta t_n = t_n - t_{n-1} \). We assume \( \Delta u_n \) is serially uncorrelated. The market maker observes
the total order flow but does not know which orders come from which traders when he sets the price

\(^6\)The assumption of market maker being risk neutral makes it possible to derive analytical solutions. Another reason
to make this assumption is that market making is usually performed by large financial institutions. These institutions
are thought to have large capacity to bear risk. Thus it is reasonable to model market makers as risk-neutral agents.

\(^7\)The reasons could be portfolio rebalancing or tax avoidance.
for trade in current period. After order executed in current period, but before the next period, the informed trader discloses his trade to the public by the requirement of the mandatory disclosure regulations.

The risk-averse informed trader maximizes his expected utility by backward induction to determine his optimal trading strategy given his conjectures about the trading strategies of the other players. In equilibrium, the conjectures of each players must be correct conditional on each trader’s information set at each period. That is, the risk-averse informed trader is maximized his expected utility:

$$\max_{\{\Delta x_1, \ldots, \Delta x_2\}} E[-\exp\{-A[W_0 + \sum_{n=1}^{N} \Delta x_n(v - p_n(\Delta x_1, \ldots, \Delta x_{n-1}, \Delta x_n + \Delta u_n)))]\}[v]$$

We follow the tradition in the literature that the market maker is assumed to set informationally efficient price \(p_n\) after observing the total order flow, \(y_n = \Delta x_n + \Delta u_n\), in \(n\)-th period; thus, his expected profit is zero. Specifically,

\[p_n = E[v|y_n, \Delta x_1, \ldots, \Delta x_{n-1}]
\]

After the trades of informed trader go public, the market maker update his belief \(p^*_n\) before \((n+1)\)-th auction, using the information contained in \(\Delta x_1, \ldots, \Delta x_n\), that is,

\[p^*_n = E[v|\Delta x_1, \ldots, \Delta x_{n-1}, \Delta x_n]
\]

While \(p^*_n\) is not a part of the general requirement of an equilibrium, it is helpful for us to identify the linear equilibrium we focus on.

We follow Kyle (1985) in our definition of equilibrium. Just before period \(n\), the informed trader’s private information consists of true value of the asset, \(v\), total order flows and his own orders up to \((n-1)\)th period, while his own orders are sufficient statistics of total order flows. Let

\[\Delta x_n = \Delta X_n(v, x_1, \ldots, x_{n-1})\]

represent the optimal strategy of the informed trader.

Let

\[p_n = P_n(y_n, x_1, \ldots, x_{n-1})\]

represent the optimal strategy of the market maker.

Let \(X = (X_1, \ldots, X_N)\) and \(P = (P_1, \ldots, P_N)\) represent the vector of strategy functions. Let \(W_n\) denote the wealth of the risk-averse informed trader at the end of \(n\)-th period and \(J(W_n)\) denote the indirect utility from \(W_n\), then the indirect utility in \(n\)-th period, \(J(W_n)\) is defined as

\[J(W_n(X, P)) = E_n[-\exp\{-A[W_n-1 + \sum_{k=n}^{N} (v - p_k)\Delta x_k)]\}]

\[\text{We can assume there are Bertrand competition between at least two identical risk neutral market makers.}\]
among which, $E_n$ is the conditional expectation operator given insider’s information set at the start of $n$-th period.

The following definition defines formally the equilibrium in our model.

**Definition 1** A Bayesian Nash equilibrium of the trading game is given by the vector of strategies $(X, P)$ such that:

1. **Expected Utility Maximization:** For all $n = 1, \cdots, N$ and for all $X' = (X'_1, \cdots, X'_N)$ such that $X'_1 = X_1, \cdots, X'_{n-1} = X_{n-1}$, we have
   \[ J(W_n(X, P)) \geq J(W_n(X', P)) \]

2. **Market Efficiency:** For all $n = 1, \cdots, N$, we have
   \[ p_n = E[v|y_n, \Delta x_1, \cdots, \Delta x_{n-1}] \]

We are interested in finding the optimal trading strategies for the informed trader and the pricing rules, in particular, we are interested in a linear equilibrium, in which pricing rules are linear as a consequence of the Projection Theorem for normally distributed random variables.

### 3. Characterization of Linear Equilibrium

#### 3.1 A Special Case of Two-Period Model

Now we look at a special case of two-period model to give us insight about the equilibrium. In the first period, the informed trader chooses his optimal $\Delta x_1$, the noise traders submit their orders and the market maker observes total order flow, $\Delta x_1 + \Delta u_1$, without knowing which orders come from which traders and he sets the price $p_1$ for the first period. After trades in first period clear, the informed trader discloses $\Delta x_1$ and the market maker updates his belief of the true value of the asset. In the second period, the informed trader chooses his optimal $\Delta x_2$, noise traders submit their demand $\Delta u_2$ and the market maker observes the total order flow and set the price to be the expected value of the asset conditional on the information available to him. Trades happen and the risky asset liquidates at the end of second period.

The risk-averse informed trader’s objective in a two-period setting becomes

\[
\max_{\Delta x_1, \Delta x_2} E_{\Delta u_1, \Delta u_2} \left[ -\exp\left\{ -A[W_0 + \Delta x_1(v - p_1(\Delta x_1 + \Delta u_1)) + \Delta x_2(v - p_2(\Delta x_1, \Delta x_2 + \Delta u_2))] \right\} | v \right]
\]

In the first period, the market maker sets price and updates his belief according to:

\[
p_1 = E[v|\Delta x_1 + \Delta u_1]
\]

\[
p^*_1 = E[v|\Delta x_1]
\]
In second period, market clearance price is

\[ p_2 = E[v|p_1^*, \Delta x_2 + \Delta u_2] = E[v|\Delta x_1, \Delta x_2 + \Delta u_2] \]

We make the assumption that auctions happen evenly through time, i.e. \( \Delta t_1 = \Delta t_2 = \Delta t \). We solve the equilibrium by backward induction. Though there is disclosure requirement, at the end of second period the risky asset liquidates and everybody knows everything, there is no incentive for the informed trader to “disguise” his information, so he uses a pure strategy, just as the case where there are not regulations of disclosure in Holden and Subrahmanyam (1994). We propose that the market maker use a linear pricing rule of this form:

\[ p_2 = p_1^* + \lambda_2(\Delta x_2 + \Delta u_2) \]

where \( p_1^* \) is the market maker’s conditional expectation of the risky asset at the beginning of period 2. Given his wealth \( W_1 \) at the start of second period and the linear pricing rule of the market maker, the insider maximizes his expected utility,

\[
J(W_2) = \max_{\Delta x_2} E_2 \left[ -\exp \{-A[W_1 + \Delta x_2(v - p_2)]\} \right] = \max_{\Delta x_2} E_2 \left[ -\exp \{-A[W_1 + \Delta x_2(v - p_1^* - \lambda_2(\Delta x_2 + \Delta u_2))]\} \right] = \max_{\Delta x_2} \left\{ \frac{1}{2} \Delta x_2^2 \lambda_2^2 \sigma_u^2 \Delta t \right\}
\]

Take first order condition respect to \( \Delta x_2 \), we get the optimal demand for the insider in last period

\[
\Delta x_2 = \frac{1}{2\lambda_2 + A\lambda_2^2 \sigma_u^2 \Delta t} (v - p_1^*)
\]

That is, given a linear pricing rule, the insider uses a trading strategy that is linear in his information advantage. Denote the coefficient of information advantage as \( \beta \Delta t \), we get

\[
\beta_2 = \frac{1}{(2\lambda_2 + A\lambda_2^2 \sigma_u^2 \Delta t)\Delta t}
\]

(1)

The second order condition is

\[ 2\lambda_2 + A\lambda_2^2 \sigma_u^2 \Delta t \geq 0 \]

For market maker, he sets the price to be the conditional expectation of the value of the asset. Define \( \Sigma_2 \) and \( \Sigma_1 \) by \( \Sigma_2 = Var(v|\Delta x_1, \Delta x_2 + \Delta u_2) \) and \( \Sigma_1 = Var(v|\Delta x_1) \), respectively. Given the insider’s linear trading strategy, \( \Delta x_2 = \beta_2(v - p_1^*)\Delta t \), by applying the projection theorem for normally distributed random variables, we can see that the pricing rule used by market maker is actually linear.

\[
p_2 = \frac{E[v|p_1^*, \Delta x_2 + \Delta u_2]}{2\lambda_2 + A\lambda_2^2 \sigma_u^2 \Delta t} = \frac{E[v|p_1^*] + \frac{Cov(v, \Delta x_2 + \Delta u_2[p_1^*])}{Var(\Delta x_2 + \Delta u_2[p_1^*])} (\Delta x_2 + \Delta u_2)}{2\lambda_2 + A\lambda_2^2 \sigma_u^2 \Delta t}
\]
thus,

\[
\lambda_2 = \frac{\beta_2 \Delta \Sigma_1}{\Sigma_2} = \frac{\beta_2 \Delta^2 \Sigma_1 + \sigma_u^2 \Delta t}{\Sigma_2}
\]

\[
\Sigma_2 = \Sigma_1 - \frac{\beta_2 \Delta^2 \Sigma_1^2}{\beta_2 \Delta^2 \Sigma_1 + \sigma_u^2 \Delta t}
\]

Simplify above two expressions and we get

\[
\lambda_2 = \frac{\beta_2 \Sigma_2}{\sigma_u \Delta t}
\]

\[
\Sigma_1 = \frac{\Sigma_2}{1 - \beta_2 \lambda_2 \Delta t}
\]

In addition,

\[
J(W_2) = E_2 \left[ - \exp \left\{ -A(W_1 + \Delta x_2(v - p_2)) \right\} \right]
\]

\[
= E_2 \left[ - \exp \left\{ -A(W_1 + \beta_2(v - p_2^t) \Delta t(v - p_2^t) - \lambda_2 \beta_2 \Delta t(v - p_2^t) - \lambda_2 \Delta w_2) \right\} \right]
\]

\[
= - \exp \left\{ -AW_1 - A \left[ \beta_2 \Delta t(1 - \lambda_2 \beta_2 \Delta t) - \frac{1}{2} A \beta_2^2 \lambda_2^2 \sigma_u^2 \Delta t^2 \right] (v - p_2^t)^2 \right\}
\]

\[
= - \exp \left\{ -A(W_1 + \alpha_2(v - p_2^t)^2) \right\}
\]

Thus

\[
\alpha_2 = \beta_2 \Delta t(1 - \lambda_2 \beta_2 \Delta t) - \frac{1}{2} A \beta_2^2 \lambda_2^2 \sigma_u^2 \Delta t^3
\]

Simplifying \(\alpha_2\) we get

\[
\alpha_2 = \frac{1}{2\lambda_2 (2 + A \lambda_2 \sigma_u^2 \Delta t)}
\]

In the first period, because of the disclosure requirements, the informed trader’s strategy similar to that he uses in the second period is not in equilibrium. To see it, suppose that the informed trader follows the strategy \(\Delta x_1 = \beta_1(v - p_0)\Delta t\), then he would surrender his private information when he discloses his order to meet requirements of disclosure regulations. More specifically, after observing \(\Delta x_1\), the market maker would infer \(v = \Delta x_1/(\beta_1 \Delta t) + p_0\). Therefore, he would set the price in second period as \(p_2 = \Delta x_1/(\beta_1 \Delta t) + p_0\) and \(\lambda_2 = 0\). However, the informed trader would understand the strategy of the market maker and he would not follow \(\Delta x_1 = \beta_1(v - p_0)\Delta t\).

Using the concept of dissimulation introduced by Huddart et al (2001), we show there exits an equilibrium in which the informed trader follows trading strategies that consist of an information-based linear component and a noise component, that is \(\Delta x_1 = \beta_1(v - p_0)\Delta t + z_1\), where \(z_1\) is normally distributed with mean 0 and variance \(\sigma^2_{z_1}\), and \(z_1\) is independent with \(\Delta u_1\) and \(v\); the market maker follows the pricing rule \(p_1 = p_0 + \lambda_1(\Delta x_1 + \Delta u_1)\), and update his belief of the value of the asset using \(p_1^* = p_0 + \gamma_1 \Delta x_1\) after observing the disclosed demand from informed trader.
From the second period, we have

\[ J(W_2) = - \exp \{-A[W_1 + \alpha_2(v - p_1^\ast)]\} \]

Then

\[
J(W_1) = \max_{\Delta x_1} E_1 [J(W_2)] \\
= \max_{\Delta x_1} E_1 \left[ - \exp \left\{-A[W_0 + \Delta x_1(v - p_0) + \alpha_2(v - p_1^\ast)]\right\} \right] \\
= \max_{\Delta x_1} E_1 \left[ - \exp \left\{-A[W_0 + \Delta x_1(v - p_0 - \lambda_1 \Delta x_1 - \lambda_1 \Delta u_1) \\
+ \alpha_2(v - p_0 - \gamma_1 \Delta x_1)]\right\} \right] \\
= \max_{\Delta x_1} - \exp \left\{-A[W_0 + \Delta x_1(v - p_0 - \lambda_1 \Delta x_1) + \alpha_2(v - p_0 - \gamma_1 \Delta x_1)]\right\} \\
\exp \left\{ \frac{1}{2} A^2 \Delta x_1^2 \lambda_1^2 \sigma_1^2 \Delta t \right\} \\
\]

Take the first order condition with respect to \( \Delta x_1 \), then we have

\[
(2\lambda_1 - 2\alpha_2 \gamma_1^2 + A\lambda_1^2 \sigma_1^2 \Delta t) \Delta x_1 + (2\alpha_2 \gamma_1 - 1)(v - p_0) = 0 \\
\]

The second order condition is

\[
2\lambda_1 - 2\alpha_2 \gamma_1^2 + A\lambda_1^2 \sigma_1^2 \Delta t \leq 0 \\
\]

If our proposed mixed trading strategy \( \Delta x_1 = \beta_1 (v - p_0) \Delta t + z_1 \) is to hold in equilibrium, then the insider must be indifferent across all values of \( \Delta x_1 \). Then we have

\[
\gamma_1 = \frac{1}{2\alpha_2} \\
A \sigma_1^2 \Delta t \lambda_1^2 + 2\lambda_1 - \gamma_1 = 0 \\
\]

(5)  
(6)

For the market maker, again, she sets the price to be the conditional expectation of the value of the asset.

\[
\lambda_1 = \frac{\beta_1 \Delta t \Sigma_0}{\beta_1^2 \Delta t^2 \Sigma_0 + \sigma_1^2 \Delta t + \sigma_1^2} \\
\]

(7)

After observing \( \Delta x_1 \), the market maker update her belief of the value of the asset according to \( p_1^\ast = p_0 + \gamma_1 \Delta x_1 \). Simple application of projection theorem for normally distributed random variables we get

\[
\gamma_1 = \frac{\beta_1 \Delta t \Sigma_0}{\beta_1^2 \Delta t^2 \Sigma_0 + \sigma_1^2} \\
\Sigma_1 = \Sigma_0 - \frac{\beta_1^2 \Delta t^2 \Sigma_0^2}{\beta_1^2 \Delta t^2 \Sigma_0 + \sigma_1^2} \\
\]

(8)  
(9)

Combining Eq. (7) and Eq. (8) we get

\[
\beta_1 = \frac{\Sigma_1 - \Sigma_0}{\lambda_1 - \gamma_1} + \gamma_1 \sigma_1^2 \Delta t \\
\]

(10)
Simplifying Eq. (8) and Eq. (9) we get
\[ \sigma_{z_1}^2 = \frac{\beta_1 \Sigma_1 \Delta t}{\gamma_1} \]  
\[ \Sigma_0 = \frac{\Sigma_1}{1 - \beta_1 \gamma_1 \Delta t} \]

Eq. (1)-Eq. (6), Eq. (10) and Eq. (12) form a system of equations with 8 unknowns: \( \lambda_2, \lambda_1, \beta_2, \beta_1, \Sigma_2, \Sigma_1, \alpha_2 \) and \( \gamma_1 \). The solution to this system of equations leads to the following proposition that characterizes our equilibrium in the two-period settings:

**Proposition 1** For a risk-averse informed trader, in the setting with public disclosure of insider trades, a Bayesian Nash equilibrium exists in which

\[ \Delta x_1 = \beta_1 (v - p_0) \Delta t + z_1 \]  
\[ \Delta x_2 = \beta_2 (v - p_1^*) \Delta t \]  
\[ p_1 = p_0 + \lambda_1 (\Delta x_1 + \Delta u_1) \]  
\[ p_2 = p_1^* + \lambda_2 (\Delta x_2 + \Delta u_2) \]  
\[ p_1^* = p_0 + \gamma_1 \Delta x_1 \]  
\[ \lambda_1 = \lambda_2 \]  
\[ \gamma_1 = \frac{A \sigma_{z_1}^2 \Delta t \lambda_1^2 + 2 \lambda_1}{2 \gamma_1 \Delta t} \]  
\[ \beta_1 = \frac{1}{2 \gamma_1 \Delta t} \]  
\[ \beta_2 = \frac{1}{\gamma_1 \Delta t} \]  
\[ \sigma_{z_1}^2 = \frac{\Sigma_0}{4 \gamma_1^2} \]  
\[ \Sigma_1 = \frac{1}{2} \Sigma_0 \]  

in which \( \lambda_1 \) is a unique positive solution to the polynomial

\[(A \sigma_{z_1}^2 \Delta t)^2 \lambda_1^4 + 4A \sigma_{z_1}^2 \Delta t \lambda_1^3 + 4 \lambda_1^2 - \frac{4 \Sigma_0}{\Sigma_1} - \frac{\Sigma_0}{2 \sigma_{z_1}^2 \Delta t} = 0\]

(24)

\( z_1 \) is normally distributed with mean 0 and variance \( \sigma_{z_1}^2 \), and \( z_1 \) is independent with \( \Delta u_1 \) and \( v \).

**Proof** See Proof 1 in Appendix.

From Proposition 1, we can see some obvious properties of the equilibrium, such as, market depths are the same across two period, that is, \( \lambda_1 = \lambda_2 \), so that liquidity traders are indifferent with the timing of their trades if they were given discretion to allocate their demands across different periods, as in Admati and Pfleiderder (1988); one-half of the insider’s private information is learned by the market maker after insider’s disclosure at the end of the first period; insider trades more aggressively in the second period. Corollary 1 provide more properties of the equilibrium.
Corollary 1  (a) Insider optimizes by setting the total variance of his demand less than the variance of liquidity traders, that is, \(1 \frac{1}{\Delta t} \Sigma_0 + \sigma^2_{z_1} < \sigma^2_u \Delta t\). (b) Insider chooses his optimal noise by setting the variance of the noise equal to the variance of the information-based linear component.

**Proof**  See Proof 2 in Appendix.

### 3.2 Multi-Period Analysis

The nice properties can be generalized to a \(N\)-period model. We assume trades happen evenly through time, i.e. \(\Delta t_1 = \cdots = \Delta t_N\).

**Proposition 2** In a \(N\)-period model where there is a risk-averse informed trader and there are disclosure requirements, there exists a Bayesian Nash equilibrium in which, given \(p^*_0 \equiv p_0\),

For \(n = 1, \cdots, N-1\),

\[
\Delta x_n = \beta_n(v - p^*_{n-1})\Delta t + z_n \tag{25}
\]
\[
p_n = p^*_{n-1} + \lambda_n(\Delta x_n + \Delta u_n) \tag{26}
\]
\[
p^*_n = p^*_{n-1} + \gamma_n \Delta x_n \tag{27}
\]
\[
\Sigma_n = \text{Var}(v|x_1, \cdots, x_n) \tag{28}
\]

where \(z_n\) is a normally distributed with mean zero and variance \(\sigma^2_{z_n}\), which is independent with \(\Delta u_n\) and \(v\). In the last period

\[
\Delta x_N = \beta_N(v - p^*_{N-1})\Delta t \tag{29}
\]
\[
p_N = p^*_{N-1} + \lambda_N(\Delta x_N + \Delta u_N) \tag{30}
\]

Among which,

\[
\lambda_1 = \cdots = \lambda_N = \lambda \tag{31}
\]
\[
\gamma_1 = \cdots = \gamma_{N-1} = \gamma = A\sigma^2_u\Delta t \lambda^2 + 2\lambda \tag{32}
\]
\[
\beta_n = \frac{1}{(N - n + 1)\gamma\Delta t} \tag{33}
\]
\[
\sigma^2_{z_n} = \frac{(N - n)\Sigma_0}{N(N - n + 1)\gamma^2} \tag{34}
\]
\[
\Sigma_n = \frac{N - n}{N} \Sigma_0 \tag{35}
\]

\(\lambda\) is the unique positive solution to the polynomial

\[
(A\sigma^2_u\Delta t)^2\lambda^4 + 4A\sigma^2_u\Delta t\lambda^2 + 4\lambda^2 \frac{A\Sigma_0}{N} - \frac{\Sigma_0}{N\sigma^2_u\Delta t} = 0 \tag{36}
\]

**Proof**  See Proof 3 in Appendix.

**Proposition 3** With public disclosure, when number of trading rounds per unit time goes to infinity, market depth, \(\lambda\), in the case of risk-averse insider, converges to market depth in the case of risk-neutral insider.
Proof. See Proof 4 in Appendix.

4. Properties of the Equilibrium

As we can see in Proposition 2, the liquidity parameters, $\lambda$’s, remain constant across periods. The intensitivity of learning from the mandatory disclosure of informed trader’s orders for the market maker is also constant across periods. More strikingly, information is released to the market constantly, $\Sigma_{n−1} − \Sigma_n = \frac{1}{\sigma_0^2}$. This result is the same as the case where insider is risk neutral.

We are also interested in when the insider is risk averse, how market depth and price efficiency change when the number of trading periods increases and when the insider becomes more risk averse, and how much room the informed trader has to choose the noise he adds to the linear informational component. In particular, we are interested in how the equilibrium in our model compares to the case of a risk-averse information monopolist with no mandatory disclosure regulations, which is studied in Holden and Subrahmanyam (1994). However, the difference equation system in Holden and Subrahmanyam (1994) does not allow the derivation of analytical solutions. Thus we resort to numerical methods for comparison.

In order to make comparisons with Holden and Subrahmanyam (1994), we first assume that $A = 4$, $\Sigma_0 = 1$ and $\sigma_u^2 = 1$. Figure 1 plots the information stock, $\Sigma_n$, when insider faces and does not face mandatory disclosure regulations for cases where there are 2, 4, 5, 6, 20 and 100 trading rounds. We can see from figure 1 that when there are a few trading rounds in a fixed time horizon, in our numerical examples when there are less than 5 trading rounds per unit time, information is released more quickly when insider faces disclosure regulations, i.e. disclosure increases market efficiency, as in the case where insider is risk neutral in Huddart et al (2001). However, as trading rounds per unit time increase, disclosure regulations cause information stock of price to decrease at a slower rate in the first few rounds. When the number of trading rounds becomes large, in our numerical simulations, when there are more than 14 trading rounds, price is always more informative when there are no disclosure regulations. When the number of trading rounds becomes very large, the difference between the error variances with and without disclosure grows wider. Thus when trades happen frequently enough, disclosure regulations slow down price discovery process.

Figure 2 plots the liquidity parameter, $\lambda_n$, when insider faces and does not face mandatory disclosure regulations for cases where there are 2, 10, 12, 14, 20 and 100 trade rounds between time 0 and 1. When the number of trading rounds is sufficient small, in our numerical simulations, this number is 10, disclosure requirements of insider trading increase market liquidity, as in the case when insider is risk neutral in Huddart et al (2001). However, as the number of trading rounds increases, insider without facing disclosure regulations tends to trade much more aggressively at the
beginning, causing more severe adverse selection, while trade much less aggressively in the end since his aggressive behavior at the beginning reveals his private information to the market maker. But the insider facing disclosure regulations always trades constantly. When trades happen sufficient often, the market with disclosure regulations is more liquid at the beginning and becomes less liquid near the end.

5. Concluding Remarks

This paper studies the effect of mandatory ex post disclosure regulations on insider trading when the only informed trader in the market is risk averse. As in the case when the insider is risk neutral, the risk-averse insider adds noise to his demand so that he would not reveal his private information after first round of trading. This is called “dissimulation”. Surprisingly, the dynamic behavior of a risk-averse insider facing mandatory disclosure regulations is very similar to that of a risk-neutral insider facing same regulations, but very different from that of a risk-averse insider without facing mandatory disclosure regulations. Our comparison of a risk-averse insider with and without facing disclosure regulations shows that mandatory disclosure regulations may not benefit the liquidity traders. When the insider can keep his private information for a relatively short period, mandatory disclosure benefits the liquidity traders. However, when insider is able to keep his private information for longer period, i.e. he can trade upon his private information many times, mandatory disclosure regulations harm liquidity traders and the market is less efficient. This is contrast to the case when the insider is risk neutral which is studied in Huddart et al (2001).

The purpose of US securities laws requiring insiders to disclose their trader ex post is to protect the “public”, which is mainly composed by liquidity traders. While people generally believe that “The most potent weapon against abuse of insider information is full and prompt publicity”, our paper shows that when the insider is able to keep his private information for relatively long period, that is, when there is no other source to reveal the value of the risky asset, mandatory disclosure regulations may hurt the liquidity traders and the market is deeper when there are no such regulations.

Appendix

Proof 1 (for Proposition 1): First, Eq. (19) is the same as Eq. (6). Now we solve the eight unknowns from Eq. (1)-Eq. (6), Eq. (10) and Eq. (12). Eq. (4) to Eq. (6) gives

\[
\gamma_1 = 2\lambda_2 + A\lambda_2^2\sigma_u^2\Delta t
\]

\[
\gamma_1 = 2\lambda_1 + A\lambda_1^2\sigma_u^2\Delta t
\]

thus

\[
(\lambda_1 - \lambda_2)(2 + A\sigma_u^2\Delta t(\lambda_1 + \lambda_2)) = 0
\]
Since $A\sigma_u^2 \Delta t > 0$, and $\lambda_1, \lambda_2 > 0$, we must have $\lambda_1 = \lambda_2$, which is Eq. (18). Eq. (12) gives

$$\beta_1 \gamma_1 \Delta t = 1 - \frac{\Sigma_1}{\Sigma_0}$$

Multiply Eq. (10) by $\gamma_1 \Delta t$, we get

$$\beta_1 \gamma_1 \Delta t = \frac{\gamma_1 \sigma_u^2 \Delta t}{\lambda_1 - \Sigma_1 - \lambda_2 \gamma_1}$$

Equate above two equations and take the inverse of both sides

$$\frac{\Sigma_1}{\lambda_1} - \frac{\Sigma_1}{\lambda_1} = \frac{1}{1 - \frac{\Sigma_1}{\Sigma_0}}$$

Simplify and we get

$$\frac{1}{\lambda_1} - \frac{1}{\gamma_1} = \frac{\gamma_1 \sigma_u^2 \Delta t}{\Sigma_0 - \Sigma_1} \tag{A-1}$$

From Eq. (1) and Eq. (4), we have $\beta_2 \Delta t = 2\alpha_2 = \frac{1}{\gamma_1}$. Thus Eq. (21) follows. Plug $\beta_2 \Delta t = \frac{1}{\gamma_1}$ into Eq. (2) and Eq. (3), we have

$$\lambda_2 = \frac{\Sigma_2}{\gamma_1 \sigma_u^2 \Delta t} \tag{A-2}$$

$$\Sigma_2 = \Sigma_1 (1 - \frac{\lambda_2}{\gamma_1}) \tag{A-3}$$

respectively. Plug Eq. (A-3) into Eq. (A-2) and simplify, we get

$$\frac{1}{\lambda_1} - \frac{1}{\gamma_1} = \frac{\gamma_1 \sigma_u^2 \Delta t}{\Sigma_1} \tag{A-4}$$

Equate Eq. (A-1) and Eq. (A-4), we have $\Sigma_1 = \frac{1}{2} \Sigma_0$, which is Eq. (23).

From Eq. (12) and $\Sigma_1 = \frac{1}{2} \Sigma_0$, we have $\beta_1 = \frac{1}{2 \gamma_1 \Delta t}$, which is Eq. (20). Combine Eq. (11), Eq. (20) and Eq. (23), we have Eq. (22). From Eq. (10), Eq. (20) and Eq. (23), we have

$$\Sigma_0 \left(\frac{1}{\lambda_1} - \frac{1}{\gamma_1}\right) = 2 \gamma_1 \sigma_u^2 \Delta t$$

Plug in $\gamma_1 = 2 \lambda_1 + A\sigma_u^2 \Delta t \lambda_1^2$, we have

$$\Sigma_0 \left(\frac{1}{\lambda_1} - \frac{1}{2 \lambda_1 + A\sigma_u^2 \Delta t \lambda_1^2}\right) = 2(2 \lambda_1 + A\sigma_u^2 \Delta t \lambda_1^2) \sigma_u^2 \Delta t$$

Simply it and we get Eq. (24). Denote the four roots to this polynomial as $r_1$ to $r_4$, then we have

$$r_1 + r_2 + r_3 + r_4 = -\frac{4}{A\sigma_u^2 \Delta t} < 0$$

$$r_1 r_2 + r_1 r_3 + r_1 r_4 + r_2 r_3 + r_2 r_4 + r_3 r_4 = \frac{4}{(A\sigma_u^2 \Delta t)^2} > 0$$

$$r_1 r_2 r_3 + r_1 r_3 r_4 + r_1 r_2 r_4 + r_2 r_3 r_4 = \frac{A \Sigma_0}{2(A\sigma_u^2 \Delta t)^2} > 0$$

$$r_1 r_2 r_3 r_4 = \frac{-\Sigma_0}{2\sigma_u^2 \Delta t (A\sigma_u^2 \Delta t)^2} < 0$$
From \( r_1r_2r_3r_4 < 0 \), there is either one negative root or three negative roots. So there are three possible cases: (a) three negative roots, one positive root; (b) one negative root, one positive root, and two complex roots; (c) one negative root, three positive roots. We now prove that case (c) cannot be true. Suppose there are three positive roots, without loss of generality, \( r_2 \) to \( r_4 \) are positive. We have 
\[
-r_1 > r_2 + r_3 + r_4 \quad \text{from } \Sigma_{i=1}^4 r_i < 0. \]
Then 
\[
-r_1 > r_2 \Rightarrow -r_1r_3 > r_2r_3 \\
-r_1 > r_3 \Rightarrow -r_1r_4 > r_3r_4 \\
-r_1 > r_4 \Rightarrow -r_1r_2 > r_4r_2
\]
Add up above three inequality, we have 
\[
r_1r_2 + r_1r_3 + r_1r_4 + r_2r_3 + r_2r_4 + r_3r_4 < 0. \]
Contradiction! There is a unique positive root to this polynomial. \( \square \)

Proof 2 (for Corollary 1): (a) The total variance chosen by the insider of his demand in first period is \((\beta_1 \Delta t)^2 \Sigma_0 + \sigma_{2i}^2\). From Proposition 1, Eq. (20) and Eq. (22) gives 
\[
(\beta_1 \Delta t)^2 \Sigma_0 + \sigma_{2i}^2 = \frac{\Sigma_0}{2\lambda_1} = \frac{\Sigma_0}{2(A\sigma_u^2 \Delta t \lambda^2 + 2\lambda)^2}
\]
Transform the polynomial Eq. (24), we get 
\[
\frac{1}{\lambda^2(A\sigma_u^2 \Delta t \lambda + 2)^2} = \frac{2\sigma_u^2 \Delta t}{\Sigma_0(A\sigma_u^2 \Delta t \lambda + 1)}
\]
So 
\[
(\beta_1 \Delta t)^2 \Sigma_0 + \sigma_{2i}^2 = \frac{\sigma_u^2 \Delta t}{A\sigma_u^2 \Delta t \lambda_1 + 1} < \sigma_u^2 \Delta t
\]
(b) is obvious. \( \square \)

Proof 3 (for Proposition 2): We solve this equilibrium by backward induction. Define \( \Sigma_N \) by 
\[
\Sigma_N = Var(v|\Delta x_1, \cdots, \Delta x_{N-1}, \Delta x_N + \Delta u_N). \quad \text{The problem of last period in } N\text{-period model is the same as the one in the two-period model, so we can replace 2 by } N \text{ in Eq. (1)-Eq. (4) and get}
\]
\[
\beta_N = \frac{1}{(2\lambda_N + A\lambda_N^2 \sigma_u^2 \Delta t)\Delta t} \quad (A-5)
\]
\[
\lambda_N = \frac{\beta_N \Sigma_N}{\sigma_u^2} \quad (A-6)
\]
\[
\Sigma_{N-1} = \frac{\Sigma_N}{1 - \beta_N \lambda_N \Delta t} \quad (A-7)
\]
\[
\alpha_N = \frac{1}{2\lambda_N(2 + A\lambda_2 \sigma_u^2 \Delta t)} \quad (A-8)
\]
With the second order condition 
\[
2\lambda_N + A\lambda_N^2 \sigma_u^2 \Delta t \geq 0
\]
and the indirect utility function 
\[
J(W_N) = -\exp\{-A[W_{N-1} + \alpha_N(v - p_{N-1})^2]\}\]
For $n = 1, \cdots, N - 1$, because of the disclosure requirement, the informed trader uses mixed strategies instead of pure strategies. We propose that the insider uses strategies described in Eq. (25), market maker follows the pricing rule as in Eq. (26), and update his belief of the value of the asset as in Eq. (27) after observing the disclosed demand from informed trader.

Make the inductive hypothesis that $J(W_{n+1})$ is given by

$$J(W_{n+1}) = -\mu_{n+1} \exp \left[-A(W_n + \alpha_{n+1}(v - p_n^*))^2 \right]$$

Then

$$J(W_n) = \max_{\Delta x_n} E_n \left[J(W_{n+1})\right]$$

$$= \max_{\Delta x_n} E_n \left[-\mu_{n+1} \exp \{-A[W_{n-1} + \Delta x_n(v - p_n) + \alpha_{n+1}(v - p_n^*)^2]\}\right]$$

$$= \max_{\Delta x_n} E_n \left[-\mu_{n+1} \exp \{-A[W_{n-1} + \Delta x_n(v - p^*_{n-1} - \lambda_n \Delta x_n - \lambda_n \Delta u_n) + \alpha_{n+1}(v - p^*_{n-1} - \gamma_n \Delta x_n)^2]\}\right]$$

$$= \max_{\Delta x_n} -\mu_{n+1} \exp \left\{ -A[W_{n-1} + \Delta x_n(v - p^*_{n-1} - \lambda_n \Delta x_n) + \alpha_{n+1}(v - p^*_{n-1} - \gamma_n \Delta x_n)^2] \right\} \exp \left\{ \frac{1}{2} A^2 \Delta x_n^2 \lambda_n^2 \sigma_u^2 \Delta t^2 \right\}$$

Take the first order condition with respect to $\Delta x_n$ and make the same argument as the one for the first period in the two-period model, we have

$$\gamma_n = \frac{1}{2\alpha_{n+1}}$$

$$A\sigma_u^2 \Delta t \lambda_n^2 + 2\lambda_n - \gamma_n = 0$$

The analysis for market maker for $n = 1, \cdots, N - 1$ is very much the same as the one for the first period in the two-period setting, so we get three equations similar to Eq. (10) - Eq. (12)

$$\beta_n = \frac{\sigma_u^2}{\lambda_n - \frac{\Sigma_n}{\gamma_n} + \gamma_n \sigma_u^2 \Delta t}$$

(A-11)

$$\sigma_{n-1}^2 = \frac{\beta_n \Sigma_n \Delta t}{\gamma_n}$$

(A-12)

$$\Sigma_{n-1} = \frac{\Sigma_n}{1 - \beta_n \gamma_n \Delta t}$$

(A-13)

In addition

$$J(W_n) = E_n \left[-\mu_{n+1} \exp \{-A[W_{n-1} + \Delta x_n(v - p_n) + \alpha_{n+1}(v - p_n^*)^2]\}\right]$$

$$= E_n \left[-\mu_{n+1} \exp \{-A[W_{n-1} + (\beta_n(v - p^*_{n-1}) \Delta t + z_n)(v - p^*_{n-1} - \lambda_n \beta_n(v - p^*_{n-1}) \Delta t - \lambda_n z_n - \lambda_n \Delta u_n) + \alpha_{n+1}(v - p^*_{n-1} - \gamma_n (\beta_n(v - p^*_{n-1}) \Delta t + z_n))^2]\}\right]$$

$$= -\mu_{n+1} \exp \{-AW_{n-1}\} E_n \left[\exp \{a_n(v - p^*_{n-1})^2 + b_n(v - p^*_{n-1}) z_n + c_n(v - p^*_{n-1}) \Delta u_n + d_n \Delta u_n z_n + e_n z_n^2\} \right]$$
where \(a_n, b_n, c_n, d_n, e_n\) are defined as

\[
\begin{align*}
    a_n &= -A\beta_n \Delta t (1 - \lambda_n \beta_n \Delta t) - A\alpha_{n+1} (1 - \beta_n \gamma_n \Delta t)^2 \\
    b_n &= A(2\lambda_n \beta_n \Delta t - 1) + 2A\alpha_{n+1} \gamma_n (1 - \beta_n \gamma_n \Delta t) \\
    c_n &= A\beta_n \lambda_n \Delta t \\
    d_n &= A\lambda_n \\
    e_n &= A\lambda_n - A\alpha_{n+1} \gamma_n^2
\end{align*}
\]

In particular,

\[
E_n \left[ \exp \left\{ b_n (v - p_{n-1}^*) z_n + c_n (v - p_{n-1}^*) \Delta u_n + d_n \Delta u_n z_n + e_n z_n^2 \right\} \right]
\]

\[
= E_n \left[ \exp \left\{ (c_n (v - p_{n-1}^*) + d_n z_n) \Delta u_n + b_n (v - p_{n-1}^*) z_n + e_n z_n^2 \right\} \right]
\]

\[
= E_n \left[ \exp \left\{ \frac{1}{2} (c_n (v - p_{n-1}^*) + d_n z_n)^2 \sigma_u^2 \Delta t + b_n (v - p_{n-1}^*) z_n + e_n z_n^2 \right\} \right]
\]

\[
= E_n \left[ \exp \left\{ (e_n + \frac{1}{2} d_n^2 \sigma_u^2 \Delta t) z_n^2 + [b_n + c_n d_n \sigma_u^2 \Delta t] (v - p_{n-1}^*) z_n + \frac{1}{2} c_n^2 (v - p_{n-1}^*)^2 \sigma_u^2 \Delta t \right\} \right]
\]

Using Lemma 1 in Holden and Subrahmanyan (1994), Let \(Y \sim N(0, \sigma^2)\), Then, \(E[\exp(aY^2 + bY)] = \frac{1}{\sqrt{1 - 2a\sigma^2}} \exp \left( \frac{b^2 \sigma^2}{2(1 - 2a\sigma^2)} \right)\), if \(2a\sigma^2 < 1\); if \(2a\sigma^2 > 1\), the above expectation is infinite, we have

\[
= \frac{1}{\sqrt{1 - 2\alpha_n \sigma_z^2 - d_n^2 \sigma_u^2 \sigma_z^2 \Delta t}} \exp \left\{ (v - p_{n-1}^*)^2 \left[ \frac{1}{2} c_n^2 \sigma_u^2 \Delta t + \frac{(b_n + c_n d_n \sigma_u^2 \Delta t)^2 \sigma_u^2}{2(1 - 2\alpha_n \sigma_z^2 - d_n^2 \sigma_u^2 \sigma_z^2 \Delta t)} \right] \right\}
\]

Thus

\[
J(W_n) = -\frac{\mu_{n+1}}{\sqrt{1 - 2\alpha_n \sigma_z^2 - d_n^2 \sigma_u^2 \sigma_z^2 \Delta t}} \exp \left\{ -AW_n + (v - p_{n-1}^*)^2 \left[ a_n + \frac{1}{2} c_n^2 \sigma_u^2 \Delta t + \frac{(b_n + c_n d_n \sigma_u^2 \Delta t)^2 \sigma_u^2}{2(1 - 2\alpha_n \sigma_z^2 - d_n^2 \sigma_u^2 \sigma_z^2 \Delta t)} \right] \right\}
\]

thus

\[
\alpha_n = -\frac{1}{A} \left\{ a_n + \frac{1}{2} c_n^2 \sigma_u^2 \Delta t + \frac{(b_n + c_n d_n \sigma_u^2 \Delta t)^2 \sigma_u^2}{2(1 - 2\alpha_n \sigma_z^2 - d_n^2 \sigma_u^2 \sigma_z^2 \Delta t)} \right\}
\]

\[
\mu_n = \frac{\mu_{n+1}}{\sqrt{1 - 2\alpha_n \sigma_z^2 - d_n^2 \sigma_u^2 \sigma_z^2 \Delta t}}
\]

A closer inspection much simplifies the above expressions. First,

\[
1 - 2\alpha_n \sigma_z^2 - d_n^2 \sigma_u^2 \sigma_z^2 \Delta t
\]

\[
= 1 - 2\sigma_z^2 \Delta t (2\lambda_n - 2\alpha_{n+1} \gamma_n - A\sigma_u^2 \Delta t \lambda_n^3)
\]

\[
= 1 \quad \text{(by Eq. (A-9) and Eq. (A-10))}
\]

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So we have \( \mu_1 = \cdots = \mu_N = 1 \). In addition,

\[
    b_n + c_n d_n \sigma_u^2 \Delta t = \frac{A(2 \lambda_n \beta_n \Delta t - 1) + 2 A \alpha_{n+1} \gamma_n}{\lambda_n + 1 - \beta_n \gamma_n \Delta t} + (A \beta_n \lambda_n \Delta t) (A \lambda_n) \sigma_u^2 \Delta t
\]

\[
    = \frac{A(2 \lambda_n \beta_n \Delta t - 1) + 2 A \alpha_{n+1} \gamma_n}{\lambda_n + 1 - \beta_n \gamma_n \Delta t} + (A \beta_n \lambda_n \Delta t) \sigma_u^2 \Delta t^2
\]

\[
    = 0
\]

So when \( n = 1, \cdots, N - 1 \)

\[
    \alpha_n = -\frac{1}{A}(a_n + \frac{1}{2} c_n^2 \sigma_u^2 \Delta t)
\]

\[
    = \beta_n \Delta t (1 - \lambda_n \beta_n \Delta t) + \alpha_{n+1} (1 - \lambda_n \beta_n \Delta t)^2 - \frac{1}{2} A \lambda_n^2 \sigma_u^2 \Delta t^3
\]

\[
    = \beta_n \Delta t - \lambda_n \beta_n \Delta t + \frac{1}{2} \gamma_n^2 \sigma_u^2 \Delta t (\beta_n \Delta t)^2 + \alpha_{n+1} - 2 \alpha_{n+1} \gamma_n \beta_n \Delta t + \alpha_{n+1}^2 (\beta_n \Delta t)^2
\]

\[
    = \alpha_{n+1} + (\beta_n \Delta t)^2 (\frac{1}{2} \gamma_n - \lambda_n - \frac{1}{2} A \lambda_n^2 \sigma_u^2 \Delta t)
\]

\[
    = \alpha_{n+1}
\]

Thus \( \alpha_1 = \cdots = \alpha_N \). Since \( \gamma_n = \frac{1}{2 \alpha_{n+1}} \), \( \gamma_1 = \cdots = \gamma_{N-1} \equiv \gamma \). For \( n = 1, \cdots, N - 1 \), from \( \gamma_n = A \sigma_u^2 \Delta t \lambda_n^2 + 2 \lambda_n \) and \( \gamma_n = \gamma_{n+1} \), we have \( \lambda_n = \lambda_{n+1} \). Denote all \( \lambda_n \) as \( \lambda \). Transform Eq. (A-13) and plug in \( \beta_n \)

\[
    \frac{\Sigma_n}{\Sigma_{n-1}} = 1 - \beta_n \gamma \Delta t = 1 - \frac{\alpha_n}{\lambda - \frac{1}{2} A \lambda_n^2 \sigma_u^2 \Delta t} \gamma \Delta t = \frac{\Sigma_n (\frac{1}{\lambda} - \frac{1}{2})}{\Sigma_n (\frac{1}{\lambda} - \frac{1}{2}) + \gamma \sigma_u^2 \Delta t}
\]

Simplify and we get for \( n = 1, \cdots, N - 1 \),

\[
    \Sigma_{n-1} - \Sigma_n = \frac{\gamma \sigma_u^2 \Delta t}{\frac{1}{\lambda} - \frac{1}{2}} = \frac{\lambda^2 (2 + A \sigma_u^2 \Delta t \lambda) \sigma_u^2 \Delta t}{1 + A \sigma_u^2 \Delta t \lambda}
\]

So

\[
    \Sigma_0 - \Sigma_{N-1} = (N - 1) \frac{\lambda^2 (2 + A \sigma_u^2 \Delta t \lambda) \sigma_u^2 \Delta t}{1 + A \sigma_u^2 \Delta t \lambda}
\]

While in the last period, plug Eq. (A-5) into Eq. (A-6), we get an expression of \( \Sigma_N \): \( \Sigma_N = \frac{\lambda \sigma_u^2}{\beta_n} = \lambda^2 (2 + A \lambda \sigma_u^2 \Delta t) \sigma_u^2 \Delta t \)

Eq. (A-7) gives

\[
    \Sigma_{N-1} = \frac{\Sigma_N}{1 - \lambda \beta_n \Delta t} = \frac{\lambda^2 (2 + A \lambda \sigma_u^2 \Delta t) \sigma_u^2 \Delta t}{1 - \frac{1}{2} \lambda \lambda \sigma_u^2 \Delta t} = \frac{\lambda^2 (2 + A \lambda \sigma_u^2 \Delta t) \sigma_u^2 \Delta t}{1 + A \sigma_u^2 \Delta t \lambda}
\]

Thus

\[
    \Sigma_0 = N \frac{\lambda^2 (2 + A \sigma_u^2 \Delta t \lambda) \sigma_u^2 \Delta t}{1 + A \sigma_u^2 \Delta t \lambda}
\]

(A-14)

so \( \Sigma_{N-1} = \frac{1}{N} \Sigma_0 \) and \( \Sigma_{n-1} - \Sigma_n = \frac{1}{N} \Sigma_0 \), hence Eq. (35) follows. Simplify Eq. (A-14), we get the polynomial in Eq. (36). We can see that Eq. (24) is a special case of Eq. (36). By the same argument
in the proof of Proposition 1, Eq. (36) has a unique positive root. Eq. (32) is Eq. (A-10), Eq. (33)
comes from Eq. (A-13) and Eq. (35), and Eq. (34) comes from Eq. (A-12) after we replace \( \beta \)
and \( \Sigma_n \). □

Proof 4 (for Proposition 3): Rearrange Eq. (A-14) and replace \( \Delta t \) with \( \frac{1}{N} \) we get

\[
\frac{\Sigma_0}{\sigma_u^2} = \lambda^2 \left( 2 + A \sigma_u^2 \lambda \lim_{N \to \infty} \frac{1}{N} \right)^2 > 2 \lambda^2
\]

So \( \lambda(N) \) is bounded, and \( \lim_{N \to \infty} \lambda(N) \) exists. Denote the limit as \( \bar{\lambda} \), then take the limit of both sides

\[
\frac{\Sigma_0}{\sigma_u^2} = \bar{\lambda}^2 \left( 2 + A \sigma_u^2 \bar{\lambda} \lim_{N \to \infty} \frac{1}{N} \right)^2 = 4 \bar{\lambda}^2
\]

Hence \( \lim_{N \to \infty} \lambda(N) = \frac{1}{2} \sqrt{\frac{\Sigma_0}{\sigma_u^2}} \), which is the market depth when the insider is risk neutral and there
are disclosure regulations in Huddart et al (2001). □

References

Figure 1 – Information stock when there are 2, 4, 5, 6, 20 and 100 rounds of trading. These figures compare information stock of price, $\Sigma_n$, when the risk-averse insider faces and does not face mandatory disclosure regulations. Exogenous parameters are: $A=4$, $\Sigma_0=1$; $\sigma_u^2=1$. 

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Figure 2 – Liquidity parameter when there are 2, 10, 12, 14, 20 and 100 rounds of trading. These figures compare the inverse of market liquidity, $\lambda_n$, when the risk-averse insider faces and does not face mandatory disclosure regulations. Exogenous parameters are: $A=4, \Sigma_0 = 1; \sigma^2_u = 1$. 

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