BASLE II CAPITAL ADEQUACY: COMPUTING THE ‘FAIR’ CAPITAL CHARGE FOR
LOAN COMMITMENT ‘TRUE’ CREDIT RISK

By

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our own responsibility.
This research makes two contributions. The first one is to price analytically put option and extension premiums embedded in rollover commitments, and the second to combine this put and the exercise-cum-takedown proportion in computing the ‘fair’ capital charge corresponding to the commitment ‘true’ credit risk. In doing so, the procedure proposes to do away with the BIS accounting-based concepts of conversion factor, principal-risk factor, and commitment term-to-maturity dichotomy. They are replaced by the exercise-cum-takedown proportion and the put value implicit in borrower-extendible commitment contracts, respectively. This option-based procedure has the advantage that (i) put values constitute a finer credit-risk grid than the two artificial values of the conversion and principal-risk factors, and (ii) capital charges computed from risk-weighted balances are quite moderate and internally consistent for all types of commitments. Finally, the approach is developed one step further to account for the borrowers’ risk ratings by external credit agencies; this results in a matrix of new standard risk weights that applies to all off-balance-sheet commitments.
I. INTRODUCTION AND SUMMARY

According to the Bank for International Settlement (BIS), the computation of the regulatory capital charge of off-balance-sheet credit commitments relies on the following parameters: the credit conversion factor (0% or 50%), the principal risk factor (0% or 100%), and two distinctions, e.g., between revocable and irrevocable commitments with the latter having either initial term to maturity up to one year and longer than one year. This procedure is criticized for allowing a well-known regulatory arbitrage: 364-day credit irrevocable commitments (even those benefiting from a rollover option) as well as all revocable commitments without distinction of maturity date are not subject to any capital charge whereas the positive, risk-weighted balance of longer-term irrevocable commitments is used to compute the banks’ positive capital charge. The bank’s interest in using this arbitrage is both financial and strategic: the absence of a capital charge for short-term irrevocable commitments and all revocable commitments allows the bank to increase its return on the regulatory capital committed for other instruments. To link more directly the commitment “true” credit risk to its regulatory capital charge, this research proposes to substitute two market-based concepts to the BIS accounting-based coefficients. The credit-conversion factor makes way for an empirically relevant exercise-cum-takedown proportion, namely the average amount of the credit line drawn down when the line is exercised. And the principal risk factor is replaced by the value of the put option embedded in credit commitments with an extendible maturity date. Valuing this implicit credit-risk derivative then raises three questions: 1) Does the put option embedded in an extendible commitment captures commitment credit risk irrespective of the latter initial term to maturity? 2) What is the magnitude of line funding when the commitment is exercised? And 3) how is the “fair” capital charge for commitment credit risk computed?

Tkakor *et al.* (1981) have shown that a commitment contract can be viewed as a put option sold by the bank to its borrower. When the commitment interest rate is lower than that on an equivalent spot loan, the borrower receives the line face value but is only indebted for its lower marked-to-market value --henceforth to be referred to as the *indebtedness value*. The borrower's claim on the lending bank constitutes an embedded, yet valuable, commitment put option.
option, which has to be fitted into the BIS regulatory time frame. The aggregate face value of still unused commitments is reported as an off-balance-sheet entry to the bank's annual consolidated balance sheet and is subject to at least annual audits. Yet, at the annual reporting date, the time remaining to commitment expiry is less than the initial term for many of the outstanding commitments. To account for this, the average time remaining to the commitment first expiry date, $T_1$, has been standardized at $T_1 - s$, with $s = 0$ denoting the (BIS) date at which banks’ auditing for capital adequacy takes place. Within this time frame, the commitment put option is European and its value captures the bank's notional liability for carrying the commitment at the audit date. In this research, we examine the most prevalent type of loan commitments, those with a floating prime-rate\footnote{According to Duca and Vanhoose (1990) or a more recent Federal Reserve survey (2000), about eighty percent of U.S. commercial and industrial lending is done via loan commitments, with the vast majority being of the floating-rate type. Typically, commitments with an upfront fee only are sold to high-credit-quality firms. But longer-term commitments with upfront and rear-end fees are sold to medium size firms whose credit quality is poorer (see Berger and Udell [1995] and Petersen and Rajan [1994]). This most prevalent type is examined here.} formula devised as "fixed markup over a stochastic index cost of funds".

In recent years, Ergungor (2001) and Kashyap, Rajan, and Stein (2002) have stressed the central role played by commitments in bank intermediation, and several researchers have derived alternative formulas for valuing bank credit line commitments. Thakor et al. (1981) and Ho and Saunders (1983) derived option-like expressions for fixed-rate commitments, Thakor (1982) and Chateau (1990) obtained valuation formulas for variable-rate commitments, and Hawkins (1982) priced revolving credit lines. To the best of our knowledge, extendible or rollover commitments, namely those in which the initial commitment period is extended for another time period, have not yet been priced. Within the BIS regulatory framework, pricing borrower-extendible commitments has the advantage to circumvent the artificial dichotomy between irrevocable commitments with a one-year initial term to maturity and those with an initial term to maturity longer than one year. Fortunately, there have been advances in research on derivatives with extendible maturities: Anathanarayan and Schwartz (1980) have priced extendible bonds and Longstaff (1990) European call and put options with extendible maturities. Here the analytical value of the extension premium...
results from a risk-neutral argument; although our closed-form solution differs from Longstaff’s arbitrage-based formula for the extendible put option, both approaches yield the same computed put values. Granted this short literature review, the research sets out: (i) to value the European commitment put implicit in extendible commitment contracts; (ii) to determine in simulation experiments the magnitude of the single- or multiple-year extension privilege by comparison with a one-year non-extendible commitment; (iii) to use the simulated values to compute the fair capital charge corresponding to the commitment true credit risk; and (iv) to propose new standard risk weights for extendible and non-extendible commitments as well. All this is worked out for commitment puts generated by the fixed markup of credit lines with a floating prime-rate formula.

Here is a summary of the paper. Section II models the ‘true’ credit risk of a commitment contract; the latter corresponds to the put option embedded in commitments in which the borrower can postpone the contract maturity upon paying an extension fee. We derive analytically the value of the extension privilege comprised in the borrower-extendible put option. A special case nested in the model is also examined: the put option embedded in a one-year straight commitment is priced as in Thakor (1981). An exercise-cum-takedown proportion is next defined: it combines an exercise-indicator function that captures the line exercise decision to a takedown or funding proportion that increases with the length of the extension period. As commitment puts are but notional values of embedded credit-risk derivatives, simulations are used in Section III to validate the robustness of the extendible-commitment model. Two patterns emerge from the simulations: (i) put values, and hence credit risks, are increasing when the indebtedness value moves progressively in the money or when the extension period grows longer, and (ii) the extension premiums (expressed as a percentage of the put values) increase with the length of the extension period but decline with the indebtedness value moving deeper in the money.

Based on these simulations, the regulatory implications of extendible commitments are presented in Sections IV and V. To do this, the proposed option-based procedure does away with the BIS conversion factor, principal-risk factor, and the two distinctions, e.g., between revocable and irrevocable commitments and for the later, between commitments with an initial term to maturity less-than or longer-than one year. These parameters are replaced by the exercise-cum-takedown proportion and the put value implicit in borrower-extendible commitment contracts;
these are then combined together so as to directly link commitment credit risk to its market-based or ‘fair’ capital charge. According to this procedure, the risk-weighted balance of all types of commitments is positive and does attract a positive capital charge. The procedure also has two additional advantages: (i) put values constitute a finer credit-risk grid than the two artificial values of the BIS principal risk factor, and (ii) capital charges computed from risk-weighted balances are moderate and internally consistent for all types of commitments. In Section V finally, the procedure goes one step further to account for the borrowers’ credit-risk ratings. This yields a matrix of new standard risk weights sensitive to three parameters: the borrowers’ risk ratings of external credit agencies, the duration of the extension period, and the proportion of line funding, respectively.

The layout of the paper is as follows. Section 2 provides the analytical value of the European put embedded in extendible and non-extendible commitments; it also determines the exercise-cum-takedown proportion. Simulation results are presented in Section 3 and used in Sections 4 and 5 to quantify the link between commitment credit risk and banks’ capital charge. Concluding remarks close the paper in Section 6.

2. VALUATION OF EXTENDIBLE CREDIT COMMITMENTS

2.1 The borrower-extendible commitment

The salient features of a (no-default) commitment with a fixed markup are stylized in the decision chart below. In part (a) of the chart, the bank writes at date 0 an off-balance-sheet commitment contract for a credit line (CL) with the following features: (i) the initial one-year commitment period, \([0, T_1]\), is extended at \(T_1\) for another year\(^2\), \([T_1, T_2]\), at the borrower’s option (if the commitment is exercised at date \(T_1\), it is a one-year straight commitment examined at the end of this subsection), (ii) loan duration\(^3\), \([T_2, T^*]\), is one year from date \(T_2\) if the credit line is drawn down; (iii) the CL face value, standardized at \(L = \$100\) maximum, remains constant over both

\(^2\) \(T_2 - T_1\) can be a multiple-year extension period, as will be the case later on.
commitment and **DECISION CHART of the borrower’s extendible credit commitment with a fixed forward markup in its floating-rate formula.**

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a) Initial situation at t=0: The contractual terms of reference abstract from compensating balances, bank reserve requirement and annual fees\(^4\).

<table>
<thead>
<tr>
<th>Initial one-year CL commitment period</th>
<th>One-year extension at the borrower’s option</th>
<th>Duration of potential loan</th>
</tr>
</thead>
<tbody>
<tr>
<td>t=0</td>
<td>s</td>
<td>T1</td>
</tr>
<tr>
<td>CL max. of $100, MAC clause, and fixed markup</td>
<td>T2</td>
<td>T*</td>
</tr>
<tr>
<td>(m_o = 1.5%\text{-p.a.}) remain in force from t=0 to T2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(f^c_0 = 25\psi\): commitment fee \(f^E_{T_1} = 25\psi\): extension fee \(f^U_{T_2} = 25\psi\): non-usage fee

b) BIS regulatory time frame: Valuation takes place at the annual audit date, s. \(T_1 - s = 6\) months is the standardized time left to commitment first expiry.

$100 one-year corporate loan + “vulnerable” repayment call

\[ \begin{align*}
\text{Audit = valuation date} & \quad f^E_{T_1} = 25\psi: \text{extension fee} \\
\text{The borrower triggers the agreed upon 1-yr extension} & \quad \uparrow \quad \text{T} \\
\text{\(s=0\)} & \quad \text{or} \quad \downarrow \\
\text{or} & \quad f^U_{T_2} = 25\psi: \text{is paid by the borrower as the line is NOT drawn down}
\end{align*} \]

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\(^3\) Loan duration, \(T_2 - T^*\), can also be more than one year.

\(^4\) Compensating deposit balances (James [1981] and Hawkins [1982]) constitute a disguised cost (or deadweight loss) to be treated here as a scaling problem. U.S. reserve requirement for transaction deposits ranges from 3 to 10\% depending on the size of the financial institution (Rose [2000]). There also exist minor annual expense and annulation expense, in the order each of 3 to 5 basis points per annum (0.03 to 0.05 \% p.a.).
extension periods, and (iv) the floating prime-rate formula is $c_{T_2} + \bar{m}_0$. Its first component, $c_{T_2}$, is the bank's stochastic cost of funds at exercise date $T_2$, with the rate on certificates of deposits (CDs) being
generally used as exogenous index (it is explained later on why $T_2$ is the exercise date here). The other component, $\bar{m}_0$, is the **fixed forward markup** that is determined when the commitment contract is written at date $t=0$. For instance, the borrower-extendible commitment for a $100$-maximum CL has a time-$0$ (time-$T_2$) prime rate of $6\%$ p.a. ($6.5\%$ p.a.) made up of a $4.5\%-p.a. (5\%-p.a.)$ stochastic cost of funds plus at both dates a fixed forward markup $\bar{m}_0$ of $1.5\%$ p.a. The fixed markup thus only hedges credit risk as the borrower bears the funding risk, $c_{T_2}$. Most commitments also comprise a material-adverse-change (MAC) clause, that remains in force during both commitment and extension periods. This escape clause allows the bank to limit or even deny credit funding if the borrower’s financial condition deteriorates over both periods --the degree of deterioration being judged solely by the bank from its private information about the borrower. The fact that the borrower’s financial condition is ambiguously defined prevents us from defining a lower bound to the marked-to-market value of the credit line. Below this lower bound the commitment would automatically vanishes, a sort of down-and-out put option.

Thakor and Udell (1987) provide the economic rationale for the bank's optimal deployment of up-front and rear-end fees in a one-year straight commitment. In their competitive equilibrium model, the screening device resolves the bank-borrower asymmetries of information and the presence of adverse selection gives rise to split fees at the commitment end-dates. Upfront but especially back-end fees are related to the borrower's probabilities of future line utilization (formalized in subsection 2.5 as a takedown-sensitive proportion). When their sorting variables are adapted to the borrower-extendible commitment, three fees are deployed at dates $t=0$, $T_1$, and $T_2$ in part (a) of the decision chart. The first one is an upfront commitment fee of $1/4$ of $1\%$ per annum, $f_0^{C}$, or here 25 cents per $100$ of line face value, the next one is a middle or extension fee, $f_{T_1}^{E} = 25$ cents, of the same magnitude as the initial commitment fee, and the last one is a

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5 Borrower self-selection as a screening and risk-sharing device with optimal fee mix is also examined in Fery et al. (2003), Shockley and Thakor (1997), Thakor (1989), and reviewed in Ergungor (2001) or Greenbaum and Thakor (1995).
rear-end or usage fee, \( f_{T_2} \). This 25-cent payment is really a non-usage fee, as it is paid if and when the credit line is not drawn down.

It now remains to explain why extendible loan commitments are considered as European put options within the BIS regulatory time frame. The aggregate value of all unused commitments is reported as an off-balance sheet entry to the bank's annual consolidated statement. Yet, at the annual reporting date \( s \), the time remaining to the extension date \( T_1 \) is less than the initial one-year period for many of the outstanding rollover commitments. So, in part (b) of the chart, the average time remaining to the extension date \((T_1 - s)\) has been standardized at 6 months, with \( s=0 \) denoting the valuation (= audit) date (see also Merton [1977] for a similar argument). Two outcomes are possible at date \( T_2 \) for a commitment that has been extended at time \( T_1 \). It is either exercised and line funding results in an on-balance-sheet corporate loan; the bank then holds a "vulnerable" repayment call on the borrower's assets since the latter may default on loan principal and/or loan interests. Or, alternatively, the commitment simply expires and the borrower pays the rear-end fee, \( f_{T_2} \), as the line had not been drawn down --line partial takedown is dealt with in subsection 2.5 of the model. Thakor (1982) was the first to price the components "split fees + commitment put" of non-extendible floating-rate commitments. For the sake of continuity, we shall refer to the above problem statement in the rest of the paper.

To be complete, the decision chart can be altered when some of its basic assumptions are changed. Two changes are relevant to the subsequent developments: the extension and loan-duration periods can be lengthened to two or more years\(^7\), and the forward markup can be

\(^6\) According to Shockley and Thakor (1997, Table 1) for the years 1989 and 1990, the mean upfront fee on short-term (liquidity, working capital, and trade and finance) commitments was 24.2 basis points while the mean annual usage fee was 22.8 basis points. Regarding commitments used for general corporate purposes, the mean upfront and usage fees are 18.6 and 19.6 basis points, respectively. Angbazo et al. (1998) note that both fees are declining since the mid-90s due to strong competition; the usage fee also has a tendency to be somewhat lower than the upfront fee.

\(^7\) According to Table 1 in Shockley and Thakor (1997), the average duration of liquidity commitments and general-corporate-purposes commitments is 28.4 and 38 months, respectively. It thus makes sense to model one-year commitments with extensions up to four or even five years.
adjusted by add-ons or discounts (± 25 basis points, ±50 basis points, …) for non-prime rate commitments. In addition, it also makes sense to use the risk ratings used by external credit agencies to assess the credit risk of off-balance-sheet commitments and on-balance-sheet loans. We now examine a particular case nested in the borrower’s extendible commitment.

**Nested case**

The one-year non-extendible commitment is a special case nested in the extendible-commitment model. In that case and always within the BIS time frame, the borrower draws on the line at date T₁, with the one-year corporate loan, (T₁, T*), becoming outstanding immediately. It ensues that the middle section of part (a) of the decision chart (relating to the extension fee and extension period) is omitted and part (b) of the chart is adjusted accordingly. However, the emergence of a one-year straight commitment does not prevent the borrower from writing subsequently a separate new commitment.

### 2.2 Indebtedness value

Thakor et al. (1981) were the first to define the marked-to-market value of a credit line, an economic value usually referred to as the borrower’s debt or **indebtedness value**, X. With regard to a fixed markup commitment, the indebtedness values, at date T₁ for a one-year straight commitment and at date T₂ for a borrower’s extendible commitment, are computed respectively as

\[
X_{T_1} = L_1 \exp\left\{ (\bar{m}_0 - m_{r_1}) (T_1 - T*) \right\}, \quad \text{or} \quad X_{T_2} = L_2 \exp\left\{ (\bar{m}_0 - m_{r_2}) (T_2 - T*) \right\},
\]

where L₁ = L₂ is the line par value, (T₁ – T*) = (T₂ – T*) is loan duration once the commitment has been exercised at T₁ or T₂, and (\bar{m}_0 – m_{r_i}) is the difference between \( \bar{m}_0 \), the fixed forward markup set at date 0 when the commitment was written, and \( m_{r_i} = (l_{r_i} - c_{r_i}) \), the date-T₁ stochastic spot

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8 For non-prime commitments, the rate is rescaled for prime-rate add-ons or discounts; as for the magnitude of such spreads over the floating prime rate, consult Angbazo et al. (1998), Elsas (1998) or Shockley and Thakor (1997).
markup defined as the difference between the spot prime rate, \( l_{T_i} \), and the funding rate in the CD market, \( c_{T_i} \). The same holds true for \( (\bar{m}_0 - m_{T_i}) \). At date \( T_1 \) for one-year commitments (and at date \( T_2 \) for extendible commitments), the commitment holder decides to draw on the line only if \textit{ceteris paribus} \( \bar{m}_0 < m_{T_1} \) (or \( \bar{m}_0 < m_{T_2} \)), namely when the fixed markup is less than the stochastic spot markup computed from primary credit and funding rates. For instance, when our illustrative 1.5-% forward markup is combined with, say, a 2.5% spot markup, the markup differential in eq. (1) is negative at -1%. It follows that the inequality \( X_{T_1} < L_1 \) (or \( X_{T_2} < L_2 \)) gives rise to an implicit commitment put option as the borrower’s debt value is less than the option strike price. We assume that the law of motion of the indebtedness value in eq. (1) is

\[
dX(t) = X[\mu dt + \sigma dz_X(t)],
\]

where the constant terms \( \mu \) and \( \sigma \) are the instantaneous drift and instantaneous standard deviation of the indebtedness-value lognormal distribution, and \( dz_X(t) \) the differential of the Wiener process \( z_X(t) \).

We next move from the real world to the risk-neutral world by a change of probability measure, as this will be useful in the pricing developments below. So

\[
dX(t) = X[(r - .5\sigma^2)dt + \sigma d.z^*_X(t)],
\]

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9 Any banking decision taken at the margin considers the upfront fee as a fixed, and thus sunk, cost. To maintain here the neutrality of the trade-off between spot loan and credit under a commitment, we assume that the usage fee due at exercise date \( T_1 \), \( f^{U}_{T_1} \), matches the administrative cost, \( c^{a}_{T_1} \), that the borrower will pay for a spot loan at the same date. Otherwise, the markup differential \( (\bar{m}_0 - m_{T_i}) \) becomes \( (\bar{m}_0 + f^{U}_{T_i} - m_{T_i} - c^{a}_{T_i}) \) with eq. (1) adjusted accordingly. The same adjustment holds true for date \( T_2 \). The alternative approach is the all-in-cost basis, in which markup + fees are computed and compared for credit commitments and spot loans.

10 In other terms, the benefit received at exercise date is borrower specific: the subsidy is with respect to the markup of the class of floating prime-rate borrowers, and not some sort of cost index or interest rate.
where \( r \) is the constant instantaneous riskless rate of interest, \( \mu = r - 0.5\sigma^2 \) and the starred differential denotes the risk-adjusted version of the original Wiener process.

### 2.4 Valuing the European put implicit in extendible and non-extendible commitments

We now price in turn the European put option implicit in one-year straight commitments and commitments with single- or multiple-year extension periods.

#### 2.4.a Pricing the put embedded in one-year straight commitments

The value of the put option embedded in a one-year commitment, labelled \( P_0 \) since non-extendible, is isomorphic to the value of the Black-Scholes (1973) European put option. Namely

\[
P_0(X, L_1, T_1) = L_1 e^{-rt_1} N(-d_-) - X N(-d_+)
\]

where

\[
d_{\pm} = \{ \ln(X/L_1) + (r \pm 0.5\sigma^2)T_1 \}/(\sigma \sqrt{T_1}),
\]

and \( N(d_{\pm}) \) is the cumulative standard normal distribution at \( d_{\pm} \), the other terms having been defined previously. In eq. (2), the value of the implicit commitment put depends on the indebtedness value \( X \), the credit-line par (= exercise) value \( L_1 \), and the maturity date \( T_1 \).

#### 2.4.b Pricing the borrower-extendible commitment put

We denote the value of the European commitment put embedded in borrower-extendible commitments as \( EP_i(X, L_1, T_1, L_2, T_2, f_{T_1}^E) \); it depends on the indebtedness value \( X \), the credit-line par (= exercise) value \( L_1 = L_2 \), the maturity dates \( T_1 \) and \( T_2 \), and the extension fee, \( f_{T_1}^E \). The postscript \( i \) to \( EP \) indicates the length of the extension period, \( T_2 - T_1 \), with \( i: 1, 2, \ldots, 5 \). For instance, \( EP_1(\ldots) \) refers to a two-year commitment contract with a one-year extension period. The value of the extension privilege is thus

\[
EP_i\text{-premium} = EP_i(X, L_1, T_1, L_2, T_2, f_{T_1}^E) - P_0(X, L_1, T_1),
\]
where the second term is defined in eq. (2). We now call upon a risk-neutral argument to value the extension privilege. This premium is the expected value of the extension privilege at time \(T_1\) in a risk-neutral world, discounted to the valuation time \(s=0\) at the risk-free rate of interest, \(r\). Namely

\[
\text{EPi-premium} = e^{-rT_1} \hat{E} (\text{EPi-premium at } T_1),
\]

(3)

where \(\hat{E}\) denotes expectations in the risk-neutral world. At the first expiry date of the borrower-extendible commitment, the payoff of the implicit put option is:

\[
\max\{L_1 - X_{T_1}, 0\} + \max\{0, \text{EPi}(X_{T_1}, L_2, T_2 - T_1) - f_{T_1}^E - \max (L_1 - X_{T_1}, 0)\}.
\]

(4)

In eq. (4), the second term is the extension privilege or a call option on the extendible commitment put. Its exercise value obtains by adding the extension privilege to the intrinsic value of the straight one-year commitment put, \(\max\{L_1 - X_{T_1}, 0\}\). As the bank charges a positive extension fee, \(f_{T_1}^E = 25\) cents, the commitment contract is extended at \(T_1\) only if the indebtedness value lies in the so-called extension interval, \([I_2, I_1]\) with \(I_2 < I_1\). If \(X < I_2\), the commitment put is exercised at time \(T_1\), but if \(X > I_1\), the put is allowed to expire at time \(T_1\). The interval upper and lower bounds, \(I_1\) and \(I_2\), can be found by solving the following maturity conditions:

\[
P(I_1, L_2, T_2 - T_1) - f_{T_1}^E = 0
\]

(5)

and

\[
P(I_2, L_2, T_2 - T_1) - f_{T_1}^E - (L_1 - I_2) = 0.
\]

(6)

Once \(I_1\) and \(I_2\) have been found, the final payoff of the extension privilege is written:
where \(1_{\text{condition}}\) is equal to one if the condition is verified and zero otherwise. Following some tedious but straightforward developments collected in the Appendix, the closed-form expression of the EP-premium is:

\[
\text{EP-premium} = XN_2(-x^*, z_2; -\rho) - L_2 e^{-rT_2} N_2(-x^* + \sigma\sqrt{T_2}, z_2 - \sigma\sqrt{T_1}; -\rho) - \left[XN_2(-x^*, z_1; -\rho) - L_2 e^{-rT_1} N_2(-x^* + \sigma\sqrt{T_2}, z_1 - \sigma\sqrt{T_1}; -\rho)\right] - \left[f_{T_1}^E e^{-rT_1} N(-z_1 + \sigma\sqrt{T_1}) + f_{T_1}^E e^{-rT_1} N(-z_2 + \sigma\sqrt{T_1}) - P_0(X, L_1, T_1)\right] + \left[-X N(-z_2) + L_1 e^{-rT_1} N(-z_2 + \sigma\sqrt{T_1})\right]
\]  

(8)

where the terms \(x\), \(x^*\), \(z_1\), \(z_2\) and \(\rho\) are defined as follows:

\[
x = \frac{\ln(X/L_1) + (r + .5\sigma^2)T_1}{\sigma\sqrt{T_1}} \quad x^* = \frac{\ln(X/L_2) + (r + .5\sigma^2)T_2}{\sigma\sqrt{T_2}} \quad z_1 = \frac{\ln(X/I_1) + (r + .5\sigma^2)T_1}{\sigma\sqrt{T_1}} \quad z_2 = \frac{\ln(X/I_2) + (r + .5\sigma^2)T_2}{\sigma\sqrt{T_1}} \quad \rho = (T_1/T_2)^{\frac{1}{2}}
\]

In eq. (8), \(N[\cdot \cdot]\) is the cumulative probability of the standard normal density and \(N_2(\cdot \cdot \cdot ; -\rho)\) is the cumulative probability of the standard bivariate normal density with correlation \(-\rho\). Note that the extension privilege comprises the extension fee, \(f_{T_1}^E\), explicitly --its discounted value is the first term on the right-hand side (RHS) of eq. (8). The RHS second term is the value of the one-year straight commitment put, \(P_0\) from eq. (2); so, if the latter is added to the extension premium, we verify that:

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11 Using a standard arbitrage argument, Longstaff previously derived a different expression for the European put option with an extendible maturity (Longstaff, 1990, eq. (12), p 943). Although our derivation and closed-form solution differ from that of Longstaff, both approaches yield the same computed put values.
or borrower-extendible put value =

\[
\text{one-year put value + value of the single-or-multiple-year extension privilege.}
\]

\[\text{EPi}(X, L_1, T_1, L_2, T_2, f_{i}^{E}) = P_0(X, L_1, T_1) + \text{EPi-premium with } i: 1, \ldots, 5\quad (9)\]

\[\text{2.5. Modelling the exercise-cum-takedown proportion}\]

Once the embedded put value is computed, it remains to determine the proportion of the credit line taken down. This is done in two steps: the first step formalizes the exercise (or absence thereof) of a commitment with a given extension period, and the second one determines the amount of line funding when a commitment with a given extension period is exercised.

Commitment exercise is captured by an indicator function that is equal to one if exercise occurs and zero otherwise; namely \(I_i = 1\{X_i < L\}\), where \(i: 0, \ldots, 5\) denotes the length of the extension period -- from zero year for straight commitments to five years for six-year commitments with a five-year extension period. Once the commitment with a given extension period is exercised, the exercise indicator is combined with a draw-down parameter, \(d_i\), that captures the amount of the credit line actually taken down. This combination defines the average proportion of the commitment taken down in the \(i\)th extension period. That is

\[
\pi_i = E[d_i \times I_i | I_i = 1] = E[d_i | I_i = 1] = E[d_i | X_i < L],\quad (10)
\]

where \(E\) is the mathematical expectation and \(d_i\) the takedown parameter, with \(i: 0, \ldots, 5\) denoting the length of the extension period. When there is full takedown, \(d_i = 1\); in the absence of exercise and thus commitment takedown, the complementary proportion is \((1 - \pi_i) = E[1 - d_i | X_i < L]\). In the BIS regulatory context, this average proportion applies at the bank level to the aggregate amount of commitments within each maturity class or extension period, namely to the dollar total of each class of commitments at the audit date. In addition, from the empirical evidence reported in Morgan [1993]\(^{12}\) and later on in panel B of Table 2 for a large international bank, we

\[^{12}\text{Morgan (1993) indicates that between 1988 and 1990, the fraction of the loan limit actually}\]
select a fixed proportion, $\pi_i$, for each extension period, but this proportion increases with the extension duration. To wit, a proportion of $\pi_0 = 50\%$ means that 50\% of the aggregate contractual value of all commitments with an initial maturity less than one year is taken down; this proportion increases to $\pi_5 = 75\%$ for commitments with at least a five-year extension period. The percentage chosen for straight commitments is low because: (i) there is little time left to drawn down these credit lines and (ii) the borrower intentionally refrains from taking down full funding so as to avoid being charge higher commitment fees in the next period (Ergungor [2001]). The funding proportion is likely to increase, however, as borrowers have more time and opportunities to draw down (even cumulatively) longer-term irrevocable credit lines.

The exercise-cum-takedown proportion just defined along with the commitment put value is all that we need to compute banks’ capital charge corresponding to commitment credit risk. Equations (1) to (10) form the credit-risk valuation programme of loan commitments, which is estimated in the next section.

3. **SIMULATION RESULTS**

3.1 *Simulation*

As embedded credit-risk derivatives, the put values implicit in extendible and non-extendible commitments are but notional values. We thus rely on simulations to compute their values, and our simulation parameters are based on the empirical evidence presented in Exhibit 1. The latter shows that historically the indebtedness value, $X$, varies in the value range $97$ to $103$: it is thus sensible to set $X$ at $100$, $99.5$, $99$, $98.5$ and $98$, for a commitment put that moves progressively in the money. With a line par value of $100$, the slightly in-the-money borrowed by prime-rate borrowers is about 55\%; unfortunately, he is not reporting the number of commitments left unexercised. There is another reason why borrowers do not draw their commitment down completely, and so do not exploit their implicit put option fully: if they do, the bank will charge them higher future fees.
indebtedness values simulate small increases in the spot markup of the class of floating prime-rate borrowers over the commitment and/or extension periods. Granted these indebtedness values, simulation experiments are performed for one-year non-extendible commitments, $P_0$, and borrower-extendible commitments, $EPI_s$, with extension durations from one to five years. The time to

\begin{tabular}{cccccc}
\textbf{EXHIBIT 1} \\
Statistical analysis of $X$, the indebtedness-value monthly time series computed from eq. (1) for the period 1966.01 to 2001.12. In eq. (1), spot markups and markup differentials are computed from Statistics Canada monthly time series $B14020$ and $B14043$ of the prime rate and 90-day deposits of chartered (commercial) banks, respectively.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100.01$</td>
<td>$0.7418$</td>
<td>$97.07$</td>
<td>$103.87$</td>
<td>$-0.038$</td>
<td>$6.122$</td>
</tr>
</tbody>
</table>

commitment maturity at valuation date $s = 0$ is $T_1 - s = 0.5$ year for the one-year straight commitment and $T_2 - s = 1.5, \ldots, 5.5$ years for the other commitment contracts. In addition, the following parameters are common to all simulations: the credit-line strike price remains constant through time, $L_1 = L_2 = \$100$, the risk-free interest rate, $r = 0.04$, is consistent with the 4.5% CD rate introduced in subsection 2.1, and $\rho = (T_1/T_2)^{1/2}$. The indebtedness-value volatility, $\sigma$, computed above is 0.7418 or 2.57% on an annual basis; we have chosen $\sigma = 0.03$ in the simulations.

Before reporting on the simulations, we first clarify the meaning of computed put values. Consider the very plausible scenario (represented by the entries in column (3) of Table 1) in which the indebtedness value is slightly in-the-money, $X = \$99$. According to the first boldfaced entry in
column (3), the estimate \( P_0 = 0.434 \) means that the European put embedded in a one-year straight commitment has an equilibrium value of 0.43% of the $100 par value if: (i) our floating prime-rate commitment with a 1.5%-p.a. fixed forward markup is priced when the stochastic markup on spot loans is 2.5% p.a.; and (ii) the remaining life of contract is six months. By way of contrast, when the time remaining to commitment expiry is 18 months, the put value of the borrower-extendible (EP1) commitments in column (3) is $0.733. Put values of longer-term commitments are also computed, with $4.62 corresponding to a commitment with a five-year extension period. The magnitude of the extension premiums in borrower-extendible commitments from eq. (8) is shown in entries 3a to 3e of column (3). For our reference scenario again, the premium is but a moderate 40.8% when the extension duration is one year but much steeper (up to 90.6%) for longer-term extendible commitments.

3.2 Commitment put values and their extension premiums

Two revealing tendencies of commitment put values are emerging from matrices 2 and 3 in Table 1. The first tendency from matrix 2 is that all commitment put values (and hence credit risks) are increasing when either (i) the indebtedness value is moving progressively deeper in the money or (ii) the extension period is growing longer. To wit, in row (2a) for a commitment that offers a one-year extension only, the put value increases from approximately 34 cents for an at-the-money indebtedness value to about $1.19 for \( X \) in the money at $98. Similarly, from entries (2a) to (2e) in column (5), put values are increasing from $1.19 for a straight commitment to $7.56 for a commitment with a 5-year extension period. The other pattern from matrix 3 is that extension premiums expressed as a percentage of the EPi-values are (i) increasing with the length of the extension period, but (ii) declining when the indebtedness value moves deeper in the money. In the latter case and according to entries on row (3a), the one-year extension privilege as a percentage of EP1 declines from 43.5% to 29.4% when the indebtedness values move deeper in the money. This
declining pattern is duplicated in each of the other rows of matrix 3, but from a higher starting point.
In other terms, the extension premiums implicit in longer-term extendible commitments are
percentagewise much larger than those embedded in short-term commitments; yet the declining
pattern becomes smoother for longer extension periods.

The simulation results are used in the next two sections to quantify the link between the
credit risk of off-balance-sheet commitments and the bank’s risk-weighted capital charge.

4. THE ‘FAIR’ CAPITAL CHARGE FOR COMMITMENT ‘TRUE’ CREDIT RISK

The Basle credit risk rules (see BIS [1988] or Santos [2001]) require that standard
risk-adjusted balances be determined for each off- as well as on-balance-sheet instrument and
their aggregate value be weighted against a definition of regulatory capital. To calculate
risk-adjusted values, off-balance-sheet contractual amounts are initially converted by way of
credit conversion factors to on-balance-sheet “credit equivalent amounts”; which in turn are
weighted by appropriate principal risk factors to determine “risk-adjusted balances”. Since the
end of 1992, a minimum total capital requirement of 8% applies to such balances. Regarding off-
balance-sheet commitments more specifically, the 1999 Amendments to the Accord do not
recognize anymore the commitment original term to maturity (less than or over one year) as the
criterion on which to differentiate conversion and risk factors13. Yet before implementing the
1999 Amendments,
the BIS is still entertaining proposals as to how to compute the capital charge of such off-
balance-sheet instruments.

----------------------------------------
Insert Table 2 about here

----------------------------------------

13 These amendments will be implemented by November 1, 2006 (see BIS [1999a, b, 2000 and
2001]). An analysis and critique of the BIS new guidelines can be found, among others, in Andre
et al. (2001), Benink and Wihlborg (2002), Fisher (2001), Hammes and Shapiro (2001), or
The information regarding commitments is presented in Panel A of Table 2, for a large international bank, the Royal Bank of Canada as at October 31, 2002. Consider first how irrevocable commitments with a one-year initial term to maturity and all revocable commitments without distinction of initial term to maturity are treated under the BIS present guideline.

According to line (3) of Panel A, the commitment credit-equivalent amount of both types is nil, since the credit-conversion factor applied to their contractual amount ($40.9 billion and 46.0 billion respectively on line (1)) is 0%; and their risk-adjusted balance on line (5) is accordingly also nil as their risk factor is also 0%. The resulting regulatory capital charge (line (5) \times 8\%) is thus also nil. **There is thus no link between actual and/or potential credit risk and the bank’s capital charge, as short-term irrevocable commitments and all revocable commitments do not affect the risk-adjusted capital requirement at any point in time or on a continuous basis.** The situation is not same however for longer-term irrevocable commitments and on-balance sheet loans. On line (5), the risk-adjusted balance of over-one-year commitments is $15.6 billion and that of other (mainly corporate) loans is $89.8 billion, and in both cases, the theoretical principal risk weight is 100% according to line (4). Yet for computation, the actual weight is less than 100% since it is a weighted average of counterparty risk weights within each category. On line (1) also, the aggregate of all commitment contractual amounts, $121 billion, is of the same order of magnitude as the total amount of on-balance-sheet corporate loans, $121.9 billion shown on line (3). But $86.9 billion or 71.8% of all off-balance-sheet commitments is deemed riskless according to the BIS **accounting-based** valuation of commitment credit risk. This nil risk-weighted balance should be contrasted with the extremely large risk-adjusted balance (over 15 billions) for longer-term irrevocable commitments. Panel B of Table 2 complements the previous panel. According to Panel B, the revocable and irrevocable commitments are classified in four time ranges defined on the basis of the commitment initial term to maturity. These ranges are particularly relevant since we intend to consider all commitments as borrower-extendible commitments. Visual inspection of Panel B reveals that most commitments are irrevocable, except for the majority of the 1-to-3-year commitments which are of the revocable type. In aggregate, over 80% of all commitments have an initial maturity up to 3 years, and only 13.7 % of them has an initial term to maturity longer than 5 years.
We are now in a position to offer an alternative to the BIS valuation of commitment credit risk. It is based on three premises. Firstly, the use of borrower-extendible commitments allows us to circumvent the BIS dichotomy of revocable and irrevocable commitments, with the latter being split on the basis of only two initial term to maturity. Secondly, the proportion of [off-balance-sheet] commitments that is likely to become [on-balance-sheet] outstanding loans is captured by the exercise-cum-takedown proportion, the latter being dependent on the commitment maturity ranges introduced in Panel B of Table 2. And thirdly, the commitment credit risk is determined by the put value implicit in borrower-extendible commitments. In other terms, the exercise-cum-takedown proportion and the extendible-commitment put value play the role of the BIS credit conversion factor and principal risk factor, respectively. The approach is illustrated in Table 3 where the benchmark scenario $X = $99 from Table 1 is combined with data from Table 2. As the computation is for illustrative purpose only, we select the mid-point of the time ranges from Panel B of Table 2; for instance all 1-to-3-year commitments (contractual amount, $L = $51.7 billion) are considered as commitments with an average 2-year maturity. The computation is as follows:

\[ 51.7 \text{ billion} \times 0.55 = 28.4 \text{ billion}. \]
\[ 28.4 \text{ billion} \times 0.00733 \text{ (commitment put value per $ billion)} = 208.2 \text{ million}. \]
\[ 208.2 \times 0.08 = 16.7 \text{ million}. \]

On the first line, the exercise-cum-takedown proportion of 55% converts the off-balance-sheet contractual amount into an on-balance-sheet credit-equivalent amount --also reported on line (3) of Table 3. And since the European put value captures the credit risk embedded in commitment contracts, the risk-adjusted balance of short-term commitments is computed on the second line --the amount is also shown on line (5) of Table 3. On the third line finally, the capital charge obtains by applying the 8% capital requirement to the just-computed risk-weighted balance --this corresponds
to line (6) in Table 3. In Table 3 similarly, the fair capital charge is $7.9 million for straight commitments, $1.0 million for all commitments in the 3-to-5-year maturity range, and 4.6 million for commitments with maturities over 5 years. It thus appears that unlike the BIS accounting-based nil value in Table 2, straight commitments do attract a positive but quite moderate capital charge of approximately 7.9 million dollars. In addition, the capital charge for commitments with different original term to maturity is usually moderate and internally consistent -- all in the order of a few millions. This is to be contrasted with the BIS dichotomy of no charge for both non-extendible irrevocable commitments and all uncommitted commitments, and an extremely heavy charge (0.08 x $15.6 billion = $1.25 billion) for longer-term irrevocable commitments. We round up this first policy implication by formalizing the option-based procedure just proposed in

**PROPOSITION I:** (1) for the *i*th maturity range, apply the exercise-cum-takedown proportion, $\pi_i$, to the commitment contractual amount, $L_i$; (2) weight this amount by the put value implicit in extendible commitments, $E Pi$, to arrive at the risk-adjusted balance; and finally (3) apply the 8% capital requirement to the commitment balance. Or analytically: $\pi_i \times L_i \times E Pi \times 0.08 =$ the “fair” capital charge, expressed in dollars, corresponding to the “true” credit risk of commitments in the *i*th maturity range.

5. NEW STANDARD CREDIT RISK WEIGHTS FOR LOAN COMMITMENTS

We next propose that the risk weights applicable to all credit commitments be based on the two previously developed concepts. The extendible commitment put can be written as $E Pi = f (X, \sigma, r, L_1, L_2, T_2 - T_1, f_t^E)$ and the exercise-cum-takedown proportion as $\pi_i = g (d_i, I_i, T_2 - T_1)$. In the previous developments, most of the parameters of both variables were kept constant, with only the following parameters being really variables: $X$, $\sigma$, and $T_2 - T_1$. As the indebtedness-value volatility is low and relatively constant, we can assume that $E Pi$ and $\pi_i$ be dependent on $X$ and $T_2 - T_1$, respectively. In what follows, the matrix of new risk weights rely on two not-unreasonable assumptions: (i) the extendible put value is mainly a function of the indebtedness value, the latter
being a proxy for the borrowers’ risk ratings of external credit agencies and (ii) the exercise-cum-
takedown proportion varies with the length of the extension period.

First, regarding the extendible-put sensitivity to \( X \), we make the following observation: the
floating rate and hence forward markup of credit commitments are generally set below the rate and
markup set in spot loans. It is moreover sensible to assume that the differential between spot markup
and forward markup grows larger as the borrower’s risk rating by external credit agencies declines.
In essence, we propose to associate the progressively in-the-money indebtedness values with the
deleing rating ranges proposed for on-balance-sheet loans in the Second Consultative Document
(BIS 2000 or Fischer 2001). The argument runs as follows. For prime-rate borrowers (say, those in
the risk range \([\text{AAA to AA}]\)), the bank is likely to charge a spot markup that is equal to the forward
markup of credit commitment: in that case the indebtedness value, \( X \), is equal to the line par value,
\( L \). But for spot loans and credit lines of borrowers with a rating in the range or risk bucket \([\text{A}^+ \text{ to A}^-]\),
the loan spot markup is slightly higher than the corresponding forward markup charged on credit
lines. Hence, according to expression (1), the indebtedness value corresponding to this risk bucket is
lower than the line $100 par value, say $99.5. To lower risk ranges correspond deeper in-the-money
indebtedness values. This holds true up to the lowest risk range, defined as less than \( B^- \), which
 corresponds to the indebtedness value $98.

Second, regarding the funding proportion, it was already observed in subsection 2.5 that the
proportion of line funding is likely to be somewhat greater the longer the commitment extension
period. Generally speaking, borrowers have more opportunities to draw cumulatively on the credit
line if the extension period is longer than one year. The computation of the new risk weights is based
on the following scale: the funding proportion increases progressively from 50\% of the initial $100
maximum for one-year straight commitments to 75\% for six-year commitments – namely those with
a five-year extension period beyond the initial one-year commitment. Given the above assumptions,
the proposed matrix of standard risk weights has the advantage to be a function of three parameters:
the risk rating ranges of external credit agencies, the length of the extension period, and the variable
takedown proportion. The granularity of this matrix is indeed richer (although it could be improved
by increasing the number of risk grades in a bank’s internal rating system) than the present BIS
coefficients characterized by the superficial time-to-maturity dichotomy for irrevocable commitment
and only two principal-risk factors for commitments.

The new risk weights per $100 of borrower’s extendible commitment are presented in Table 4: columns of this two-entry table refer to rating ranges from external credit agencies and rows to the proportion of line funding\(^{14}\). Not unexpectedly, the table rows reveal that, for a given level of line takedown, the risk weights increase linearly as the indebtedness value moves progressively deeper in the money. To wit, for a commitment with a two-year extension period on the matrix third row, the risk weights vary from $0.517 for top investment-grade borrowers to $1.556 for below-investment-grade borrowers. By the same token for a given credit rating, the risk weights also increase with the duration of the commitment extension. For top credit borrowers in the first column, the risk weights increase exponentially from $0.098 for a one-year straight commitment to $1.801 for a commitment with a five-year extension period. More concretely, the risk weights are increasing linearly with lower credit ratings (for a given takedown proportion), but exponentially with the takedown proportion for a given risk bucket. Finally, the proposed weights are simply multiplied by 8%, the Cooke ratio, to determine the credit-risk capital charge per $100 of CL commitment. We round up this second policy implication by formalizing it in

**PROPOSITION II:** (1) compute the sensitivity of put values, the \(EPi\), to the indebtedness value, \(x\), and the extension duration, \(T_2 - T_1\); (2) multiply the rows of the resultant matrix by the varying takedown proportions, the \(\pi_is\); and (3) apply the 8% regulatory capital requirement to the new standard credit-risk weights. Or analytically, \([EPi = f(x, T_2 - T_1)] \times [\pi_i = g(T_2 - T_1)] \times 0.08 = \) the credit-risk capital charge per $100 of borrower-extendible loan commitments. The same procedure holds also true for straight commitments.

6. CONCLUDING REMARKS

This research makes two contributions. The first one is to price analytically put option

\(^{14}\) The matrix captures the sensitivity of the extendible put value to the indebtedness value and the extension duration, namely \(\partial^2 EPi/\partial X \partial (T_2 - T_1)\), the second cross-partial argument derivative of \(EPi\). Each row of the resultant matrix is next multiplied by a given exercise-cum-takedown proportion, \(\pi_i\).
and extension premiums embedded in rollover commitments; and the second is to combine extendible put and the exercise-cum-takedown proportion in computing the ‘fair’ capital charge corresponding to the commitment ‘true’ credit risk. In doing so, the procedure proposes to do away with the BIS accounting-based concepts of conversion factor, principal-risk factor, and commitment term-to-maturity dichotomy. They are replaced by the exercise-cum-takedown proportion and the put value implicit in borrower-extendible commitment contracts, respectively. The fair-value procedure has the advantage that (i) the put value constitutes a finer credit-risk grid than the two artificial values of the conversion and principal-risk factors, and (ii) capital charges computed from risk-weighted balances are quite moderate and internally consistent for all types of commitments. Finally, the paper provides new standard commitment risk weights that account for the borrower’s rating ranges of external credit agencies. Further work will consider expressing rollover commitments as multiple shout options, as was done for equity options in Cheuk and Vorst (1997) and Windcliff et al. (2003). For extendible commitments, shout options have the advantage to allow resetting at the end of each commitment or extension period the indebtedness value equal to the line par value; it also allows accounting for multiple extension fees. Another point to elaborate further is the change of markup class by the bank’s borrower: a matrix of transition probability between markup states seems a promising start.
APPENDIX

In eq. (7), the payoff of the extension privilege was written:

\[
[P( X_1, L_2, T_2 - T_1) - f^{E}_{\alpha}(L_1 - X_1)]1_{t_2 \leq X_1 \leq \alpha_l} + [P( X_1, L_2, T_2 - T_1) - f^{E}_{\alpha}]1_{L_2 \leq X_1 \leq \alpha_l}, \tag{7}
\]

This expression can be rearranged further as:

\[
P(X_1, L_2, T_2 - T_1)1_{t_2 \leq X_1 \leq \alpha_l} - f^{E}_{\alpha}1_{t_2 \leq X_1 \leq \alpha_l} - (L_1 - X_1)1_{L_2 \leq X_1 \leq \alpha_l},
\]

or

\[
P(X_1, L_2, T_2 - T_1)1_{X_1 \geq t_2} - P(X_1, L_2, T_2 - T_1)1_{X_1 \geq t_1} - f^{E}_{\alpha}1_{X_1 \geq \alpha_l} + f^{E}_{\alpha}1_{X_1 \geq \alpha_l} - (L_1 - X_1)1_{X_1 \geq \alpha_l}
+ (L_1 - X_1)1_{X_1 \geq \alpha_l} \tag{A.1}
\]

The present value of the expected value of the extension privilege can be decomposed into six components:

\[
\text{EP-premium} = \text{EP}(1) - \text{EP}(2) - \text{EP}(3) + \text{EP}(4) - \text{EP}(5) + \text{EP}(6) \tag{A.2}
\]

Each of these components are then further developed to arrive at the final expression. The six final expressions are respectively:

\[
\text{EP}(1) = e^{-rT_1} \hat{\mathbb{E}}[P(X_1, L_2, T_2 - T_1)1_{X_1 \geq t_1}]
= XN_2(-x^*, z_2; -\rho) - L_2 e^{-rT_2} N_2(-x^* + \sigma \sqrt{T_2}, z_2 - \sigma \sqrt{T_1}; -\rho) \tag{A.3}
\]

\[
\text{EP}(2) = e^{-rT_1} \hat{\mathbb{E}}[P(X_1, L_2, T_2 - T_1)1_{X_1 \geq t_1}]
= XN_2(-x^*, z_1; -\rho) - L_2 e^{-rT_2} N_2(x^* - \sigma \sqrt{T_2}, z_1 - \sigma \sqrt{T_1}; -\rho) \tag{A.4}
\]
\[ EP(3) = e^{-rT_1} \hat{E} \left[ f_{T_1}^E 1_{X_{T_1} \leq L_1} \right] \]
\[ = f_{T_1}^E e^{-rT_1} N(-z_1 + \sigma \sqrt{T_1}) \]
(A.5)

\[ EP(4) = e^{-rT_1} \hat{E} \left[ f_{T_1}^E 1_{X_{T_1} \leq L_2} \right] \]
\[ = f_{T_1}^E e^{-rT_1} N(-z_2 + \sigma \sqrt{T_1}) \]
(A.6)

\[ EP(5) = e^{-rT_1} \hat{E} [(L_1 - X_{T_1}) 1_{X_{T_1} \leq L_1}] \]
\[ = -XN(-x) + L_1 e^{-rT_1} N(-x + \sigma \sqrt{T_1}) \]
\[ = P_0(X, L_1, T_1) \]
(A.7)

\[ EP(6) = e^{-rT_1} \hat{E} (L_1 - X_{T_1}) 1_{X_{T_1} \leq L_2} \]
\[ = -XN(-z_2) + L_1 e^{-rT_1} N(-z_2 + \sigma \sqrt{T_1}) \]
(A.8)

where \( \hat{E} \) denotes expectations in the risk-neutral world and \( x, x^*, z_1, z_2 \) and \( \rho \) are defined as follows:

\[
\begin{align*}
x &= \frac{\ln(X/L_1) + (r + 0.5 \sigma^2)T_1}{\sigma \sqrt{T_1}} \\
x^* &= \frac{\ln(X/L_2) + (r + 0.5 \sigma^2)T_2}{\sigma \sqrt{T_2}} \\
z_1 &= \frac{\ln(X/I_1) + (r + 0.5 \sigma^2)T_1}{\sigma \sqrt{T_1}} \\
z_2 &= \frac{\ln(X/I_2) + (r + 0.5 \sigma^2)T_1}{\sigma \sqrt{T_1}} \\
\rho &= (T_1/T_2)^{\frac{1}{2}}
\end{align*}
\]

By collecting the results in (A.3) to (A.8), we obtain the analytical value of the extension premium at date \( s = 0 \):

\[ \text{EP-premium} = XN_2(-x^*, z_2; -\rho) - L_2 e^{-rT_2} N_2(-x^* + \sigma \sqrt{T_2}, z_2 - \sigma \sqrt{T_2}; -\rho) - [XN_2(-x^*, z_1; -\rho) - L_2 e^{-rT_1} N_2(-x^* + \sigma \sqrt{T_1}, z_1 - \sigma \sqrt{T_1}; -\rho)] - f_{T_1}^E e^{-rT_1} N(-z_1 + \sigma \sqrt{T_1}) + f_{T_1}^E e^{-rT_1} N(-z_2 + \sigma \sqrt{T_1}) - N_0(X, L_1, T_1)
\]
\[ + [-XN(-z_2) + L_1 e^{-rT_1} N(-z_2 + \sigma \sqrt{T_1})]
\]
(A.9)
This is expression (8) in the body of the text, with the i-postscript to EP indicating the length of the extension period, $T_2 - T_1$. 
REFERENCES


**TABLE 1: European put values embedded in extendible and non-extendible commitments**

Valuation at date $s = 0$. Entries on line 1: PO, European put values implicit in one-year straight commitments, from eq. (2). Entries in matrix 2: put values implicit in borrower-extendible commitments, $EP_i$, from eq. (7) with $i: 1, \ldots, 5$ the length of the extension period. Entries in matrix 3: extension premiums, $[EP_i - PO]/EP_i$, with $i: 1, \ldots, 5$, respectively.

Parameter definition: $L_1 = L_2$: credit line exercise value in $\$; $r = \text{short-term rate of interest, in } \% \text{ per annum}; \sigma = \text{indebtedness-value volatility in } \% \text{ per annum}; T_1 = \text{commitment initial maturity date}; T_2 = \text{commitment terminal maturity date, with } T_2 \text{ varying from 2 to 6 years}; T_2 - T_1 = \text{extension duration, from 1 to 5 years}; T^* = \text{loan maturity date}; \text{and } X = \text{indebtedness value in } \$ \text{ computed from eq. (1)}.$

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<th>#</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<td>0.296</td>
<td><strong>0.434</strong></td>
<td>0.613</td>
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</table>

**Matrix 2**

a) EP1

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</table>

**Matrix 3**

a) EP1-Premium

<p>| | | | | | | | | | | | | | | | |</p>
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<td>43.5</td>
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<td>86.1</td>
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<td><strong>90.6</strong></td>
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</tr>
</tbody>
</table>

Common parameter values: $L_1 = L_2 = 100; \ r = 0.04; \ \sigma = 0.03; \ T_1 - s = 0.5; \ T_2 - s = 1.5, \ldots, 5.5; \ T_2 - T_1 = 1, \ldots, 5$. 

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PANEL A

<table>
<thead>
<tr>
<th></th>
<th>Off-balance sheet commitments</th>
<th>On-balance-sheet loans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Irrevocable(^1)</td>
<td>revocable(^1)</td>
</tr>
<tr>
<td>With an original term to maturity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Contractual amount, C$ in billions</td>
<td>40.9</td>
<td>34.1</td>
</tr>
<tr>
<td>(2) \textbf{Credit conversion} factor, in %</td>
<td>0</td>
<td>50%(^0)</td>
</tr>
<tr>
<td>(3) Credit-equivalent amount, C$ in billions</td>
<td>nil</td>
<td>17.1</td>
</tr>
<tr>
<td>(4) \textbf{Principal risk} factor, in %</td>
<td>0</td>
<td>100%/92(^3)</td>
</tr>
<tr>
<td>(5) BIS risk-adjusted balance, C$ in billions</td>
<td>nil</td>
<td>15.6</td>
</tr>
</tbody>
</table>

PANEL B

<table>
<thead>
<tr>
<th>BIS time ranges</th>
<th>within 1 yr</th>
<th>1 to 3 yrs</th>
<th>over 3 to 5 yrs</th>
<th>over 5 yrs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irrevocable commitments(^1)</td>
<td>44.8</td>
<td>10.6</td>
<td>4.2</td>
<td>15.4</td>
<td>75.00</td>
</tr>
<tr>
<td>Revocable commitments(^1)</td>
<td>0.8</td>
<td>41.1</td>
<td>2.8</td>
<td>1.2</td>
<td>46.00</td>
</tr>
<tr>
<td>Sub-total</td>
<td>45.6</td>
<td>51.7</td>
<td>7.1</td>
<td>16.6</td>
<td>121.00</td>
</tr>
<tr>
<td>% of previous line sub-total</td>
<td>37.72</td>
<td>42.76</td>
<td>5.84</td>
<td>13.69</td>
<td>100.00</td>
</tr>
</tbody>
</table>

1 Irrevocable commitments are unused portions of firm authorizations to extend credit and revocable commitments are offers but no obligations to extend credit.
2 n.a. = not applicable.
3 The first figure refers to the BIS-set percentage and the second to the actual (after netting out) weighted average of counterparty risk within this class. The latter figure is used to compute (5).

Source: Royal Bank of Canada, 2002 annual report. For Panel A: Table 25, p 60 and Note 18, p 91, and for Panel B, Table 27, p 63.
TABLE 3: Fair (option-based) capital charge of off-balance-sheet commitments; all commitments are assumed to be borrower-extendible commitments.

<table>
<thead>
<tr>
<th>Original term to maturity$^1$</th>
<th>1 yr</th>
<th>2 yrs</th>
<th>4 yrs</th>
<th>6 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Contractual amount, C$ in billions</td>
<td>45.6</td>
<td>51.7</td>
<td>7.1</td>
<td>16.6</td>
</tr>
<tr>
<td>(2) Exercise-cum-takedown proportion, $\pi_i$ in %</td>
<td>50</td>
<td>55</td>
<td>65</td>
<td>75</td>
</tr>
<tr>
<td>(3) Credit-equivalent amount, C$ in billions</td>
<td>22.8</td>
<td>28.4</td>
<td>4.6</td>
<td>12.45</td>
</tr>
<tr>
<td>(4) Commitment put value, EPi per billion$^2$</td>
<td>.00434</td>
<td>.00733</td>
<td>.00270</td>
<td>.00462</td>
</tr>
<tr>
<td>(5) Risk-weighted balance, C$ in millions</td>
<td>98.9</td>
<td>208.2</td>
<td>12.5</td>
<td>57.5</td>
</tr>
<tr>
<td>(6) Fair capital charge, C$ in millions</td>
<td>7.9</td>
<td>16.7</td>
<td>1.0</td>
<td>4.6</td>
</tr>
</tbody>
</table>

1 The original term to maturity corresponds to the mid-point of the maturity ranges introduced in Panel B of Table 2. To wit, the two-year commitment (one-year commitment period + one-year extension period) characterizes commitments with an initial term from 1 to 3 years.

2 These EPi values are from column (3) in Table 1, but per one billion of commitment face value. They correspond to our representative scenario in which the indebtedness value is slightly in the money at X = $99.
**TABLE 4: Proposal for new standard credit risk weights: Weights per $100 of straight and borrower-extendible credit commitments (up to a five-year extension)**

<table>
<thead>
<tr>
<th>Borrowers’ risk bucket or indebtedness value, x in $</th>
<th>[AAA to AA']</th>
<th>[A+ to A']</th>
<th>[3B+ to 3B']</th>
<th>[BB+ to B']</th>
<th>&lt; B'</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$99.5</td>
<td>$99.0</td>
<td>$98.5</td>
<td>$98.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line takedown, $\pi_i$ in %:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_0 = 0.50$</td>
</tr>
<tr>
<td>$0.098$</td>
</tr>
<tr>
<td>$0.148$</td>
</tr>
<tr>
<td>$0.217$</td>
</tr>
<tr>
<td>$0.306$</td>
</tr>
<tr>
<td>$0.412$</td>
</tr>
<tr>
<td>$\pi_1 = 0.55$</td>
</tr>
<tr>
<td>$0.190$</td>
</tr>
<tr>
<td>$0.285$</td>
</tr>
<tr>
<td>$0.403$</td>
</tr>
<tr>
<td>$0.508$</td>
</tr>
<tr>
<td>$0.654$</td>
</tr>
<tr>
<td>$\pi_2 = 0.60$</td>
</tr>
<tr>
<td>$0.517$</td>
</tr>
<tr>
<td>$0.707$</td>
</tr>
<tr>
<td>$0.980$</td>
</tr>
<tr>
<td>$1.275$</td>
</tr>
<tr>
<td>$1.556$</td>
</tr>
<tr>
<td>$\pi_3 = 0.65$</td>
</tr>
<tr>
<td>$0.911$</td>
</tr>
<tr>
<td>$1.276$</td>
</tr>
<tr>
<td>$1.756$</td>
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<td>$2.287$</td>
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<tr>
<td>$2.852$</td>
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<tr>
<td>$\pi_4 = 0.70$</td>
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<td>$1.341$</td>
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<td>$3.360$</td>
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<td>$4.218$</td>
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<td>$\pi_5 = 0.75$</td>
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<td>$1.801$</td>
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<td>$3.465$</td>
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<td>$4.531$</td>
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<td>$5.674$</td>
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</table>

Common parameter values: $L_1 = L_2 = 100; r = 0.04; \sigma = 0.03; T_1 - s = 0.5; T_2 - s = 1.5, ..., 4.5.$

1 The takedown proportion varies with the length of the extension period, denoted by the $\pi$-subscript.