Regime Dependent Conditional Volatility in the U.S. Equity Market

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Abstract

This study develops the Regime Dependent Generalized Auto Regressive Conditional Heteroskedasticity (RD-GARCH) model and applies it to a daily index of returns on U.S. equities covering the period 1926 to 2000. The model is based on the ARCH methodology first proposed by Engle (1982) and refined by subsequent researchers. The RD-GARCH model is different from previous models in that it combines the ARCH methodology with a general approach which allows model parameters to vary across periods or regimes of differing volatility.

Likelihood ratio tests of the within-sample properties of the RD-GARCH methodology demonstrate superiority of the model over a variety of models from the conventional GARCH family, both for the full sample period and for a number of sub-periods.

The results of this study indicate that combining a model which allows conditional volatility to change in an evolutionary manner with a model which allows for abrupt changes in unconditional volatility at discrete points in time provides a significant improvement over previously available models. The RD-GARCH model provides a general approach to combining these two alternative modeling methodologies, thereby providing a valuable addition to the GARCH family of conditional volatility models.
1 Introduction

A number of techniques exist for specifying the nature of the expected volatility of financial series. Some of these techniques, such as the Auto Regressive Integrated Moving Average (ARIMA) approach of Box and Jenkins (1976), make the assumption that the variance of the underlying series is constant through time or that simple transformations such as taking logs or differencing will render the variance constant. The assumption of constant variance is problematic, however, as inspection of stock return series over long time periods will indicate.

Alternatively, the model may be specified in such a way that the variance of the series being modeled is allowed to change over time. The number of available specifications for models with time varying volatility, or heteroskedasticity, is large, but one particularly useful family of such models has grown out of the Auto Regressive Conditional Heteroskedasticity (ARCH) specification first suggested by Engle (1982). Models based on this specification allow the conditional volatility of the series to evolve predictably through time as a function of current and prior unexpected shocks, prior conditional volatility, and possibly other exogenous variables. These models make the implicit assumption that the volatility “clumping” commonly observed in financial time series, such as stock returns, is the result of persistence in volatility from period to period; periods of high volatility tend to be followed by periods of high volatility and periods of low volatility tend to be followed by periods of low volatility. Since the parameters of these models do not change through time, there is an implicit assumption that the unconditional volatility, $\sigma_u^2$, is constant.\footnote{For the simple ARCH specification of Engle (1982):}

$$\text{var}(u_t|I_{t-1}) = h_t^2 = \omega + \sum_{i=1}^{q} \alpha_i u_{t-i}^2$$

the unconditional volatility of the residuals from the mean equation, $\sigma_u^2 = E(u_t^2) = E(h_t^2)$, is given by:

$$\sigma_u^2 = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i}$$

If the $\alpha_i$ are time-invariant, the unconditional volatility will also be constant through time.
an ARIMA model. If such a shift is detected, the original series is transformed to remove the impact of the variance shift and the process repeated until no new variance shifts are detected. Hamilton (1989) provides a somewhat different approach through a Markov switching model applied to the series. This model generates estimates of the probabilities that the series has switched into one of several predefined volatility states.

Both of these alternative explanations for time-varying volatility have merit, and some attempts have been made at combining the techniques to address specific modeling situations. In particular, Hamilton and Susmel (1994) combine the ARCH framework with Hamilton’s Markov switching model. This approach uses the Markov switching model to determine the probability that the economy is in a particular volatility state (regime) and then allows some of the parameters of the ARCH conditional volatility equation to vary depending upon the regime. Hamilton and Susmel’s approach only works with ARCH models, however, as the lagged conditional volatility from the GARCH model would require the Markov chain to be maintained for the total number of observations over which the model is estimated. The approach would be numerically intractable for GARCH models covering a large number of observations.

Gray (1996) extends the Markov switching process to include a special case of the GARCH(1,1) model by enforcing an assumption of conditional normality for the series being modeled. This assumption of conditional normality is problematic, however, as even conditional stock returns are generally found to be leptokurtic. In addition, the model is specified only for the simple GARCH(1,1) form of conditional volatility.

Baldauf and Santoni (1991) investigate the hypothesis that the advent of program trading has caused a shift in the level of volatility in the underlying stock market. They propose a model in which the parameters of the ARCH conditional volatility equation are allowed to change at a discrete point:

\[ h_t^2 = \omega + \sum_{i=1}^{q} \alpha_i u_i^2 + \beta_0 D_f + \sum_{i=1}^{q} \beta_i u_i^2 D_f \]  

(1)

In this specification, \( D_f \) is a \((0,1)\) dummy variable which is zero prior to the date corresponding to the advent of program trading and one otherwise. They find no evidence of a change in the

\(^2\text{(See, for example, Campbell et al., 1997, pp. 488–489)}\)
parameters of the ARCH equation corresponding to the advent of program trading. They do note, however, that “Identifying the advent of program trading is problematic.” (p. 198)

These approaches to combining the evolutionary nature of GARCH models with models that allow for changing unconditional volatility provide potential solutions in specific situations. This paper combines the two techniques in a generalized manner to generate better estimates of expected volatility. In particular, the ARCH approach is combined with a technique that allows model parameters to depend upon the current exogenously determined unconditional volatility regime to form the Regime Dependent Generalized Auto Regressive Conditional Heteroskedasticity (RD-GARCH) model.

The remainder of this paper is organized as follows. The next Section identifies the model which will form the base for the RD-GARCH model and outlines the graphical tools that will be used to interpret the models developed. Section 3 describes the data that will be used in the development and application of the model. Section 4 provides the formal development of the RD-GARCH model and Section 5 presents the results from the application of the model to both the full sample period and to a number of sub-periods. Section 6 provides concluding remarks.

2 The Model

Since Engle (1982) first proposed the ARCH approach to modeling conditional volatility, a wide variety of alternative, closely related specifications have appeared. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model of Bollerslev (1986) addresses the problem of the long and arbitrary fixed lag structure frequently required for satisfactory model fit of the simple ARCH specification by including lagged values of the conditional variance directly in the conditional variance specification. The Exponential GARCH (EGARCH) model of Nelson (1991) avoids the need for parameter restrictions to ensure non-negativity of the conditional variance by allowing the conditional volatility to evolve as the log of the conditional variance, \( \ln h_t^2 \). This specification also allows the conditional variance to be an asymmetric function of the residuals from the mean equation—that is, positive and negative shocks of equal size can have a different impact on subsequent conditional volatility. The Nonlinear ARCH (NARCH) model of Higgins and Bera (1992) provides a more general functional form of which allows for nonlinear evolution of the conditional
volatility through the use of a Box-Cox power transformation. The Asymmetric Power ARCH (APARCH) model of Ding et al. (1993) also incorporates the Box-Cox transformation of the conditional volatility and allows for asymmetry in the response of conditional volatility to unexpected shocks. As with the EGARCH and APARCH models, the GARCH(1,1) model of Glosten et al. (1993) (GJR-GARCH) provides for asymmetry. In addition, the model allows the risk-free interest rate to impact the conditional volatility directly as an additional explanatory variable. The Non-linear Asymmetric GARCH (N-A GARCH) specification of Engle and Ng (1993) also allows the conditional variance to be an asymmetric function of the residuals from the mean equation, however the form of asymmetry provided in the N-A GARCH model is different than the form provided in the EGARCH, APARCH, and GJR-GARCH models. There are many other members of this family of conditionally heteroskedastic models. Engle and Rothschild (1992) and Bollerslev et al. (1992) provide reviews of this group of models. Unfortunately, it is exactly this richness of functional forms that makes applying the GARCH methodology problematic; it is frequently unclear which one of the particular competing and often partially overlapping specifications is appropriate in a given modeling situation.

2.1 Nested GARCH

The nested GARCH specification of Hentschel (1995) seeks to resolve this problem by providing a single specification which nests several of the more popular extensions to the GARCH model directly into a single equation. The form of this equation for the GARCH(1,1) family of models is:

$$\frac{h_t^\lambda - 1}{\lambda} = \omega + \alpha h_{t-1}^{\lambda} f'(v_{t-1}) + \beta \frac{h_{t-1}^\lambda - 1}{\lambda}$$

(2a)

$$f(v_t) = |v_t - b| - c(v_t - b)$$

(2b)

where $h_t$ is the conditional volatility, $v_t$ is the standardized residual from the mean equation\(^3\), and $(\lambda, \omega, \alpha, \beta, \nu, b, c)$ are coefficients to be estimated. The sub-models which are nested in this spec-

\(^3\)The GARCH specification makes the distributional assumption $u_t \sim (0, h_t^2)$. Because the variance evolves through time, a useful decomposition of the residual is: $u_t = h_t v_t$ or $v_t = \frac{u_t}{h_t}$ where $v_t$ has the useful property that $v_t \sim iid(0,1)$.  

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ification obtain from equality and inequality constraints on the parameters. The appropriateness of each nested model to in a particular situation can then be assessed using Likelihood Ratio tests of the parameter restrictions.

### 2.2 The News Impact Curve

While the nested GARCH specification is attractive in that it allows simple likelihood ratio tests to determine the most appropriate model for a particular situation, the large number of parameters provided can make the results difficult to interpret. A tool which allows for graphical interpretation is the *News Impact Curve* (NIC) of Engle and Ng (1993). The NIC relates the conditional volatility, $h_t$, to the lagged standardized residuals from the mean equation, $v_{t-1}$, providing a graphical indication of the impact of unexpected shocks, or news, on subsequent conditional volatility. The curve indicates how shocks of different magnitudes and signs are impounded into conditional volatility. The various parameters of the model affect this curve in different ways.

The parameters $\omega$ and $\alpha$ in Equation (2a) correspond to the intercept and the coefficient of the lagged residual in Engle’s original ARCH conditional variance specification. The intercept locates the NIC in space and represents unconditional volatility. The parameter $\alpha$ represents the short term memory of the conditional volatility specification by controlling the curvature of the NIC. Smaller values of $\alpha$ are associated with a flatter NIC, which indicates that the immediate impact on conditional volatility for a given shock is smaller for models characterized by a small value for $\alpha$ than for models with large values for the parameter.

[Figure 1 about here.]

For the simple ARCH and GARCH specifications of Engle and Bollerslev respectively, the

\[ h_t^2 = \omega' + \alpha' u_{t-1}^2 + \beta h_{t-1}^2 \]

where

\[ \omega' = 1 + 2\omega - \beta \]
\[ \alpha' = 2\alpha \]

which is Bollerslev’s GARCH(1,1) specification. Other models can be similarly constructed by imposing parameter restrictions.
impact of news on volatility is symmetric. Positive and negative shocks of the same magnitude have the same impact on conditional volatility in the next period. As shown in Figure 1, different GARCH specifications allow for asymmetry in the response of conditional volatility to news in two ways: The parameter $c$ in Equation (2b) allows for rotation of the NIC, which is the method provided in the EGARCH, APARCH, and GJR-GARCH models. For $c > 0$ the curve is rotated clockwise as shown. In this case a negative shock will have a larger impact on subsequent conditional volatility than an equally large positive shock. Because of the rotation of curve this effect is magnified for progressively larger shocks. The parameter $b$ also provides for asymmetry in the response of conditional volatility to shocks through a shift of the NIC, which is the method provided in the N-A GARCH model. For $b > 0$ the NIC is shifted to the right as shown. A negative shock will have a larger impact on subsequent conditional volatility than a positive shock of the same magnitude. Of course, these asymmetry effects can be combined—the NIC can be both shifted and rotated simultaneously.

The parameter $\nu$ controls the shape of the NIC. For $\nu = 1$ the relationship between small and large shocks is linear; the difference in impact for shocks with a given difference in magnitude is the same, regardless of the magnitude of the shocks. For $\nu < 1$ the NIC is concave and the difference in impact for shocks with a given difference in magnitude is greater for small shocks than for large shocks. Similarly, when $\nu > 1$ the curve is convex and difference in impact for shocks with a given difference in magnitude is greater for large shocks than for small shocks.

### 2.3 The History Impact Curve

The parameter $\beta$ does not directly affect the NIC. Instead, it describes the impact of prior period conditional volatility on current conditional volatility. In essence, this parameter defines the long-term memory of the conditional volatility model. As it is not possible to interpret this effect in relation to the NIC, an interpretation is afforded by a new device entitled the *History Impact Curve* (HIC) because it represents the contribution of the conditional volatility from the previous period to the current conditional volatility. As the impact of the shock in the immediately prior period does not directly impact this curve, it represents longer term effects, or the history of the conditional volatility. As Figure 2 indicates, $\beta$ controls the slope of the HIC in a manner similar to the effect
of $\alpha$ on the NIC. In essence, $\beta$ controls the degree of persistence in the conditional volatility. The greater the slope of the HIC, the more persistent the conditional volatility.

[Figure 2 about here.]

[Figure 3 about here.]

The parameter $\lambda$ defines a Box-Cox transformation of the conditional volatility series. As such it can be interpreted in terms of specifying the functional form for the conditional volatility. While the impact of $\lambda$ cannot be defined simply in terms of how it affects either the NIC or the HIC, its effect can readily be demonstrated in both of these devices. The left-hand panel of Figure 3 shows that the effect of $\lambda$ on the NIC is similar to the combined effect of the parameters $\omega$ and $\alpha$. Increasing values for $\lambda$ are associated with a NIC which is located higher in space and exhibits less curvature. The right-hand panel shows that $\lambda$ affects the shape of the HIC in similar manner to the effect of $\nu$ in the NIC. The parameter $\lambda$ allows for a non-linear response of persistence to small and large values of conditional volatility.

3 Data

The sample comprises logged daily returns on U.S. equities covering the period January 2, 1915 through December 29, 2000. The daily stock returns for the period January 2, 1915 through July 2, 1962 are from Schwert (1989), and for the period July 3, 1962 through December 29, 2000 are from the Center for Research in Security Prices (CRSP) value weighted stock indices for NYSE and AMEX combined.

4 Model Development

Because it nests many of the more popular GARCH models into a single equation and allows the most appropriate model to be selected via Likelihood Ratio tests, the Nested GARCH model of Hentschel (1995) forms an excellent foundation for extensions to the GARCH framework. The particular model employed by Hentschel was the GARCH-in-Mean or GARCH-M specification:
\[ r_t = \gamma_0 + \gamma_1 h_t^2 + u_t \]  \hspace{1cm} (3a)  
\[ v_t = \frac{u_t}{h_t} \]  \hspace{1cm} (3b)  
\[ \frac{h_t^\lambda - 1}{\lambda} = \omega + \alpha h_{t-1}^\lambda f'(v_{t-1}) + \beta \frac{h_{t-1}^\lambda - 1}{\lambda} \]  \hspace{1cm} (3c)  
\[ f(v_t) = |v_t - b| - c(v_t - b) \]  \hspace{1cm} (3d)

where \( \text{var}[u_t|I_{t-1}] = h_t^2 \). As the form of Equation (3c) is not convenient for estimation purposes, it is rewritten:

\[ h_t = \begin{cases} 
(\omega + \alpha h_{t-1}^\lambda f'(v_{t-1}) + \beta h_{t-1}^\lambda)^{\frac{1}{\lambda}}, & \lambda \neq 0 \\
\exp(\omega + \alpha f'(v_{t-1}) + \beta \log h_{t-1}), & \lambda = 0 
\end{cases} \]  \hspace{1cm} (4)

4.1 Mean Equation Development

The GARCH-M specification is an attractive starting point for development of the mean equation portion of the model both because it provides a standard against which to compare alternative specifications and because of its popularity in the finance literature (see, for example, Engle et al., 1987; French et al., 1987; Nelson, 1991; Glosten et al., 1993). It is important to ensure that the conditional mean equation is properly specified to avoid misleading results when interpreting the conditional variance, however. One potential source for misspecification is suggested by French et al. (1987), who note that stock portfolio returns tend to be autocorrelated, particularly at lag one. If such autocorrelation is present, attempting to impose a model which does not include lagged values of the dependent variable would result in apparent non-linear dependence in the conditional variance of the series. A second potential source of model misspecification is the so-called “weekend effect” first identified by French (1980) in which daily returns exhibit a smaller than average value on Mondays. This effect can be thought of as a simple but predictable shifting of the mean, which again causes apparent nonlinear dependencies in the conditional volatility if not accounted for in the model. By adding additional terms to Equation (3a), it is possible to construct a mean equation that corrects for these possible sources of model misspecification. The general form of the
The augmented conditional mean equation is:

\[ r_t = \gamma_0 + \gamma_1 h_t^2 + \gamma_2 r_{t-1} + \gamma_3 r_{t-2} + \gamma_4 D_M + u_t \]  

(5)

where, as before, \( u_t \) has zero mean and conditional variance \( h_t^2 \), \( r_{t-i} \) represents the return lagged \( i \) periods,\(^5\) and \( D_M \) is a \((0, 1)\) dummy which is one if the observation \( r_t \) occurred on a Monday, and zero otherwise.

Table 1 reports likelihood ratio tests for misspecification of the conditional mean equation. The table compares the unrestricted conditional mean equation to the restricted equations comprising those with all reasonable subsets of the parameters included. The test results show that the null hypothesis of the simple GARCH-M mean equation (\( \gamma_2 = \gamma_3 = \gamma_4 = 0 \)) can be rejected in favour of the unrestricted model which includes in addition the two lagged returns and the Monday dummy. Similarly, the mean equations which include a single lagged return, both lagged returns alone, and a single lagged return plus the Monday dummy can be rejected at the 5% level in favour of the unrestricted mean equation. The mean equation which includes both lagged returns and the Monday Dummy but excludes the GARCH-M term cannot be rejected in favour of the unrestricted mean equation. A conditional mean equation which includes the daily excess return lagged one and two periods and a Monday dummy but does not include the GARCH-M term is superior to the simple GARCH-M specification.

4.2 Conditional Volatility Equation Development

If the specification for conditional volatility is correct, the standardized residual from the conditional mean equation, \( v_t = u_t / h_t \), should be homoskedastic with unit variance. Levene’s method provides a convenient test for homogeneity of variance which is robust to non-normality. The computed value test statistic for the standardized residuals from the conditional variance equation selected

\(^5\)To select a value for the number of lagged returns to include in the conditional mean specification, the excess return on the daily stock index series was regressed on the first three lags of the excess return and a constant. Only the first and second lagged values of the return are significantly different from zero. A likelihood ratio test comparing the unrestricted model which included all three lagged returns with the restricted models containing one and two lagged returns confirmed the decision to include the return lagged one and two periods.
in Section 4 and the mean equation developed in Section 4.1 is 10.333 and the 5% critical value is 1.88; the null of homogeneous variance for the standardized residuals is strongly rejected. This is problematic as the GARCH specification under investigation, which is the preferred specification from the broad ranged of models nested in Equation (3c), does not adequately compensate for the heteroskedasticity apparent in the data.

Conditional volatility specifications from the GARCH family provide a mechanism for the variance of residuals from the mean equation to change through time. A key assumption underlying conventional GARCH models is that the nature of these changes in variance is evolutionary; the conditional volatility in the current period is a function of prior values of the conditional volatility and unexpected shocks in prior periods. Because each parameter of the conditional volatility equation is time-invariant, the structure carries with it an implicit assumption that the determinants of volatility are static; there is no allowance for shifts in the underlying unconditional volatility which might be precipitated by structural changes in the economy or by changes in the structure of the market itself.

The specification for the variance equation can be extended to allow for changes the underlying unconditional volatility by defining two (0, 1) dummy variables, $D_L$ and $D_H$ which are zero in periods of normal volatility and one in periods of low or high volatility respectively. These dummy variables condition each of the parameters $\omega, \alpha, \beta, b, c, \lambda,$ and $\nu$, allowing the parameters to vary from the base regime, $(B)$, for both the low, $(L)$, and high, $(H)$, volatility regimes:6

\[
\frac{h_t^\lambda - 1}{\lambda} = \omega + \alpha f_t^\nu(v_{t-1}) + \beta \frac{h_{t-1}^\lambda - 1}{\lambda} + \frac{h_t^\lambda - 1}{\lambda} \\
f(v_t) = |v_t - b| - c(v_t - b)
\]

This substitution allows for a rich parameterization of the model since it allows each estimated parameter to vary across the regimes of interest. It does introduce two complications, however.

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6Parameters which are conditioned by the regime dummy variables are indicated in bold. Thus, for example:

$$\lambda = \lambda_B + \lambda_L D_L + \lambda_H D_H$$
proposed volatility regimes. The approach employed sequentially locates and isolates regimes of unusually high or low volatility from the return series using the method of Bauer (2002). In essence, a window of 60 observations is moved through the return series and the ratio of the variance of the observations inside that window to the variance of the observations outside the window is computed. The largest value for the variance ratio, in excess of an empirically determined critical value of 3.0, indicates a regime. Observations are iteratively added to the beginning or end of the window as appropriate to determine the extent of the regime. The largest window of observations with a variance ratio (or its inverse) greater than 3.0 is isolated and excluded. The process is repeated until no additional regimes are detected.

A second issue arises from the rich parameterization of the variance equation provided by the regime dependent framework. Since each of the seven explanatory variables in the variance equation is conditioned by the same pair of (0, 1) dummies, the 14 regime specific variables appear to be in two highly correlated groups. This has the effect of greatly overstating the standard errors reported for these variables, making standard t-tests for significance of the coefficients invalid. As an alternative to using the asymptotic standard errors calculated from the ML estimation, a stepwise backwards elimination procedure is employed which removes one of the parameters conditioned by a regime dummy in each iteration until the likelihood ratio for removal of any of the remaining parameters is significant at the 1% level.

Together, Equations (5) and (6a) define the Regime Dependent GARCH (RD-GARCH) specification. The model provides a total of 10 parameters in the conventional form and, as the seven conditional volatility parameters $\omega, \alpha, \beta, b, c, \lambda$, and $\nu$ each become three parameters when conditioned by the regime dependent dummy variables, a total of 25 parameters in the regime dependent form. Because the specification of Equation (6a) is recursive in nature and requires a value for $h_{t-1}$ to compute a value for $h_t$, $h_0$ is estimated as a nuisance parameter. This provides a total of 12 parameters in the conventional form and 26 in the regime dependent form.

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7A window with a variance ratio greater than the critical value indicates a high volatility regime. Similarly, a window with the inverse of the variance ratio greater than the critical value indicates a low volatility regime.
5 Model Performance

The parameter estimates from the model for the full sample are presented in the first rows of Tables 2 through 5. In addition, Table 4 presents likelihood ratio results for a test of the null that all regime-dependent coefficients are zero.

The likelihood ratio test strongly rejects the null that all regime-dependent coefficients are zero (LR=826.51, \( p < 0.001 \)); the performance improvement of the regime dependent model is highly statistically significant.

Comparison of the mean equation parameters, \( \gamma_0, \gamma_2, \gamma_3, \) and \( \gamma_4 \) presented in Tables 2 and 4 indicates that almost no difference in the conditional mean results from allowing the conditional variance to depend upon the regime. In both cases, the prior period return has predictive power for the current return, although the negativity of \( \gamma_3 \) indicates the possibility of a slight systematic over-reaction of the current return to the impact of the prior period return. The small negative coefficient of the Monday dummy is supportive of the weekend effect.

Because of the interactions between parameters, it is difficult to interpret the numerical results for the conditional volatility equation presented in Tables 3 and 5. Figure 4 presents the comparison graphically, in the form of NICs and HICs. For both curve types, the left-hand panel shows the curve which results from the conventional version of the model and the right hand panel shows the three curves which result when the parameters conditioned by the regime dependent dummy variables are included.

In the NIC for the model in which regime detection and isolation was not performed, the single curve is shifted and rotated somewhat to the right and exhibits a small amount of convexity.
Because the curve is asymmetric, a positive shock \((v_{t-1} > 0)\) will have a smaller impact on \(h_t\) than a negative shock \((v_{t-1} < 0)\) of the same magnitude. If the volatility shocks are viewed as a response to news, a positive news event has a smaller effect on subsequent conditional volatility than a negative news event of the same magnitude. As the NIC exhibits convexity, the response of the conditional volatility to shocks of the same sign but different magnitudes is not linear. The difference in impact between a shock of \(v_{t-1} = 2^8\) and a shock of \(v_{t-1} = 3\) is greater than the difference in impact between a shock of \(v_{t-1} = 1\) and a shock of \(v_{t-1} = 2\).

The separation of the curves in the regime dependent model reflects the difference in unconditional volatility found in the three regimes. By construction, the low volatility regimes exhibit the smallest level of unconditional volatility, the base regime a larger level, and the high volatility regimes the largest level. The convexity exhibited by the NIC differs markedly across the three regimes. High volatility regimes are characterized by a NIC which has a much higher degree of curvature than either the base or low volatility regimes. Conditional volatility is more responsive to large shocks in periods of high volatility and this effect is not simply due to the higher unconditional volatility that exists in these regimes; the difference in impact between small and large shocks is greater in periods of high volatility. If volatility is already unusually high, the market tends to react to large news events more aggressively than they would otherwise in periods of lower volatility. Low volatility regimes exhibit a slightly sharper impact curve than the base regime, although it exhibits less curvature. For a range of magnitudes of shocks, the difference in impact for larger shocks in low volatility periods is greater than for normal volatility periods. Once outside that range, however, the reverse is true: the difference in impact for larger shocks is greater in normal volatility periods than in low volatility periods. This difference is quite small, however, and seems to largely mimic the effect noted in the high volatility case; in the base volatility regime investors tend to react more aggressively to large news events than they do in low volatility regimes. The curvature of the NIC for the conventional version of the model appears largely to be an average of the curvature of the three NICs from the regime dependent model. Allowing the model parameters to vary across regimes has provided a mechanism for identifying a difference in market reaction to news items of different magnitude. While the low and base regimes exhibit a very similar form of

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\(^8\)Recall that \(v_t\) represents the residual from the mean equation standardized to have unit variance. Thus, \(v_{t-1} = 1\) represents a shock in the prior period with a magnitude of one standard deviation.
asymmetry to each other, the curve for high volatility regimes is shifted to the right and rotated to the left, providing an overall suggestion of a stronger impact for positive news events than for negative events.

Initial comparison of the $\beta$ coefficient between the conventional and regime dependent models suggests a notable decrease in the persistence of conditional volatility. Comparison of the HICs, however, reveals that the large decrease in persistence suggested by the change in value of $\beta$ overstates the true change in persistence resulting from regime dependence. The level of persistence in the base regime is similar to the overall level of persistence observed from the model which does not allow for regime dependence. There does appear to be a small reduction in the level of persistence in low and base volatility regimes, with high volatility regimes exhibiting slightly more persistence.

5.1 Subsample Results

Allowing conditional volatility equation parameters to vary across regimes of differing unconditional volatility has demonstrated responses of notably different natures for regimes of differing unconditional volatility. To assess and interpret inter-temporal differences, the data are divided into four sub-periods. The first subsample break is at the end of the third quarter of 1929, which isolates the period of extreme volatility associated with the Great Depression from the period surrounding the First World War and represents a point in time at which the economy of the United States begins to become a more dominant force internationally. A second subsample break is made at the end of 1951 following Campbell and Hentschel (1992) and serves to separate the Great Depression from the postwar period. Finally, the subsample break at the end of 1973 with the oil crisis, represents another shift in the international positioning of the economy of the United States.

5.1.1 Conventional Models

Table 2 presents the parameter estimates for the version of the model which does not allow regime detection and separation. The lagged return, $\gamma_2$, is significantly different from zero in all cases and takes on values of 0.088, 0.087, 0.273, and 0.155 in the sub-periods 1915–1929, 1929–1951, 1952–1973, and 1974–2000 respectively. The return lagged two periods, $\gamma_3$, exhibits somewhat similar behaviour, becoming significantly different from zero in the 1929–1951 sub-period with a value of
-0.032, increasing in magnitude to a value of -0.073 in the 1952–1973 sub-period, and becoming not significantly different from zero in the final 1974–2000 sub-period. In addition, the Monday dummy, \( \gamma_4 \), is significantly different from zero in the three sub-periods prior to 1974, increasing in magnitude from a value of -0.001 in the 1915–1929 sub-period to a value of -0.002 in both the 1929–1951 and 1952–1973 sup-periods. It is not significantly different from zero in the final 1974–2000 sub-period.

The apparent changes in autocorrelation structure may reflect changes in the construction of the data series itself rather than changes in the structure of the market. As outlined in Section 3, the data come from two sources: prior to July 2, 1962, returns are from Schwert (1989); for the period since July 3, 1962, returns are from the CRSP value weighted stock indices for NYSE and AMEX combined. Prior to 1928, the Schwert index is based on a weighted average of the Dow Jones Industrial and Railroad averages which comprised 22 stocks in the beginning, rising through time to 50 stocks. Beginning in 1928 the index is based on the S&P 90 index, changing to the S&P 500 index in 1957. The CRSP index, which replaces the Schwert index in 1962, is a value-weighted index of all issues on the NYSE and AMEX exchanges.

Lo and MacKinlay (1990) suggest that nonsynchronous trading of the stocks in a portfolio contributes to the autocorrelation commonly noted in portfolio returns. If nonsynchronous trading is contributing to the autocorrelation, an increase in autocorrelation is to be expected following the change from the S&P 90 index to the S&P 500 index in 1957 as some portion of the 410 added stocks are likely more thinly traded than the 90 original stocks. A second increase would be expected following the change from the S&P 500 index to the index comprising the full set of stocks trading on the NYSE and AMEX exchanges. The decrease in autocorrelation between the 1952–1973 subsample and the 1974–2000 subsample is then attributable to a general increase in trading volume through time, with a corresponding decrease in thinness of trading and a decrease in the impact of nonsynchronous trading.

Table 3 presents the conditional volatility equation parameters for the subsamples. Given the difficulty in interpretation of the parameters in isolation, however, comparison of conditional volatility parameters for the subsamples will be conducted based on the NICs and HICs. In each case, the top left panel of the relevant figure presents the NIC for the conventional version of the model,
and the lower left panel presents the corresponding HIC.

Figures 5, 6, 7, and 8 present the NICs and HICs for the sub-periods 1915–1929, 1929–1951, 1952–1973, and 1974–2000 respectively. Considering first the unconditional volatility indicated by the location of the curves in space, it is apparent that the 1929–1951 sub-period exhibits the greatest unconditional volatility, and that the 1952–1973 period exhibits the least. Given the contribution of the extreme volatility associated with the Great Depression to the 1929–1952 sub-period and the low volatility associated with the 1960's to the 1952–1973 sub-period these observations are both expected and appropriate. It is interesting to note, however, that although the shape of the NIC is markedly different for the 1915–1929 and 1974–2000 sub-periods, the location in space of these two curves are similar; unconditional volatility in the market is at a similar level. The curvature of the NIC is most pronounced in the 1929–1951 sub-period. This suggests that during this period investors reacted more strongly to large news events, both positive and negative, than during the other periods. The earliest sub-period, 1915–1929, is characterized by a NIC which is also more curved than the periods after 1952, and in addition this curve is shifted much more to the right than any other period. The parameter $b$ presented in Table 3 reflects this large shift, taking on a value of 1.739 in the 1915–1929 sub-period and values of 0.384, 0.517, and 0.370 in the 1929–1951, 1952–1973, and 1974–2000 sub-periods respectively. Although the rotation effect measured by the parameter $c$ differs across the sub-periods and makes interpretation somewhat more problematic, it appears that during the period prior to the Crash of 1929, the asymmetry in the reaction of investors to negative versus positive news is stronger than during any other period—even during the period immediately following the Crash. This raises interesting questions regarding possible structural changes in the market coincident with the Crash. Finally, the curvature of the NIC decreases from the 1929–1951 to the 1952–1973 sub-period, and again from the 1952–1973 to the 1974–2000 sub-period.

[Figure 5 about here.]  
[Figure 6 about here.]  
[Figure 7 about here.]  
[Figure 8 about here.]
Prior to the Crash, investors were much more concerned about negative news than about positive news. Following the Crash, the overall volatility of the market increased substantially. In addition, large news events of both signs resulted in proportionately greater subsequent volatility than small news events—not only is the market more volatile following the Crash, investors appear to react more strongly to large news events during this period. In general, since the Great Depression both the overall volatility of the market and the extent to which the market reacts proportionately more strongly to large news events than small news events has lessened. The overall volatility was lower during the period which includes the Korean and Vietnam wars than during the 1974–2000 period. The reverse is true for the difference in the reaction of the market to large and small news events—large news events have a proportionately greater impact on subsequent volatility in the 1952–1973 period than in the 1974–2000 period. The former period was characterized by relatively low volatility but exhibited stronger reactions to large news events, while the latter period was characterized by somewhat higher overall volatility but less aggressive reactions to large news events. Asymmetry also appears to have increased between the 1952–1973 and 1974–2000 sub-periods, with negative news providing a stronger impact on subsequent volatility in the final sub-period.

Finally, the HIC indicates that for much of the sample period there has been little difference in the persistence of conditional volatility. The exception to this was during the 1952–1973 sub-period where the slope of the curve is less, indicating that the persistence during this was notably less than during any other period.

5.1.2 Regime Dependent Models

Tables 4 and 5 presents the parameter estimates which result when regimes are detected and the parameter values are permitted to vary across regimes.

In all cases, likelihood ratio tests strongly reject the null that all regime-dependent coefficients are zero. For the 1915–1929 sub-period LR= 105.28, for the 1929–1951 sub-period LR= 58.49, for the 1952–1973 sub-period LR= 261.46, for the 1974–2000 sub-period LR= 194.90. In all cases, $p < 0.001$. Again, for all sub-periods the performance improvement of the regime dependent model is highly statistically significant.
As was the case in Section 5, comparison of the mean equation parameters, $\gamma_0$, $\gamma_2$, $\gamma_3$, and $\gamma_4$ between the conventional models presented in Table 2 and the regime dependent version of the models presented in Table 4 indicates that almost no difference in conditional mean results from allowing the conditional variance to depend upon the regime. In all cases the parameter values are similar for the conventional and regime dependent versions of the model. While numerical differences do exist, the differences are small and in no case is there a change in the sign or the significance of parameter estimates.

The conditional volatility equation parameters exhibit considerably more activity than the mean equation parameters. Following the procedure employed above, the discussion of the conditional volatility equation parameters will be conducted with reference to the NICs and HICs rather than the tabulated parameter estimates because of the difficulties involved in interpreting the interrelationships between the various parameters. In each case, the top left panel of the relevant figure presents the NIC for the conventional version of the model, and the lower left panel presents the corresponding HIC.

Figure 5 presents the NICs and HICs for the 1915–1929 sub-period. The separation of the three NICs resulting from the regime dependent version of the model reflects the difference in unconditional volatility present in the three regimes. The asymmetry noted in the conventional version of the model is also present all three NICs in the regime dependent version, with the shift appearing to be greater in the regime dependent version of the model than in the conventional version. In addition, large positive shocks show dramatically increased response of subsequent conditional volatility and this effect is similar for all three regime types. There is somewhat greater curvature present in the base regime than in low volatility regimes, and slightly more curvature in high volatility regimes than in the base regime. This indicates that during this period higher unconditional volatility in the market was associated with an increased impact of news on subsequent conditional volatility and, as the curves are all shifted to the right, the impact was more significant for negative news events. The differences between HICs for the conventional and regime dependent versions of the model are unremarkable. There is a slight increase in the persistence of

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This dramatic increase in the response of subsequent conditional volatility is associated with positive shocks of greater than 4 standard deviations which, under conditional normality would represent extremely unlikely events. As stock returns are highly leptokurtic, even conditionally, shocks of this magnitude are rare but not unusually so.
conditional volatility in high volatility regimes, but no change in the base or low volatility regimes associated with allowing parameters to vary across regime types.

The NICs and HICs for the 1929–1951 sub-period, presented in Figure 6, are different from the other three sub-periods in that only base and low volatility regimes were detected in this sample period. The reason for this stems from the dominance of the period of extremely high volatility associated with the Great Depression.10 Because the high volatility period dominates the sample period, it is detected as belonging to the base regime rather than a high volatility regime. As a result, it would be more appropriate to consider the curves labeled “Base Regime” in the two right panels of Figure 6 as representing high volatility regimes. Similarly, the curves labeled “Low Volatility” should be interpreted as representing the base regime. No true low volatility regimes have been detected and isolated. The curvature of the NIC for high volatility regimes in the regime dependent version of the model is much greater than the curvature of the NIC for the base regime. As the major portion of the high volatility regimes are in the early part of the sample period,11 the greater curvature of the NIC observed in high volatility regimes suggests that the impact of news on subsequent conditional volatility followed a decreasing trend through time during this period. The HIC exhibits very little change when regime dependence is modeled, with the persistence in both high and base volatility regimes decreasing marginally from the level exhibited by the conventional model. In addition, the base regime exhibits slightly less persistence than do high volatility regimes.

The NICs for the regime dependent version of the model for the 1952–1973 sub-period are presented in Figure 7. The curves for the base regime and for low volatility regimes are unremarkable, with low volatility regimes providing a slightly greater level of asymmetry. High volatility regimes, however, exhibit one of the largest levels of asymmetry of any of the sub-periods. The HICs for this sub-period are also interesting. The level of persistence in conditional volatility is substantially less for all three regime types than it was for the conventional version of the model. In addition, there are considerable differences in the level of persistence exhibited by the three regime types. The high volatility regimes exhibit the greatest level of persistence, followed closely by the base regime. Low volatility regimes exhibit notably less persistence than either high volatility regimes

10 See, for example, Bauer (2002).
11 906 of 1,309 days, or 69% of high volatility regimes occur prior to April 24, 1935 which represents the first quarter of the sample period. 1,307 of the 1,309 days of high volatility regimes occur prior to November 15, 1940 which represents the first half of the sample period.
or the base regime.

The NICs for the 1974–2000 sub-period, presented in Figure 8, exhibit the greatest differences between the three regime types of any sub-period considered. The location in space of the curve for the base regime is similar to that obtained with the conventional version of the model, however it does exhibit substantially more curvature. In addition to being located higher in space, the NIC for high volatility regimes exhibits the second greatest curvature of any NIC from any of the sub-periods examined. The only period where the difference in impact between large and small news events, both positive and negative, was greater than during the high volatility periods in the 1974–2000 sub-period was during the 1929–1952 sub-period. In addition, since the high volatility regimes associated with the 1929–1952 sub-period are restricted almost entirely to the first half of that period, the only period which exhibited a NIC with a greater curvature than that exhibited during high volatility regimes of the 1974–2000 sub-period is in fact the period immediately following the Crash of 1929, encompassing the Great Depression and ending just over a year prior to the entry of the United States into World War II. The NIC for low volatility regimes is very similar in shape to the curve for the base regime but located lower in space as a result of the lower level of unconditional volatility.

The HIC for the regime dependent version of the model shows a slight difference in the degree of persistence of conditional volatility among the three regime types. As was the case with the NIC, the HIC for the base regime is similar to the HIC which resulted from the conventional version of the model. Both high and low volatility regimes exhibit slightly less persistence, with high volatility regimes exhibiting more persistence than low volatility regimes, but less than the base regime.

5.1.3 Summary

Initial examination of the NICs and HICs for the conventional version of the model would seem to indicate that, with the exception of the extreme volatility period associated with the 1930’s, the nature of the impact of news on subsequent conditional volatility is reasonably stable across sample periods. While there are differences between the NICs in terms of location in space, curvature, and asymmetry (shift), the curves for the overall 1915–2000 period are grossly similar to those for the 1915–1929, 1929–1951, 1952–1973, and 1974–2000 sub-periods. In particular, the results for the
1951–2000 sub-period are quite similar to those for both the 1974–2000 sub-period.

The results from the application of the RD-GARCH model, however, are interesting in their diversity. The curves from the regime dependent model applied to the 1915–1929, 1929–1951, 1952–1973, and 1974–2000 sub-periods are all notably different from each other and are all substantially different from the curves obtained from the full sample period.

6 Conclusions

The Regime Dependent Generalized Auto Regressive Conditional Heteroskedasticity model combines the evolutionary approach to modeling heteroskedasticity provided by the GARCH methodology with a general model that allows for abrupt shifts in unconditional volatility at discrete points in time. The resulting regime dependent model provides substantially improved within-sample fit as measured by likelihood ratio tests. This is a particularly interesting result as the model is an extension of the Nested GARCH specification of Hentschel (1995) which nests many of the more popular GARCH specifications into a single equation. Hence, the RD-GARCH model provides improved goodness of fit over a broad range of GARCH specifications. Subsample results show that, although the response of conditional volatility to shocks in prior periods appears to be reasonably constant through time when conventional non regime dependent models are employed, allowing the model parameters to vary across periods unusually high or low volatility reveals substantial inter-temporal differences in the response of conditional volatility to volatility shocks. These differences in response suggest a number of practical applications, from the interpretation of historical trends in volatility to the potential for improved out-of-sample forecasting.

References


The figure shows example News Impact Curves which demonstrate the effect of changing the asymmetry parameters, $b$ and $c$. The parameter $b$ provides for asymmetry in the response of conditional volatility to unexpected shocks in the prior period by allowing the NIC to shift. The parameter $c$ provides for asymmetry in the response of conditional volatility to unexpected shocks in the prior period by allowing for rotation of the NIC.
Figure 2: The History Impact Curve

The figure shows example History Impact Curves which demonstrate the effect of changing the parameter $\beta$ which controls the slope of the HIC.
Figure 3: The Effect of $\lambda$ on the News and History Impact Curves

The figure shows example News and History Impact Curves respectively which demonstrate the effect of changing the parameter $\lambda$, which controls the Box-Cox transformation of the conditional volatility series. This parameter does not have a simple, well-defined impact on either curve. The impact on the NIC is similar to a combination of the impact of $\omega$ and $\alpha$, which is to say that larger values for $\lambda$ are characterized by a NIC which is located higher in space and is less curved, however this appearance is associated with a transformation of the conditional volatility series rather than a simple change in the nature of the impact of current shocks on subsequent conditional volatility. The effect on the HIC is similar to the effect of parameter $\nu$ on the NIC.
The top two panels of the figure present the NIC relating volatility shocks in the previous period to the conditional volatility in the current period for the raw return series covering the dates January 2, 1915 through December 29, 2000 (23,301 observations). The left-hand panel shows the NIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. The range of volatility shocks over which the figure is presented corresponds to the range $\pm 3$. Since $v_t \sim iid(0, 1)$ this corresponds to $\pm 3$ standard deviations. Both figures include a dashed vertical line corresponding to $v_{t-1} = 0$ for comparison of asymmetry effects. The bottom two panels of the figure present the HIC relating the conditional volatility in the previous period to the conditional volatility in the current period for the raw return series covering the same period. The left-hand panel shows the HIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. To facilitate comparison, the vertical scale of each pair of figures is the same.
Figure 5: News and History Impact Curves 1915–1929

The top two panels of the figure present the NIC relating volatility shocks in the previous period to the conditional volatility in the current period for the raw return series covering the dates January 2, 1915 through September 30, 1929 (4,406 observations). The left-hand panel shows the NIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. The range of volatility shocks over which the figure is presented corresponds to the range ±3. Since $v_t \sim iid(0, 1)$ this corresponds to ±3 standard deviations. Both figures include a dashed vertical line corresponding to $v_{t-1} = 0$ for comparison of asymmetry effects. The bottom two panels of the figure present the HIC relating the conditional volatility in the previous period to the conditional volatility in the current period for the raw return series covering the same period. The left-hand panel shows the HIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. To facilitate comparison, the vertical scale of each pair of figures is the same.
The top two panels of the figure present the NIC relating volatility shocks in the previous period to the conditional volatility in the current period for the raw return series covering the dates October 10, 1929 through December 31, 1951 (6,536 observations). The left-hand panel shows the NIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. The range of volatility shocks over which the figure is presented corresponds to the range $\pm 3$. Since $v_t \sim \text{iid}(0, 1)$ this corresponds to $\pm 3$ standard deviations. Both figures include a dashed vertical line corresponding to $v_{t-1} = 0$ for comparison of asymmetry effects. The bottom two panels of the figure present the HIC relating the conditional volatility in the previous period to the conditional volatility in the current period for the raw return series covering the same period. The left-hand panel shows the HIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. To facilitate comparison, the vertical scale of each pair of figures is the same.
Figure 7: News and History Impact Curves 1952–1973

The top two panels of the figure present the NIC relating volatility shocks in the previous period to the conditional volatility in the current period for the raw return series covering the dates January 2, 1952 through December 31, 1973 (5,531 observations). The left-hand panel shows the NIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. The range of volatility shocks over which the figure is presented corresponds to the range ±3. Since $v_t \sim \text{iid}(0, 1)$ this corresponds to ±3 standard deviations. Both figures include a dashed vertical line corresponding to $v_{t-1} = 0$ for comparison of asymmetry effects. The bottom two panels of the figure present the HIC relating the conditional volatility in the previous period to the conditional volatility in the current period for the raw return series covering the same period. The left-hand panel shows the HIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. To facilitate comparison, the vertical scale of each pair of figures is the same.
The top two panels of the figure present the NIC relating volatility shocks in the previous period to the conditional volatility in the current period for the raw return series covering the dates January 2, 1974 through December 29, 2000 (6,823 observations). The left-hand panel shows the NIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. The range of volatility shocks over which the figure is presented corresponds to the range $\pm 3$. Since $\nu_t \sim \text{iid}(0, 1)$ this corresponds to $\pm 3$ standard deviations. Both figures include a dashed vertical line corresponding to $\nu_{t-1} = 0$ for comparison of asymmetry effects. The bottom two panels of the figure present the HIC relating the conditional volatility in the previous period to the conditional volatility in the current period for the raw return series covering the same period. The left-hand panel shows the HIC for the conventional version of the model, and the right-hand panel shows the three curves which result when the regime dependent methodology is adopted. To facilitate comparison, the vertical scale of each pair of figures is the same.
Table 1: LR tests for alternative mean equations

<table>
<thead>
<tr>
<th>Model</th>
<th>$H_0$</th>
<th>$H_A: h_t^2, r_{t-1}, r_{t-2}, D_M$</th>
<th>LR</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_t^2$</td>
<td>$\gamma_2 = \gamma_3 = \gamma_4 = 0$</td>
<td>555.855</td>
<td>&lt; 0.001</td>
<td></td>
</tr>
<tr>
<td>$r_{t-1}$</td>
<td>$\gamma_1 = \gamma_3 = \gamma_4 = 0$</td>
<td>179.316</td>
<td>&lt; 0.001</td>
<td></td>
</tr>
<tr>
<td>$r_{t-1}, r_{t-2}$</td>
<td>$\gamma_1 = \gamma_4 = 0$</td>
<td>170.766</td>
<td>&lt; 0.001</td>
<td></td>
</tr>
<tr>
<td>$r_{t-1}, D_M$</td>
<td>$\gamma_1 = \gamma_3 = 0$</td>
<td>8.749</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>$r_{t-1}, r_{t-2}, D_M$</td>
<td>$\gamma_1 = 0$</td>
<td>0.501</td>
<td>0.479</td>
<td></td>
</tr>
</tbody>
</table>

The table presents the results for likelihood ratio tests for the functional form of the conditional mean equation. In each case the test compares a model obtained by restricting one or more parameters of Equation (5) to be zero to the model which imposes no restrictions. The likelihood ratio statistics are all distributed $\chi^2$ with between one and three degrees of freedom. The corresponding p-values are included in the final column. The data set comprises returns on an U.S. equity index covering the period January 2, 1915 to December 29, 2000, a total of 23,301 observations.
## Table 2: Estimates of mean equation parameters (sub-sample, no regimes)

<table>
<thead>
<tr>
<th>Model</th>
<th>$\gamma_0$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>$\gamma_4$</th>
<th>Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>full sample</td>
<td>6.126$^{[-4]}$</td>
<td>0.145</td>
<td>$-0.020$</td>
<td>$-0.001$</td>
<td>79354.757</td>
</tr>
<tr>
<td></td>
<td>(5.083$^{[-5]}$)*</td>
<td>(0.008)*</td>
<td>(0.007)*</td>
<td>(1.291$^{[-4]}$)*</td>
<td></td>
</tr>
<tr>
<td>1915–1929</td>
<td>6.954$^{[-4]}$</td>
<td>0.089</td>
<td>0.008</td>
<td>$-0.001$</td>
<td>15306.298</td>
</tr>
<tr>
<td></td>
<td>(1.246$^{[-4]}$)*</td>
<td>(0.019)*</td>
<td>(0.017)</td>
<td>(3.210$^{[-4]}$)*</td>
<td></td>
</tr>
<tr>
<td>1929–1951</td>
<td>5.486$^{[-4]}$</td>
<td>0.087</td>
<td>$-0.032$</td>
<td>$-0.002$</td>
<td>20159.114</td>
</tr>
<tr>
<td></td>
<td>(1.251$^{[-4]}$)*</td>
<td>(0.015)*</td>
<td>(0.014)*</td>
<td>(3.321$^{[-4]}$)*</td>
<td></td>
</tr>
<tr>
<td>1952–1973</td>
<td>7.169$^{[-4]}$</td>
<td>0.273</td>
<td>$-0.073$</td>
<td>$-0.002$</td>
<td>20733.339</td>
</tr>
<tr>
<td></td>
<td>(7.432$^{[-5]}$)*</td>
<td>(0.022)*</td>
<td>(0.013)*</td>
<td>(1.931$^{[-4]}$)*</td>
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</tr>
<tr>
<td>1974–2000</td>
<td>4.174$^{[-4]}$</td>
<td>0.155</td>
<td>$-0.005$</td>
<td>$-3.652^{[-4]}$</td>
<td>23381.958</td>
</tr>
<tr>
<td></td>
<td>(9.988$^{[-5]}$)*</td>
<td>(0.013)*</td>
<td>(0.014)</td>
<td>(2.345$^{[-4]}$)</td>
<td></td>
</tr>
</tbody>
</table>

Entries of the form $x^{[-y]}$ represent scientific notation; that is $2.5^{[-2]}$ represents the value $2.5 \times 10^{-2}$.

The table presents the subsample mean equation parameter estimates for the conventional GARCH model. Numbers in parentheses under the parameter estimates represent asymptotic standard errors. Parameters which are significantly different from zero at the 5% level are indicated with an asterisk.
Table 3: Estimates of conditional volatility equation parameters (sub-sample, no regimes)

<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda$</th>
<th>$\nu$</th>
<th>$b$</th>
<th>$c$</th>
<th>$\omega$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>full sample</td>
<td>1.148</td>
<td>1.537</td>
<td>0.429</td>
<td>0.102</td>
<td>7.893$^{[-5]}$</td>
<td>0.069</td>
<td>0.911</td>
</tr>
<tr>
<td></td>
<td>(0.267)*</td>
<td>(0.105)*</td>
<td>(0.112)*</td>
<td>(0.074)</td>
<td>(1.007$^{[-4]}$)</td>
<td>(0.016)*</td>
<td>(0.019)*</td>
</tr>
<tr>
<td>1915–1929</td>
<td>6.938</td>
<td>3.904</td>
<td>1.739</td>
<td>−0.104</td>
<td>1.360$^{[-16]}$</td>
<td>0.058</td>
<td>0.299</td>
</tr>
<tr>
<td></td>
<td>(0.059)*</td>
<td>(0.452)*</td>
<td>(0.282)*</td>
<td>(0.109)</td>
<td>(3.603$^{[-17]}$)</td>
<td>(0.031)</td>
<td>(0.070)*</td>
</tr>
<tr>
<td>1929–1951</td>
<td>1.622</td>
<td>1.613</td>
<td>0.384</td>
<td>0.163</td>
<td>1.229$^{[-5]}$</td>
<td>0.094</td>
<td>0.882</td>
</tr>
<tr>
<td></td>
<td>(0.816)</td>
<td>(0.223)*</td>
<td>(0.145)*</td>
<td>(0.109)</td>
<td>(4.646$^{[-5]}$)</td>
<td>(0.047)</td>
<td>(0.050)*</td>
</tr>
<tr>
<td>1951–1973</td>
<td>3.190$^{[-6]}$</td>
<td>0.969</td>
<td>0.517</td>
<td>−0.006</td>
<td>0.046</td>
<td>4.466$^{[-7]}$</td>
<td>0.954</td>
</tr>
<tr>
<td></td>
<td>(7.363$^{[-7]}$)*</td>
<td>(0.131)*</td>
<td>(0.055)*</td>
<td>(0.080)</td>
<td>(2.109$^{[-7]}$)*</td>
<td>(5.573$^{[-9]}$)*</td>
<td>(2.461$^{[-20]}$)*</td>
</tr>
<tr>
<td>1974–2000</td>
<td>2.246</td>
<td>1.686</td>
<td>0.370</td>
<td>0.188</td>
<td>3.913$^{[-7]}$</td>
<td>0.096</td>
<td>0.876</td>
</tr>
<tr>
<td></td>
<td>(1.768)</td>
<td>(0.567)*</td>
<td>(0.199)</td>
<td>(0.149)</td>
<td>(3.298$^{[-6]}$)</td>
<td>(0.061)</td>
<td>(0.091)*</td>
</tr>
</tbody>
</table>

Entries of the form $x^{[-y]}$ represent scientific notation; that is $2.5^{[-2]}$ represents the value $2.5 \times 10^{-2}$.

The table presents the subsample conditional volatility equation parameter estimates for the conventional GARCH model. Numbers in parentheses under the parameter estimates represent asymptotic standard errors. Parameters which are significantly different from zero at the 5% level are indicated with an asterisk.
Table 4: Estimates of mean equation parameters (sub-sample)

<table>
<thead>
<tr>
<th>Model</th>
<th>$\gamma_0$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>$\gamma_4$</th>
<th>Likelihood</th>
<th>LR</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>full sample</td>
<td>6.611[-4]</td>
<td>0.152</td>
<td>-0.023</td>
<td>-0.001</td>
<td>79768.010</td>
<td>826.51</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>(1.812[-35])*</td>
<td>(1.107[-31])*</td>
<td>(5.457[-29])*</td>
<td>(1.906[-9])*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1915–1929</td>
<td>7.299[-4]</td>
<td>0.086</td>
<td>-0.002</td>
<td>-0.001</td>
<td>15358.938</td>
<td>105.28</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>(1.198[-4])*</td>
<td>(0.018)*</td>
<td>(0.017)</td>
<td>(3.110[-4])*</td>
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<tr>
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<td>20188.358</td>
<td>58.49</td>
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</tr>
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<td>(0.014)*</td>
<td>(0.017)*</td>
<td>(3.401[-4])*</td>
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<td>1952–1973</td>
<td>7.310[-4]</td>
<td>0.274</td>
<td>-0.067</td>
<td>-0.002</td>
<td>20864.069</td>
<td>261.46</td>
<td>&lt; 0.001</td>
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<tr>
<td></td>
<td>(7.570[-5])*</td>
<td>(0.015)*</td>
<td>(0.014)*</td>
<td>(1.905[-4])*</td>
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<tr>
<td>1974–2000</td>
<td>4.333[-4]</td>
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<td>-0.007</td>
<td>-2.838[-4]</td>
<td>23479.408</td>
<td>194.90</td>
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<td>(9.595[-5])*</td>
<td>(0.013)*</td>
<td>(0.014)</td>
<td>(2.246[-4])</td>
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</table>

Entries of the form $x[-y]$ represent scientific notation; that is $2.5[-2]$ represents the value $2.5 \times 10^{-2}$.

The table presents the subsample mean equation parameter estimates for the Regime Dependent GARCH model. Numbers in parentheses under the parameter estimates represent asymptotic standard errors. Parameters which are significantly different from zero at the 5% level are indicated with an asterisk.
Table 5: Estimates of conditional volatility equation parameters (sub-sample)

<table>
<thead>
<tr>
<th>Model</th>
<th>λ</th>
<th>ν</th>
<th>b</th>
<th>c</th>
<th>ω</th>
<th>α</th>
<th>β</th>
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<td>2.586</td>
<td>2.066</td>
<td>0.644</td>
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<td>0.069</td>
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<td>(1.100e-6)*</td>
<td>(6.954e-6)*</td>
<td>(6.514e-7)*</td>
<td>(5.575e-7)*</td>
<td>(7.775e-13)*</td>
<td>(4.693e-8)*</td>
<td>(2.438e-7)*</td>
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<td>-0.775</td>
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<td>(1.483e-6)*</td>
<td>(9.980e-8)*</td>
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<td>(1.476e-8)*</td>
<td>(1.102e-8)*</td>
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<td>1915–1929</td>
<td>6.053</td>
<td>4.863</td>
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<td>(1.107)*</td>
<td>(0.197)*</td>
<td>(0.054)*</td>
<td>(2.815e-13)*</td>
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<td>0</td>
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<td>1.361</td>
<td>0.351</td>
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<td>0.015</td>
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<td>(0.281)</td>
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<td>(0.024)</td>
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<td>(7.104e-4)*</td>
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<tr>
<td>1952–1973</td>
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<td>1.295</td>
<td>0.713</td>
<td>-0.007</td>
<td>0.084</td>
<td>2.804e-7</td>
<td>0.916</td>
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<td>(8.864e-6)</td>
<td>(0.084)*</td>
<td>(0.112)*</td>
<td>(6.078e-5)*</td>
<td>(0.001)*</td>
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<td>(0.256)</td>
<td>(4.023e-6)*</td>
<td>(2.201e-12)*</td>
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<td>(1.053e-7)*</td>
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<td>(1.639e-9)*</td>
<td>(1.053e-7)*</td>
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<td>(0.112)*</td>
</tr>
</tbody>
</table>

Entries of the form $x^{[-g]}$ represent scientific notation; that is $2.5^{[-2]}$ represents the value $2.5 	imes 10^{-2}$.

The table presents the subsample conditional volatility parameter estimates for the Regime Dependent GARCH model. Numbers in parentheses under the parameter estimates represent asymptotic standard errors. Parameters which are significantly different from zero at the 5% level are indicated with an asterisk. The first pair of rows in each section present the parameter estimates and standard errors for the base regime, and the remaining four rows present the parameter estimates and standard errors for the regimes of unusually low and high volatility respectively. Parameters which are excluded by the stepwise estimation procedure are represented by a value of zero, with no corresponding standard error presented.