The Economics of Maintenance for Real Estate Investments

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Abstract

We propose in this paper a theory of urban decay. Following a negative real estate demand shock, property managers optimally suspend maintenance. Once suspended, they unlikely restart maintenance when general economic conditions improve. Because maintenance has proportionally greater risk, managers impose a more demanding profit standard on maintenance than on the initial investment. This differential in profit standards means that rather than maintain existing investments, property managers favor new investments, which, if marginally acceptable, they also leave unmaintained. Contractually required maintenance, (e.g., for publicly subsidized real estate investments), increases the minimum profit rate for the initial investment. In our model, the profit rate boundary for accepting a permanently maintained investment exceeds the profit boundary for maintenance if maintenance is not contractually required, which exceeds the minimum profit rate boundary for this investment’s acceptance. Contractually mandated maintenance discourages subsidized real estate investments in favor of unsubsidized investments. Consequently, the subsidy that induces the investment is most expensive for permanently required maintenance, slightly cheaper for optimally induced rather than permanently required maintenance, and cheapest if maintenance is neither prescribed nor induced. All of our findings are strongest for poorer quality properties.

Keywords: Maintenance, Real Estate, Real-Asset Options.
The Economics of Maintenance for Real Estate Investments

In its 1999 study “Places Left Behind in the New Economy,” the U.S. Department of Housing and Urban Development (HUD) notes that even at the peak of the last economic expansion many urban areas saw no substantial revitalization or economic growth.\(^1\) In this paper, we propose a theory of maintenance for real estate investments that explains this disturbing finding. Property managers suspend maintenance following a fall in operating profits for real estate investments. More importantly, once suspended, they unlikely resume maintenance even if general economic conditions improve. Managers optimally neglect maintenance in favor of new investment because maintenance has proportionately greater incremental risk, and therefore, managers impose a more demanding profit standard on maintenance. This result holds even if managers can suspend and recommence maintenance at no cost. We interpret our findings to mean that urban areas that see decay during an economic downturn unlikely recover when economic conditions improve. HUD’s finding is therefore not surprising, although still disturbing.

We find that managers’ inclination to neglect maintenance is most pronounced for poorer quality real estate properties. To the extent that initial quality is related to the economic means of an area, our model implies that low-income neighborhoods are both at higher risk of maintenance suspension and have less likelihood of maintenance resumption. The most vulnerable segments of the population are the first to experience urban decay and the last to benefit from general economic expansion.

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\(^1\) The complete report is available at: http://www.hud.gov/library/bookshelf18/pressrel/execsumm.html
Our research has important implications for various assisted housing programs of HUD and other government agencies around the world.² The goal of these programs is to provide a subsidy, usually in a form of a loan guarantee, for financially marginal projects that achieve important social goals such as affordable housing to low-income families, minorities, or seniors. These subsidies can improve cash flows for an investment above the profit boundary for investment acceptance, but may be insufficient to meet the maintenance boundary. Our model suggests that if these real estate properties are to be maintained, the subsidy must be large enough to induce not only the initial investment, but also subsequent maintenance.

Alternatively, the government agency providing the subsidy may contractually force the investor to permanently maintain the property. Our analysis of permanent maintenance indicates that such a requirement increases the minimum profit required for the initial investment. For reasonable parameter values, the profit rate boundary for accepting a permanently maintained investment exceeds the profit rate boundary for maintenance when maintenance is not contractually required. In other words, the subsidy is most expensive for permanently required maintenance, slightly cheaper if maintenance is not required but optimally induced, and cheapest if maintenance is neither required nor induced. This cheapest subsidy, however, virtually guarantees that the manager leaves the investment unmaintained, which, in turn, leaves the subsidy’s original purpose unfulfilled.

In section 1, we characterize economic depreciation and maintenance for our analysis. We also review the relevant existing literature. In section 2, we describe the economic environment and

the real estate investment. In section 3, we compare the real estate investment when the manager chooses to either permanently maintain it or leave it permanently unmaintained. In section 4, we consider a manager who has a dynamic option to suspend and recommence maintenance depending upon profitability. We investigate the minimum profit rate for investment acceptance and the probability that the manager recommences maintenance after suspension. Finally, section 5 summarizes the empirical and policy implications of our model. Section 6 concludes. See the appendix for all technical details.

**Depreciation and Maintenance**

Economic depreciation is the reduced ability of an asset to generate future cash flows. For real estate investments, if property managers do not maintain their properties, operating profits and/or lease rates fall. Recognizing this possibility, if they anticipate adequate rates of return, managers make maintenance investments that offset this depreciation. For example, the manager of an apartment building replaces a fraction of carpets and appliances every year. More long-term building components like roofs and heating systems require both annual maintenance and periodic replacement within a building’s life span. Maintenance is a service that tenants pay for in their rents. If a building is not maintained, the quality of services consumed by tenants declines, and a manager must reduce rents to retain occupancy. One can think of redevelopment, where a building is either replaced or substantially replaced, as an extreme form of maintenance.

Management science and operations research contain the bulk of the academic business literature on maintenance. This literature focuses on preventing cost increases for industrial, manufacturing, production, and processing firms through repair and replacement. Repair and
replacement is a significant issue for industrial firms because their equipment has rapid
depreciation and relatively short useful life. Industrial firms maintain equipment to forestall
replacement. There are several differences between real estate and industrial firms in
controlling economic depreciation.

First, the bulk of the capital expenditures made by real estate firms, buildings, have long rather
than short useful lives. While replacement, which is redevelopment for a real estate firm, is an
important issue when it occurs, it occurs rarely for individual properties. Managers construct
buildings to last decades into the future. Because of the long time between redevelopment,
research in real estate economics studies maintenance and redevelopment separately. For
example, Williams (1997) and Childs, Riddiough, and Triantis (1996) investigate redevelopment
without maintenance, while Vorst (1986) investigates maintenance without redevelopment. For
real estate firms, the association between maintenance (including component replacement,
renovations, and repairs) and redevelopment is not nearly as immediate as the association
between repair and replacement for industrial firms.

Second, a manager in real estate maintains a property not principally to forestall redevelopment,
which might occur decades in the future even without maintenance, but to forestall revenue
contraction, which is immediate. Of course, revenue contraction over the long-term leads to
redevelopment, but in real estate, the relation between maintenance and redevelopment is distant.
Rents depend on building quality, which depends on a manager’s maintenance and renovation
program. For real estate firms, maintenance has a stronger marketing orientation than for
industrial firms and revenue maintenance is relatively more important, for a manager’s

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3 See Hsieh and Chiu (2002) for a recent example of the repair versus replacement problem in the operations
research literature.

4 Vorst (1986) investigates this motive for maintenance in real estate.
maintenance decision, than is cost control. Further, because real estate investments have large EBITDA margins (EBITDA is often more than 60% of revenue), maintaining revenue is essentially the same as maintaining profit. Because of this large margin, once a manager constructs a building, the principal element of investment return is revenue. The importance of revenues to return, in the near-term and the long-term, makes maintenance an important real estate decision.

Vorst (1986) investigates a real estate manager’s maintenance decision, but his focus is on the optimal stochastic control problem. He does not compare the maintenance decision with the initial purchase decision. Furthermore, while mathematically elegant, Vorsts’s model exogenously assumes the functional form of the building’s selling price and assumes that this selling price is constant through time and independent of rent. Also, he does not consider a utility maximizing agent and assumes a very specific and unrealistic stochastic process for the evolution of state variables.

In the current paper, we investigate the minimum profit rate required for acceptance of a real estate investment depending upon whether or not a manager prevents economic depreciation by maintaining the investment, and also, depending upon whether or not the manager dynamically suspends maintenance due to inadequate profit. In either case, for realistic parameter values that one might use for our model, the manager imposes a more demanding profit standard on maintenance than on the initial investment. This difference means that a manager leaves a marginally acceptable investment unmaintained until operating profit increases and that managers generally favor new investments over maintenance investments.

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5 The acronym EBITDA stands for earnings before interest, tax, depreciation, and amortization. The EBITDA margin is EBITDA divided by revenue.
6 The manager faces a purchase decision for an existing building or a construction decision otherwise.
The Firm

The Real Estate Investment

Let \( \hat{X}_t \) be the investment’s operating profit\(^7\) at time \( t \), before maintenance expenditure. Let \( \hat{x}_t \) denote the natural logarithm of \( \hat{X}_t \), \( \hat{x}_t = \ln(\hat{X}_t) \). To distinguish between these two variables we refer to \( \hat{X}_t \) as “operating profit” and \( \hat{x}_t \) as the “profit rate.” As a matter of mathematical convenience, most of our analytical expressions are in terms of \( \hat{x}_t \). However, we present all figures and examples with the more intuitive operating profit measure, \( \hat{X}_t \).

Operating profit \( \hat{X}_t \) follows a geometric Brownian motion:

\[
d\hat{X} = \begin{cases} 
-\delta dt + \sigma d\tilde{z}, & \text{if not maintained, or} \\
\sigma d\tilde{z}, & \text{if maintained}
\end{cases}
\]

(1)

where \( d\tilde{z} \) represents a standard Wiener process and \( \delta \) and \( \sigma \) are known constants.\(^8\) The profit rate, \( \hat{x}_t \), follows an arithmetic Brownian motion:

\[
d\hat{x} = \begin{cases} 
\left(-\delta - \frac{\sigma^2}{2}\right)dt + \sigma d\tilde{z}, & \text{if property unmaintained, or} \\
-\frac{\sigma^2}{2} dt + \sigma d\tilde{z}, & \text{if property maintained}
\end{cases}
\]

(2)

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\(^7\) The manager’s accounting representation of “income,” does not affect the cash inflow, \( \hat{X}_t \).

\(^8\) The drift in the profit is assumed zero for a fully maintained investment. This assumption is motivated by our empirical estimation of the drift described in Appendix B. The geometric Brownian motion implies that operating profit is always positive. For real estate firms, which have large EBITDA profit margins, this property of operating profit is reasonable. It is unreasonable for industrial firms, which we do not investigate, which are much more likely to have negative operating profit.
We define economic depreciation as the reduced ability of an investment to generate operating profit, \( \tilde{X}_t \). Because this paper investigates economic depreciation and maintenance expenditures that offset this decline, we presume that \( \delta \geq 0 \). In this case, expected operating profit, \( \tilde{X}_t \), declines if the property is unmaintained. If the property is maintained, then expected operating profit before maintenance remains unchanged.

Without maintenance expenditure, the expected dollar amount of economic depreciation in the next instant, \( \Delta t \), is \( X_0 - X_0 e^{-\delta \Delta t} \approx \delta X_0 \Delta t \), which is proportional to current operating profit, \( X_0 \). An interpretation of this proportionality is that economic depreciation depends not only on time, but also on the intensity of asset utilization. If the manager employs assets more intensely (less vacancy, for example) to produce greater profit, then the dollar amount of economic depreciation, \( \delta X_0 \Delta t \), increases.

Maintenance investments maintain the expectation of revenue and, thus, they maintain expected operating profit. Let \( I (I > 0) \) denote the initial investment, which generates per annum profit \( X_0 \). Without maintenance, expected profit declines to \( \approx X_0 - \delta X_0 \Delta t \) at the end of the upcoming instant. A maintenance investment of \( \delta I \Delta t \) at the end of this instant generates per instant profit \( \delta X_0 \Delta t \), which offsets economic depreciation on the initial investment. Absent further maintenance, this profit and that of the initial investment decline, thereafter, at the per annum rate \( \delta \). The capitalization rate on the initial investment, \( X_0/I \), equals the capitalization rate on the incremental maintenance investment, \( \delta X_0 \Delta t / \delta I \Delta t = X_0/I \). The efficiency of
maintenance at generating operating profit equals that of the initial investment.\(^9\) Maintenance investments of \(\delta I\) per annum offset economic depreciation and maintain expected future profit at the present amount, \(X_0\). In this case, only random changes in facets of the business other than building quality determine random operating profit, \(\tilde{X}_t\).

We model maintenance as a continuous activity with no significant extraordinary unexpected costs. We argue that because property manager time and disperse major maintenance events over the life of a building, maintenance is a regular and ongoing activity and the likelihood of unexpectedly large expenses in any particular year is minimized. While any specific maintenance activity, such as roof replacement or exterior renovation, can represent a large expenditure and occurs at discrete intervals, the composite of all maintenance expenditures are continuous and occur at a relatively constant rate for a fully maintained building.

We justify our assumption that maintenance expenditures are proportional to the initial investment, \(I\), by appealing to the fact that higher quality buildings are more expensive to both build and maintain. Furthermore, other than maintenance that is suspended due to low profitability, maintenance expenditures are largely independent of current profit rates. For example, low vacancy rates, which increase rents, have no impact on the cost of replacing flooring.

Generically, we describe maintenance expenditures as a function, \(g(x_0)\delta I\), of the rate of profit, \(x_0\), where \(g(x_0)\) is an indicator function that has value 1 if the manager maintains the building.

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\(^9\) One can also investigate maintenance investments when they are more or less efficient at generating operating profit compared to the initial building. We do not do so in this paper.
and 0 otherwise. The precise form of this function depends on the real-asset options that the manager has available for the investment.

Once undertaken, the manager always operates the investment, but he/she might or might not maintain it. We consider a number of alternatives for how the manager might make these expenditures. In section 2, we compare the investment with and without maintenance expenditures. The manager permanently maintains the investment or leaves it permanently unmaintained. When maintained, the manager maintains the investment even with negative free cash flow,\(^{10}\) which, in our case, is operating profit, \(\bar{X}_t\), less maintenance expenditure, \(\delta I\). In other words, the manager has no temporal option for maintenance investments. Alternatively, in section 3, we consider the possibility that the manager has the option to dynamically suspend maintenance due to inadequate profit, \(\bar{X}_t\).

Free cash flow, at time \(t\), is,

\[
f(\bar{x}_t) = e^{x_t} - g(x_t)\delta I
\]

\(\text{(3)}\)

*The Financial Investment*

To avoid the complications of debt financing, we presume that the manager finances the real estate investment entirely with equity and this equity trades in an organized financial market. In Rubinstein (1973), the value of an asset depends on the marginal utility of consumption, \(\bar{C}_t\), of a

\(^{10}\) See White, Sondhi, and Fried (1994) for a more general definition of free cash flow that is commonly used in practice for investment valuation.
representative investor. We presume that this investor has a power utility function. That is,

\[ U(\tilde{C}_t) = \frac{\tilde{C}_t^{1-\theta}}{1-\theta}, \text{ where } \theta \geq 0 \text{ is relative risk aversion.} \]

If consumption follows a lognormal distribution, \( \tilde{C}_t = e^{\tilde{\epsilon}_t} \), where \( \tilde{\epsilon}_t \) is normal with mean \( c_0 \) and variance \( \sigma_c^2 t \), then, marginal utility, \( U'(\tilde{C}_t) = \tilde{C}_t^{-\theta} = e^{-\theta \tilde{\epsilon}_t} \), also follows a lognormal distribution.

Because the profit rate, \( \tilde{x}_t \), and the rate of consumption, \( \tilde{c}_t \), follow a bi-variate normal distribution, then \( \text{cov}(\tilde{x}_t, \tilde{c}_t) = \sigma_{x,c} t \), where \( \sigma_{x,c} \) is the instantaneous rate of covariance.

In theorems 1 and 3 of Rubinstein (1973), risk-adjusted value is the sum (integral) of discounted expected cash flow plus the discounted value of the covariance of cash flow with marginal utility of aggregate consumption divided by the expected marginal utility of consumption.

Applying this general result, risk adjusted value, \( V(x_0) \), after the expenditure, \( I \), is

\[
V(x_0) = \int_0^\infty e^{-rt} \left( E \left[ f(\tilde{x}_t) \mid x_0 \right] + \frac{\text{cov} \left[ f(\tilde{x}_t), e^{-\theta \tilde{\epsilon}_t} \mid x_0 \right]}{E \left[ e^{-\theta \tilde{\epsilon}_t} \mid x_0 \right]} \right) dt
\]

\[
= \int_0^\infty e^{-rt} E \left[ f(\tilde{x}_t) e^{-0(\tilde{x}_t-c_0) - \frac{1}{2} \theta^2 \sigma_c^2 t} \mid x_0 \right] dt
\]

(4)

where \( r \) is the riskless rate of interest. Risk-adjusted value depends on the profit function, \( f(\tilde{x}_t) \), which depends on the real-asset options available to the manager for maintenance of the real estate investment. Define the net value of the investment prior to the expenditure, \( I \), as

\[ NV(x_0) = V(x_0) - I. \]
The Permanently Maintained Versus the Permanently Unmaintained Investment

In this section, we compare the real estate investment when the manager chooses to either permanently maintain it or leave it permanently unmaintained. The main purpose of this analysis is to provide a benchmark for the more relevant case of dynamic maintenance choice presented in Section 3. Nonetheless, the static permanent maintenance decision represents numerous realistic situations in which once taken the maintenance decision can hardly be reversed. This may arise from contractual obligations, government regulations and standards, or reputation externalities for other properties managed by the same firm. Many of the government subsidies for real estate investments that achieve important social goals require, for instance, some level of permanent maintenance.\textsuperscript{11} Furthermore, in certain situations it may be technically unfeasible to resume maintenance of a property following an extended period of neglect. If this is the case, the manager may be unable to dynamically switch between the two regimes and the one-time maintenance decision has to be made at the start of the project.

The Unmaintained Investment

If the manager leaves the investment permanently unmaintained, then \( g(\bar{x}_t) = 0 \) in equations (3) and (4). The net value of the investment, \( NV_p(x_0) \), which is the solution to the integral in equation (4) less the initial investment, \( I \), is

\[
NV_p(x_0) = \frac{e^{\gamma_0}}{r^*+\delta} - I
\]  

\textsuperscript{11} We discuss these subsidies in greater detail in Section 4 of the paper.
This value is finite when \( r^* \equiv r + \theta \sigma_{x,c} > -\delta \). The rate \( r^* \) is the risk adjusted discount factor for expected operating profit.

Setting the net value of the investment to zero, we find the manager’s minimum profit rate for acceptance of a permanently maintained investment, denoted as, \( \xi_p \),

\[
\xi_p = \ln(I) + \ln(r^* + \delta)
\]  

This operating profit “covers” both risk-adjusted rate of return on the initial investment and the depreciation rate. For notational convenience, we denote minimum profit as \( \xi_p \equiv \exp(\xi_p) \).

**The Maintained Investment**

If the manager permanently maintains the investment, then \( g(\bar{x}) = 1 \) in equations (3) and (4).

He/she maintains the investment even with negative free cash flow, which, in our case, is operating profit, \( \bar{X} \), less maintenance expenditure, \( \delta I \). The net value of the investment, \( NV_p(x_0) \), which is the solution to the integral in equation (4) less the initial investment, \( I \), is

\[
NV_p(x_0) = \frac{e^{\xi_0}}{r^*} - \frac{\delta I}{r} - I
\]  

This value is finite when \( r^* = r + \theta \sigma_{x,c} > 0 \). For a positive risk-premium, \( \theta \sigma_{x,c} \geq 0 \), a risk-adjusted rate, \( r^* \), that exceeds the risk-less rate, \( r \), discounts expected operating profit, \( e^{\xi_0} \). The discount factor in (7) does not reflect economic depreciation, \( \delta \), because the manager permanently maintains the investment. The riskless rate, \( r \), discounts non-random maintenance, \( \delta I \).
The manager requires an operating profit of at least

$$\ln(I) + \ln(r + \delta) + \ln(r^* / r)$$

(8)
to accept the maintained investment.

**The Maintained Versus the Unmaintained Investment**

Net value creation of the maintained investment over the unmaintained investment is,

$$\frac{e^{x_0}}{r^*} - \frac{e^{x_0}}{r^*+\delta} - \delta I = \frac{\delta}{r} \left[ \frac{e^{x_0}}{r^*+\delta} \right]$$

(9)

This amount is positive when $$\frac{e^{x_0}}{(r^*+\delta)} r^* - I \geq 0$$. For a positive risk premium, $$\theta \sigma_{x,s} > 0$$, the risk adjusted rate, $$r^*$$, exceeds, the riskless rate, $$r$$, and, therefore, maintenance creates incremental value only for investments that have more than marginal initial net value,

$$\frac{e^{x_0}}{r^*+\delta} - I$$. That is, if the initial investment is marginally acceptable, $$\frac{e^{x_0}}{r^*+\delta} - I = 0$$, then because $$\frac{r}{r^*} \leq 1$$, net value creation of maintenance is negative. Further, this expression,

$$\frac{e^{x_0}}{(r^*+\delta)} r^* - I$$, decreases in the rate of economic depreciation, $$\delta$$. Other things equal, maintenance creates incremental net value for properties least in need of maintenance (low $$\delta$$).

For a positive risk premium, $$\theta \sigma_{x,s} \geq 0$$, maintenance investments add proportionately more to discounted required expenditures than to discounted expected operating profits. The percentage increase in the value of operating profit from maintenance is

$$\left( \frac{e^{x_0}}{r^*} - \frac{e^{x_0}}{r^*+\delta} \right) / \frac{e^{x_0}}{r^*+\delta} = \frac{\delta}{r^*}$$. 

13
which is independent of the profit rate, \( x_0 \). Because this term is constant, maintenance investments add more to the dollar value of operating profit for high profit rates, \( x_0 \). On the other hand, the percentage increase in discounted expenditures, composed of the initial expenditure, \( I \), and maintenance expenditures, is \( \frac{I+\delta I-I}{I} = \frac{\delta}{r} \). This percentage increase, \( \frac{\delta}{r} \), exceeds the percentage increase in the value of operating profit, \( \frac{\delta}{r^*} \). Consequently, for a marginal unmaintained investment, which only breaks even in terms of net value, maintenance makes the overall investment unacceptable. Proportionality is important for maintenance investments because they are incremental to the initial investment and cannot be made on their own; a building cannot be maintained if it is not first purchased or built. Maintenance does not improve the net value creation of an investment that does not create positive net value without maintenance. Profitability of the initial investment must be more than minimal before maintenance enhances value.

Let \( \xi_p^* \) be the critical operating profit rate for permanent maintenance and \( \Omega_p^* \equiv \exp(\xi_p^*) \) the critical operating profit. This profit rate makes the expression in equation (9) zero,

\[
\xi_p^* = \ln(I) + \ln(r^* + \delta) + \ln(r^*/r) \geq \xi_p
\]  

(10)

Figure 1 depicts the relations between the values of the permanently maintained and the unmaintained investments (equations (7) and (5), respectively). Point “a” represents the minimum operating profit for the unmaintained investment, point “b” is the minimum profit for
the maintained investment. At the profit rate “c,” the net values of the unmaintained and maintained investments equal one another.\textsuperscript{12}

For a positive risk premium, $\theta \sigma_{x,c} \geq 0$, the critical maintenance profit, $\Omega_p^*$, exceeds the minimum operating profit for investment acceptance, $\Omega_p^* \geq \Omega_p$, and the manager imposes a more demanding profit standard on maintenance compared to the original investment. For profits, $X_0$, greater than, $\Omega_p$, but less than $\Omega_p^*$, the manager makes, but does not maintain, the investment. In particular, the manager makes, but does not maintain a marginal investment. For profits, $X_0$, greater than $\Omega_p^*$, the manager accepts the investment and undertakes to permanently maintain it. In addition, for a positive risk-premium, $\theta \sigma_{x,c} \geq 0$, the minimum profit in equation (8) for the maintained investment exceeds the minimum profit for the unmaintained investment, $\bar{\Omega}_p$ (points “b” and “a,” respectively, in figure 1).

\textsuperscript{12} Appendix B gives an empirical justification for the model parameters that we use in figure 1.
Figure 1 ■ Permanently Maintained Versus Permanently Unmaintained Investment.

This figure plots net value, $NV_p(x_0)$, which is prior to investment acceptance, against the operating profit, $X_0$, when the manager either permanently maintains the investment or leaves it permanently unmaintained. Point “a” represents the minimum operating profit for the unmaintained investment, point “b” is the minimum profit for the maintained investment. At the profit rate “c,” the net values of the unmaintained and maintained investments equal one another. For profits, $X_0$, above $\Omega^*_p = \exp(\xi^*_p)$, where these two curves intersect, the manager makes the investment and permanently maintains it. For profits, $X_0$, between $\bar{\Omega}_p = \exp(\bar{\xi}_p)$ and $\Omega^*_p$, the manager accepts the investment, but leaves it unmaintained. Parameter values equal $r = 0.05$, $\sigma = 0.07$, $\theta \sigma_{x,c} = 0.03$, $I = 1$, $\delta = 0.014$. 

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Parameter values equal $r = 0.05$, $\sigma = 0.07$, $\theta \sigma_{x,c} = 0.03$, $I = 1$, $\delta = 0.014$.
Finally, the first two terms of equation (10) contain minimum profit rate for investment acceptance, $\xi_p = \ln(I) + \ln(r^* + \delta)$. So, the critical maintenance profit rate, $\xi^*_p$, increases with $\xi_p$, the minimum profit rate for investment acceptance. This commonality indicates that a necessary condition for maintenance to create positive incremental net value is that the initial investment without maintenance first create positive net value. Further, because the minimum investment profit rate, $\xi_p$, increases with the rate of economic depreciation, $\delta$, the critical profit rate for maintenance, $\xi^*_p$, also increases with the rate of depreciation, $\delta$. See figure 2 for a numerical example. Economic depreciation affects both the initial investment and maintenance investments, so the profit, $X_0$, must “cover” this depreciation, $\delta$, to create positive net value for either.

The last term on the right hand side of equation (10), $\ln(r^*/r)$, is the increment to the profit rate, above investment acceptance, required to make the net value of maintenance positive. This increment is positive for a positive risk premium, $\theta \sigma_{x,c}$, and vice versa. Maintenance adds risky operating profit to an investment, but riskless maintenance costs. Therefore, beyond the initial investment, maintenance creates positive net value only if it “covers” this incremental risk “cost,” which increases with the risk premium, $\theta \sigma_{x,c}$.

For a zero risk premium, $\theta \sigma_{x,c} = 0$, maintenance investments are precisely a scaled version of the original investment both with respect to profit and net value creation. Maintenance investments create positive net value when the unmaintained investment does so and vice versa.

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13 For a positive risk premium, $\theta \sigma_{x,c} \geq 0$.  

The manager maintains an accepted investment. Points “a,” “b,” and “c” collapse upon one another in figure 1.

**The Maintenance Option**

The preceding section develops useful insights into the maintenance decision of managers. This analysis highlights, for example, the importance of the risk-premium for a manager’s decision to maintain or not. However, this analysis has the unappealing feature that a manager cannot reverse a maintenance decision. In what follows, we consider a manager who has a dynamic option to suspend and recommence maintenance depending upon profitability. We investigate the minimum profit rate for investment acceptance when the manager has this option. A comparison with results of the preceding section answers the primary question of this paper: when does a value-maximizing manager, who has a dynamic option to maintain, impose a more demanding profit standard on maintenance compared to the original investment?

**The Value Function**

The suspension boundary for maintenance equals the restart boundary because we presume that the manager suspends and recommences costlessly. Denote the maintenance boundary as \( \xi \) in terms of profit rate and \( \Omega = \exp(\xi) \) in terms of profits. The manager maintains the investment only when \( x_0 \geq \xi \). The manager always operates the building, which earns profit, \( \tilde{X} \), but it might or might not be maintained. Instantaneous maintenance expenditures equal either \( \delta l dt \) or zero. When unmaintained, the resulting economic depreciation on the building cannot be
reversed in the future by more intense maintenance. The manager bears permanent economic depreciation as a consequence of forgone maintenance.\textsuperscript{14}

After the initial investment, \( I \), instantaneous free cash flow, operating profit less maintenance, is,

\[
f(\tilde{x}_t) \, dt = \begin{cases} 
  e^{\tilde{x}} \, dt, & x_0 < \xi \\
  e^{\tilde{x}} \, dt - \delta I \, dt, & x_0 \geq \xi
\end{cases}
\]  

(11)

With this expression for free cash flow, the investment’s value function, equation (4), for an arbitrary maintenance boundary, \( \xi \), requires expressions for the discounted value of expected operating profit and for discounted expected maintenance expenditures. Both these amounts depend upon whether the manager maintains the investment, \( x_0 \geq \xi \), or not, \( x_0 < \xi \). The appendix develops these expressions.

Write the investment’s value function, \( V(x_0) \), with equations (A1) and (A2) from the appendix,

\[
V(x_0) = \begin{cases} 
  \frac{e^{x_0}}{r^* + \delta} + \frac{\delta \xi}{r^* (r^* + \delta)} \frac{1 + \alpha}{\alpha + \lambda} e^{\lambda (x_0 - \xi)} - \frac{\delta I}{r} \frac{\alpha}{\alpha + \lambda} e^{\lambda (x_0 - \xi)}, & \text{unmaintained, } x_0 < \xi \\
  \frac{e^{x_0}}{r^* + \delta} + \frac{\delta e^{\xi}}{r^* (r^* + \delta)} \frac{1 - \lambda}{\alpha + \lambda} e^{-\alpha (x_0 - \xi)} - \frac{\delta I}{r} \left[ 1 - \frac{\lambda}{\alpha + \lambda} e^{-\alpha (x_0 - \xi)} \right], & \text{maintained, } x_0 \geq \xi
\end{cases}
\]  

(12)

where \( \lambda = \left[ \frac{1}{2} + \frac{\delta + \theta \sigma_{x,c}}{\sigma^2} \right] + \sqrt{\phi^2 + \left[ \frac{1}{2} + \frac{\delta + \theta \sigma_{x,c}}{\sigma^2} \right]^2} \), \( \alpha = \left[ \frac{1}{2} + \frac{\theta \sigma_{x,c}}{\sigma^2} \right] + \sqrt{\phi^2 + \left[ \frac{1}{2} + \frac{\theta \sigma_{x,c}}{\sigma^2} \right]^2} \), and

\( \phi \equiv \frac{2r}{\sigma^2} \). It is easy to see that both \( \alpha \) and \( \lambda \) are positive, \( \alpha \geq 0 \) and \( \lambda \geq 0 \). Further, \( \lambda \)

\textsuperscript{14} One interpretation of this cost is that forgone maintenance damages the quality “reputation” of a building, which, other things equal, permanently diminishes rental income.
exceeds one. To see this result, note that $\lambda$ increases in the riskless rate, $r$. At the minimum riskless rate (recall, $r^* + \delta \equiv r + \delta + \theta \sigma_{x,c} \geq 0$), $\lambda$ equals its minimum, which is one, so, $\lambda \geq 1$.\(^{15}\)

The first term on the unmaintained branch is the value of expected operating profit if the investment is never maintained. The second term, which is positive, is the potential for increased value from operating profits if the profit rate, $x_0$, increases above $\xi$ and the manager maintains the investment. The third term, which is negative, is the value loss from maintenance costs (when the profit rate, $x_0$, increases above $\xi$ and the manager maintains the investment). The first term on the maintained branch is the value of expected operating profit if the investment is always maintained. The second term, which is negative (because $\lambda \geq 1$), is the potential value loss from permanent economic depreciation when the profit rate, $x_0$, decreases below the maintenance boundary, $\xi$, and the manager leaves the investment unmaintained. The third term, which is negative, is the discounted value of maintenance expenditures.

Note that when the economic depreciation is zero, $\delta = 0$, the value of operating profit, $P(x_0)$, which is the first two terms on either branch of equation (12), is $\frac{e^{x_0}}{r^*}$. In this case, maintenance does not increase the value of operating profit, but the cost of maintenance is also zero, $\delta I = 0$.

\(^{15}\) At its minimum, $r = -\delta - \theta \sigma_{x,c}$, and $\phi^2 = \frac{2r}{\sigma^2} = -2 \left( \frac{\delta + \theta \sigma_{x,c}}{\sigma^2} \right)$. Substituting this into the expression for $\lambda$ and rearranging provides the result that $\lambda = 1$ for the minimum risk-free rate.
To find the value-maximizing maintenance boundary, \( \xi^* \), take the derivative of the value function, \( V(x_0) \), with respect to \( \xi \) on either branch of equation (12), set the result to zero, evaluate at \( x_0 = \xi \), and solve for \( \xi^* \),

\[
\xi^* = \ln(I) + \ln(r^* + \delta) + \ln(r^*/r) + \ln \left[ \frac{\alpha \lambda}{(1 + \alpha)(\lambda - 1)} \right] \tag{13}
\]

The first three terms on the right hand side of equation (13) equal the critical profit rate, \( \xi_p^* \), for maintenance when the maintenance decision is permanent (see the prior section of this paper).

The last term in equation (13) represents the effect on the maintenance boundary, \( \xi^* \), of two mutually exclusive options for the investment: maintain or not. The effect of these competing options on the maintenance boundary, \( \xi^* \), depends on the relative significance of the two parts of free cash flow: operating profit and maintenance costs. When unmaintained, the investment avoids maintenance costs, but profit deteriorates. When maintained, the investment generates greater profit, but incurs maintenance expenditures.

The difference in parameters, \( \lambda - \alpha \) represents the relative significance of the option to avoid costs by suspending maintenance versus the option to generate extra profit through maintenance. If \( \lambda - \alpha \leq 1 \), i.e., the depreciation rate, \( \delta \), is relatively small, the manager’s relative priority in operating the investment is to avoid maintenance costs and the last term in equation (13) is greater then zero. In this case, the optimal maintenance boundary, \( \xi^* \), exceeds that for irreversible maintenance, \( \xi^* \geq \xi_p^* \).
This difference in parameters, $\lambda - \alpha$, increases in $\delta$, the depreciation rate. Consequently, if the depreciation rate, $\delta$, exceeds a certain level (given by solving the equation $\lambda - \alpha = 1$), then the dynamic boundary is smaller then if maintenance is permanent, $\xi^* < \xi^p$.

The incremental impact, $\ln\left[\frac{a\lambda}{\lambda - \alpha - 1 + a\lambda}\right]$, of the maintenance option on the maintenance boundary, $\xi^*$, depends on three related factors. First, the manager captures incrementally higher operating profits when he/she maintains the building. Second, expected maintenance costs increase with the fraction of time that the manager maintains the investment. Third, there is permanent economic depreciation on the investment when the manager leaves it unmaintained. For a high rate of depreciation rate, $\delta$, each of these factors favors an incrementally lower maintenance boundary, $\xi^*$, and vice versa. For the same parameter values as figure 1 (other than $\delta$), figure 2 depicts the optimal maintenance boundary, with and without the maintenance option, $\Omega^* = \exp(\xi^*)$ and $\Omega^p = \exp(\xi^p)$ respectively, as $\delta$ increases from $\delta = 0.0$ to $\delta = 0.015$. 
Figure 2 ■ Optimal maintenance boundary, with and without the maintenance option.

This figure plots the optimal maintenance boundary, with and without the maintenance option, \( \Omega^* = \exp(\xi^*) \) and \( \Omega^*_p = \exp(\xi^*_p) \) respectively, as \( \delta \), the depreciation rate, increases from \( \delta = 0.0 \) to \( \delta = 1.5\% \). The investment in panel A has a modest risk premium, \( \theta \sigma_{x,c} = 0 \). The investments in panels B, C, and D have increasingly higher risk premium. Other parameter values equal \( r = 0.05 \), \( \sigma = 0.07 \), \( I = 1 \).
For a high depreciation rate, $\delta$, there is a great benefit to maintenance, greater incremental operating profit, which the manager captures with a lower maintenance boundary, $\xi^*$. Second, other things equal, for a high rate of economic depreciation, $\delta$, the stochastic process for the profit rate, $\bar{x}$, spends relatively more time in the unmaintained state, which does not incur maintenance costs. Lower maintenance costs encourage the manager to maintain the building to a greater extent by lowering the maintenance boundary, $\xi^*$. Third, for a high rate of economic depreciation, $\delta$, there is a high “cost” to permanent economic depreciation, which the manager avoids by lowering the maintenance boundary, $\xi^*$.

On the other hand, for a low rate of economic depreciation, $\delta$, these factors have the opposite effect on the maintenance boundary, $\xi^*$. Modest permanent economic depreciation in the unmaintained state and modest incremental operating profit from maintenance, encourage the manager to increase the maintenance boundary, $\xi^*$, above that for permanent maintenance, $\xi_p^*$, and avoid maintenance costs, which are relatively greater because, other things equal, the investment spends more time in the maintained state.

*The Optimal Value Function*

Substitute equation (13) into each branch of equation (12) and subtract the initial investment, $I$, to find the optimal net value function, $NV^*(x_0)$,
\[
NV^*(x_0) = \begin{cases} 
\frac{e^{-x_0}}{r} + \Psi_u e^{x_0} - I, & \text{unmaintained, } x_0 < \xi^* \\
\frac{e^{-x_0}}{r^*} + \Psi_m e^{-\alpha x_0} - \frac{\delta}{r} - I, & \text{maintained, } x_0 \geq \xi^* 
\end{cases}
\]

(14)

where \(\Psi_u = \frac{\delta I \alpha}{r(\alpha + \lambda)(\lambda - 1)} \left[ \frac{r(\alpha + 1)(\lambda - 1)}{(r + \delta) \gamma \alpha \lambda I} \right]^{\gamma}
\) and \(\Psi_m = \frac{\delta I \alpha}{r(\alpha + \lambda)(\alpha + 1)} \left[ \frac{(r + \delta) \gamma \alpha \lambda I}{r(1 + \alpha)(\lambda - 1)} \right]^{\alpha} \).

The upper and lower branches of equation (14) differ from equations (5) and (7), the net value of the investment when it is never or always maintained, respectively, only by the terms \(\Psi_u e^{x_0}\) and \(\Psi_m e^{-\alpha x_0}\). These terms represent the value of the maintenance option for the investment when unmaintained, \(x_0 < \xi\), or maintained, \(x_0 \geq \xi\), respectively. Because these terms are positive, at the same profit rate, \(x_0\), the investment’s value with the maintenance option exceeds the investment’s value without this option.

**The Minimum Profit Rate for the Investment that has the Maintenance Option**

Denote the manager’s minimum profit rate to accept an investment that has the maintenance option as, \(\xi^*\). This profit rate, \(\xi^*\), makes net value zero, \(NV^*(\xi^*) = 0\), either on the upper branch of equation (14), when \(\xi^* < \xi^*\), or on the lower branch of equation (14), when \(\xi^* \geq \xi^*\).

Comparing the upper branch of equation (14) with equation (5), which is the net value of the permanently unmaintained investment, and the lower branch of equation (14) with equation (7), which is the net value of the permanently maintained investment, one can verify that \(\xi^* \leq \xi_{p}^*\).

The minimum profit rate for the investment that has the maintenance option is always lower than
the minimum profit rate for the permanently unmaintained investment.\textsuperscript{16} Because the maintenance option adds value to the investment (the second term on each branch of equation (14)), the manager accepts the investment for a lower profit rate.

The expressions on the branches of equation (14) contain sums of exponential functions. So, generally, there is no closed form solution for $\xi$, the manager’s minimum profit rate for the investment that has the maintenance option. Numerically, however, we can compare $\xi$ to the maintenance boundary, $\xi^*$, for this investment. The panels of figure 3, depict four investments, all of which have the dynamic maintenance option. The first one has a modest risk premium, $\theta \sigma_{x,c} = 0$, which increases to $\theta \sigma_{x,c} = 0.03$ in the last one. We plot the profit boundary for investment acceptance, $\bar{\Omega} = \exp(\xi)$, and the profit boundary for maintenance, $\Omega^* = \exp(\xi^*)$, against the rate of economic depreciation, $\delta$.

\textsuperscript{16} We presume in this discussion that the risk-premium, $\theta \sigma_{x,c}$, is positive. If negative, then $\xi_p$ is given in equation (8) as the profit, $X_o$, that makes equation (7) zero, but the result, $\bar{\xi} \leq \xi_p$ remains.
Figure 3: Profit Boundaries for Investment Acceptance and Maintenance Versus Depreciation.

The panels of this figure depict four investments, all of which have the dynamic maintenance option. The investment in panel A has a modest risk premium, $\theta \sigma_{x,c} = 0$. The investments in panels B, C, and D have increasingly greater risk premium. We plot, the profit boundary for investment acceptance, $\Omega = \exp(\xi)$, and the profit boundary for maintenance, $\Omega^* = \exp(\xi^*)$, against the rate of economic depreciation, $\delta$, which varies from 0% to 5%. Other parameter values equal $r = 0.05$, $\sigma = 0.07$, $I = 1$. 

![Graph of profit boundaries for different investment panels.](image)
The increase in both boundaries, $\bar{\Omega}$ and $\Omega^*$, with $\delta$ highlights economic depreciation as an undesirable feature of the investment. Both profit boundaries, for initial investment and maintenance, increase to “cover” greater depreciation.

In panel A of figure 3, which represents an investment with a low risk premium, $\theta \sigma_{x,c} = 0$, even small rates of depreciation force the profit boundary for investment acceptance to exceed the profit boundary for maintenance, $\bar{\Omega} \geq \Omega^*$. The manager maintains a new investment ($X_0 \geq \bar{\Omega}$) until, at some time in the future, operating profit falls to the maintenance boundary, $\Omega^*$. The manager imposes a relatively undemanding profit standard on maintenance to avoid permanently high economic depreciation in the unmaintained state. However, in the first instance, prior to the investment, the manager recognizes the great burden of permanent economic depreciation for an accepted investment. As compensation, the manager imposes a more demanding profit standard on the initial investment compared to maintenance, $\bar{\Omega} \geq \Omega^*$.

For either very small economic depreciation (left most portion of panel A for figure 3), or for a greater risk premium (panels B, C, and D of figure 3), the profit boundary for maintenance, $\Omega^*$, exceeds the profit boundary for investment acceptance, $\bar{\Omega}$. The manager imposes a more demanding profit standard on maintenance than on the initial investment and leaves a marginal investment ($X_0 = \bar{\Omega}$) unmaintained until operating profit increases to the maintenance boundary, $\Omega^*$.

The increasing difference between $\Omega^*$ and $\bar{\Omega}$ in panels B, C, and D of figure 3 is because of risky incremental operating profit added by maintenance. Even for a dynamic maintenance decision, profit must “cover” the incremental “cost” of this risk. The “cost” of this incremental
risk increases with the risk premium $\theta \sigma_{s,c}$. The manager imposes a relatively more demanding profit standard on maintenance compared to the initial investment as the incremental risk of the maintenance investment increases.

**Empirical and Policy Implications**

The most immediate empirical implication of the above model is that maintenance is suspended if operating profits fall below a certain critical level. Using aggregate maintenance data one could test whether expenditures decline if operating profits fall. Note that our model predicts that this relationship is highly non-linear.

More importantly, real estate properties with least need of maintenance, i.e., relatively low depreciation rate, tend to be maintained the most, i.e., have a relatively low critical profit level for maintenance. If quality of initial construction is a proxy for economic depreciation, our model implies that high-quality buildings tend to be fully maintained even in poor economic conditions. On the other hand, poor quality buildings face maintenance suspension even in the face of modest economic slowdowns. This discrepancy suggests that the divergence between high and low quality buildings increases over time. To the extent that buildings of similar quality tend to cluster together in neighborhoods, our model predicts that absent an outside intervention the segmentation of an area into high and low quality neighborhoods deepens.

Given our assumption about the evolution of operating profits, we can investigate the probability of resuming maintenance once it has been suspended. If the operating profits fall below the critical profit boundary for maintenance, $\Omega^*$, then maintenance is suspended until profits improve. Let $\gamma$ denote the percent increase in profits required for resuming maintenance,
\[ \gamma = \ln(\Omega^* / X_0). \]
Also, let \( T \) denote the time when maintenance is first resumed, i.e., when the first time operating profits cross the critical maintenance boundary. Appendix C shows that the probability of the first passage time being less then \( t \) is given by:

\[
Pr(T \leq t) = \int_0^t \frac{\gamma}{\sigma \sqrt{1 \pi s^3}} \exp \left( -\frac{(\gamma + (\delta + \sigma^2 / 2)s)^2}{2\sigma^2 s} \right) ds \tag{15}
\]

Note that if maintenance is currently suspended, i.e., \( X_0 < \Omega^* \), then \( \gamma > 0 \) and \( Pr(T \leq t) \) is a decreasing function of the rate of depreciation. In other words, high depreciation rate properties face a lower probability of resumed maintenance over any given time interval. Furthermore, Equation (15) is not a probability distribution function in the usual sense because even for \( t = \infty \), the integral does not equal 1. In other words, there is no guarantee that maintenance will even be resumed. In fact, the higher the depreciation rate, \( \delta \), the higher the probability that maintenance will never be resumed, regardless of how favorable future economic conditions become.

To illustrate the importance of the above finding, we provide three numerical examples using our parameter estimates. Figure 4 depicts the probability of resuming maintenance at any time in the future if the current profit, \( X_0 \), is

(a) 10\% below the critical maintenance boundary, i.e., \( X_0 = 0.9 \Omega^* \) (i.e., both \( X_0 \) and \( \Omega^* \) change with \( \delta \)),

(b) just high enough to keep an investment with negligible depreciation rate fully maintained, i.e. \( X_0 = \Omega^*(\delta = 0) \) (i.e., \( X_0 \) is fixed, but \( \Omega^* \) changes with \( \delta \)), and
(c) just meets the profit boundary for investment acceptance, i.e., \( X_0 = \overline{\Omega} \) (i.e., both \( X_0 \) and \( \Omega^* \) change with \( \delta \)).

Figure 4 illustrates that in scenario (a), properties with extremely small depreciation face a probability of resumed maintenance at some point in the future of over 80%. If the depreciation rate is at our best estimate of 1.4%, the probability of resuming maintenance, however, is below 50%. Properties with even slightly higher depreciation rate face a probability of resumed maintenance of 20% or lower.

This finding can be further strengthen by considering the implication of Figure 3 that the critical maintenance boundary, \( \Omega^* \), is an increasing function of the depreciation rate, \( \delta \). To illustrate this we consider an operating profit that is just sufficient to currently keep an investment with negligible depreciation rate fully maintained. In the case of panel D of Figure 3, for instance, this operating profit is 12.4%. Investments with any measurable depreciation rate would not be maintained in that case until operating profits improve. Scenario (b) of figure 4 depicts the probability of resuming maintenance in this case for various depreciation rates.

Notice how quickly the probability of resuming maintenance under all three scenarios depicted in Figure 4 drops for higher depreciation rates. An investment with depreciation rate equal to our best estimate of 1.4% has a probability of resumed maintenance at any time in the future of less then 40% under scenario (a). High depreciation rate properties have virtually no chance of seeing their maintenance resumed.
Figure 4 ■ Probability of Resuming Maintenance.

The figure depicts the probability of resuming maintenance at any time in the future if the current profit, $X_0$, is (a) 10% below the critical maintenance boundary, i.e., $X_0 = 0.9 \Omega^*$, (b) just high enough to keep an investment with negligible depreciation rate fully maintained, i.e. $X_0 = \Omega^*(\delta = 0)$, and (c) just meets the profit boundary for investment acceptance, i.e., $X_0 = \bar{\Omega}$. Other parameter values equal $r = 0.05$, $\theta \sigma_{z,c} = 0.03$, $\sigma = 0.07$, $I = 1$. 

![Probability of Resuming Maintenance](image)
The above two examples suggest how urban areas decay following a negative demand shock but never recover even if general economic conditions improve. Furthermore, since the profit hurdle rate for new investment is generally lower than that for maintenance, existing buildings may be left unmaintained even as new nearby areas are developed.

If, as discussed above, building quality is a proxy for the rate of depreciation, our analysis indicates that higher quality buildings are not only maintained longer in poor economic conditions, but also have a higher probability of seeing resumed maintenance if the economy improves. Poor quality buildings, however, tend to have their maintenance suspended even in the face of modest economic downturns. Furthermore, these buildings have a very low probability of seeing a resumed maintenance in the future. These two factors reinforce the important conclusion that absent an outside intervention the segmentation of an area into high and low quality neighborhoods deepens.

Another important empirical implication is that, marginal investment properties may be built but never maintained. This is especially true for high depreciation rate properties. Scenario (c) of figure 4 depicts the probability of starting maintenance at any time in the future for an investment whose operating profit just meets the profit boundary for investment acceptance, $X_0 = \bar{\Omega} = \exp(\zeta)$. For our parameter estimates, this probability is at most 37% for an investment with very small depreciation rate. For any measurable depreciation rate the probability of ever starting maintenance on the building quickly drops to less than 5%. Thus, a subsidy designed to induce initial investment for an otherwise unacceptable project is almost certainly not sufficient to induce maintenance. The policy implication is that if these properties
are to be maintained, the subsidy has to be large enough to induce not only initial investment but also subsequent maintenance.

Alternatively, the government agency providing the subsidy may contractually force the investor to permanently maintain the property. Our analysis of permanent maintenance depicted in Figure 1 indicates that such a requirement increases the minimum profit required for the initial investment. For our parameter values, the profit rate boundary for accepting a permanently maintained investment exceeds the profit rate boundary for maintenance in the dynamic case. This is intuitive because the dynamic case allows for suspension of maintenance in the future should profits fall. In other words, the subsidy is most expensive if permanent maintenance is required, slightly cheaper if maintenance is not required but optimally induced, and cheapest if maintenance is neither required nor induced. This cheapest subsidy, however, virtually guarantees that the investment will not be maintained, which, in turn, defeats the initial purpose of the subsidy altogether.
Conclusion

We investigate the minimum profit rate required for acceptance of a real estate investment depending upon whether or not a manager prevents economic depreciation by maintaining the investment, and also, depending upon whether or not the manager dynamically suspends maintenance due to inadequate profit. For irreversible maintenance decisions and a positive risk premium, the manager imposes a more demanding profit standard on maintenance than on the initial investment. If the manager dynamically suspends maintenance, and if the investment has both a low risk-premium and high economic depreciation, then the manager maintains a new investment until operating profit falls. For either a greater risk premium or lower economic depreciation, the manager imposes a more demanding profit standard on maintenance than on the initial investment and leaves a marginal investment unmaintained until operating profit increases.
Appendix A: Expected profits and costs in the dynamic decision case

*Expected Operating Profit*

Let \( P(x_0) \) be expected discounted operating profit. With power utility, \( P(x_0) \) satisfies the partial differential equation (PDE),\(^{17}\)

\[
0 = rP(x) - \mu P_x(x) - e^x + \frac{\sigma^2}{2} P_{xx}(x)
\]

where \( \mu \) is the risk adjusted profit drift. If the manager leaves the investment unmaintained, 

\[
x_0 < \xi, \text{ then } \mu = -\delta - \frac{\sigma^2}{2} - \theta \sigma_x. \quad \text{If maintained, } x_0 \geq \xi, \text{ then } \mu = -\frac{\sigma^2}{2} - \theta \sigma_x. \]

The solution to this PDE, eliminating the terms with no economic content, is,

\[
P(x_0) = \begin{cases} 
\frac{e^{x_0}}{r} + C_2 e^{\lambda x_0}, & \text{unmaintained, } x_0 < \xi \\
\frac{e^{x_0}}{r^*} + C_1 e^{-\alpha x_0}, & \text{maintained, } x_0 \geq \xi 
\end{cases}
\]

where \( \lambda = \left[ \frac{\mu}{\sigma^2} + \frac{\theta \sigma_x}{\sigma^2} \right] + \sqrt{\left[ \frac{\mu}{\sigma^2} + \frac{\theta \sigma_x}{\sigma^2} \right]^2 + \phi^2 + \left[ 1 + \frac{\theta \sigma_x}{\sigma^2} \right]^2}, \quad \alpha = \left[ \frac{1}{2} + \frac{\theta \sigma_x}{\sigma^2} \right] + \sqrt{\phi^2 + \left[ \frac{1}{2} + \frac{\theta \sigma_x}{\sigma^2} \right]^2}, \quad \text{and} \)

\[
\phi^2 = \sqrt{\frac{2r}{\sigma^2}}. 
\]

Determine the arbitrary constants \( C_1 \) and \( C_2 \) with value matching and smooth pasting conditions\(^{18}\) at \( x_0 = \xi \). Value matching requires that \( P(\xi) \) is the same on the maintained and

\(^{17}\) See Goldstein, Ju, and Leland (2001).
unmaintained branches. Similarly, smooth pasting requires that \( P' (\xi) \) is the same on the maintained and unmaintained branches. These two conditions yield two equations for the two parameters \( C_1 \) and \( C_2 \). Solve these two equations to find,

\[
P(x_0) = \begin{cases} \\
\frac{e^{x_0}}{r} + \frac{\delta e^{\delta}}{r^* (r^* + \delta)} \frac{1 + \alpha}{\alpha + \lambda} e^{\lambda (x_0 - \xi)}, & \text{unmaintained, } x_0 < \xi \\
\frac{e^{x_0}}{r^*} + \frac{\delta e^{\delta}}{r^* (r^* + \delta)} \frac{1 - \lambda}{\alpha + \lambda} e^{-\alpha (x_0 - \xi)}, & \text{maintained, } x_0 \geq \xi
\end{cases}
\]  

(A1)

Expected Maintenance Costs

Let \( C(x_0) \) be expected discounted maintenance costs. For arbitrary parameters \( C_1 \) and \( C_2 \),

\[
C(x_0) = \begin{cases} \\
\frac{\delta I}{r} C_1 e^{\lambda x_0}, & \text{unmaintained, } x_0 < \xi \\
\frac{\delta I}{r} \left[ 1 - C_1 e^{-\alpha x_0} \right], & \text{maintained, } x_0 \geq \xi
\end{cases}
\]

Determine the arbitrary parameters \( C_1 \) and \( C_2 \) with value matching and smooth pasting conditions at \( x_0 = \xi \). Solve the two equations,

\[
C(x_0) = \begin{cases} \\
\frac{\delta I}{r} \frac{\lambda}{\alpha + \lambda} e^{\lambda (x_0 - \xi)}, & \text{unmaintained, } x_0 < \xi \\
\frac{\delta I}{r} \left[ 1 - \frac{\lambda}{\alpha + \lambda} e^{-\alpha (x_0 - \xi)} \right], & \text{maintained, } x_0 \geq \xi
\end{cases}
\]  

(A2)

\[^{18}\text{See Dixit and Pindyck (1994) for discussion of value matching and smooth pasting.}\]
At $x_0 = \xi$, the term, $\frac{\alpha}{\alpha + \lambda}$, represents the fraction of time that the investment spends in the maintained state.
Appendix B: Data and Empirical Calibration

Throughout this paper we use the following parameters to illustrate the solution of the model:

- Risk-free rate, \( r \),
- Risk premium for a typical real estate investment, \( \theta c, c \),
- Volatility of the operating profits, \( \sigma \),
- Initial investment, \( I \), normalized to one, and
- Depreciation rate, \( \delta \).

We use apartment building transaction data from the Los Angeles Metropolitan Area (Provided by CoStar COMPS) to empirically estimate the above parameters. The data provides transaction details and property characteristics for all transactions between October 1989 and June 2001. After filtering for missing values and data errors we have 14,878 usable observations. Table 1 provides summary statistics describing the transactions that occurred during the period. The mean and median per unit price during this period were a little more than $50,000 per unit. As can be seen from the table, the typical LA County apartment complex is relatively small and the distribution of complex size is positively skewed, with a median of 10 and a mean of 18 units. The risk premium for apartment buildings is approximated by the spread of cap rate over the contemporaneous risk-free rate.
Table 1: Descriptive statistics for transactions 10/1989 – 6/2001

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>St. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per unit</td>
<td>$56,415</td>
<td>$51,875</td>
<td>$26,482</td>
</tr>
<tr>
<td>Cap rate</td>
<td>.08</td>
<td>.08</td>
<td>.023</td>
</tr>
<tr>
<td>Risk premium (Cap rate – riskless rate)</td>
<td>.03</td>
<td>.036</td>
<td>.025</td>
</tr>
<tr>
<td>NOI per unit</td>
<td>$4,677</td>
<td>$4,387</td>
<td>$1,630</td>
</tr>
<tr>
<td>Age</td>
<td>23</td>
<td>34</td>
<td>18</td>
</tr>
<tr>
<td>Parking spaces</td>
<td>23</td>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td>Number of units</td>
<td>18</td>
<td>10</td>
<td>26</td>
</tr>
</tbody>
</table>

Some of the parameters of interest are immediately estimable from this data. The risk-free rate, $r$, over that period is given by short-term treasury bills and is on average 5%. The risk premium for real estate (i.e., the spread of capitalization rate over the risk-free rate) is 3%.

The rate of economic depreciation is not available in our data set. To estimate a reasonable parameter value, we make two assumptions: the expected useful economic life of a building is 50 years, and the building (which depreciates) represents approximately 50% of the value of the entire property. The age of 87.5% of the transactions in our sample is less then 50 years. We assume the remaining 12.5% were sold with the sole purpose of redevelopment. These assumptions provide a depreciation rate of $\delta = 1.4\%$.

Finally, but very importantly, we need an estimate of the volatility of the operating profit. Using the above data we estimate an index of the net operating income. Consistently with the distributional assumptions for $\tilde{X}_t$ made in the body of the paper, we use the following semi-log model:

---

19 It is well known that the rate of accounting depreciation far exceeds the rate of economic depreciation for real estate investments.

20 Given these assumptions the current value is normalized to one, $V_0 = 1$, and the value in 50 years is $\frac{1}{2}$, $V_T = \frac{1}{2}$. The depreciation rate is then given by $\delta = \ln(V_0 / V_T) / T = \ln(1/\cdot 5) / 50 = 1.4\%$.

21 While the data set stipulates that this income is before maintenance expenditures, it is possible that at last part of the maintenance expenditures were reported as part of this income.
\[
\ln(NOI_{it}) = Constant + \sum_{t=2}^{T} \beta_t S_t + \alpha' C_i + \varepsilon_{it} 
\]  \hspace{1cm} (16)

where \( NOI_{it} \) is the net operating income per unit for property \( i \) sold at time \( t \), \( C_i \) is a vector of physical characteristics that describe the building, \( S_t \) is a matrix of indicator variables for the time of sale, and \( \beta_t \) is the marginal time effect (i.e., monthly). \( T \) is the total number of months in the sample and \( \varepsilon_{it} \) is an error term with zero expectation.\(^{22}\) Each transaction represents one observation in the above regression. Thus \( \beta_t \) is an estimate of the rate of change in operating income for time period \( t \).

Tables 2 and 3 report the parameter estimates and implied appreciation rates obtained by estimating Equation (16). The parameter estimates presented in Table 2 have the expected signs and are highly significant. From Table 3 we can see that our model estimated an average monthly rate of change of the net operating income of nearly zero with a monthly standard deviation of 4.4%.

The last column of Table 3 provides a bias corrected estimate of the monthly standard deviation of the NOI. Since the estimated rates of change in NOI are subject to sampling error, the estimated volatility contains an upward bias. Cauley and Pavlov (2002) derive a bias correction technique for similarly derived estimates of home price appreciation. We use this technique here to compute bias-corrected estimates of the standard deviation of NOI, which turns out to be 2% monthly, or 7% annually.

\(^{22}\) The \( \beta_t \) are estimated as follows. If a transaction occurred during January 1989 (i.e., \( t=1 \)), all time indicator variables are assigned a value of zero. If a transaction occurred during the second month, the first time indicator variable is assigned a value of one and all other time indicator variables are set to zero. If a transaction occurred during the third month, the first two indicators are set to one and all others are set to zero.
Table 2: Parameter Estimates

<table>
<thead>
<tr>
<th>Estimate</th>
<th>St. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>-.012</td>
</tr>
<tr>
<td>Age²</td>
<td>8.2 E-5</td>
</tr>
<tr>
<td>.005</td>
<td>.0002</td>
</tr>
<tr>
<td>3.6 E-6</td>
<td>2.3 E-7</td>
</tr>
<tr>
<td>-.009</td>
<td>.0003</td>
</tr>
<tr>
<td>1.2 E-5</td>
<td>6.5 E-7</td>
</tr>
</tbody>
</table>

Table 3: Monthly rates of change in the NOI

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average change in NOI</td>
<td>4.5 E-4</td>
</tr>
<tr>
<td>Monthly standard deviation</td>
<td>4.4%</td>
</tr>
<tr>
<td>Median change in NOI</td>
<td>-2.5 E-4</td>
</tr>
<tr>
<td>Skewness of the change in NOI</td>
<td>-.22</td>
</tr>
<tr>
<td>Bias-Corrected Monthly Standard Deviation</td>
<td>2%</td>
</tr>
</tbody>
</table>

Figure 5 depicts the time series of estimated Net Operating Income (October, 1989=1) of a typical apartment building implied by the parameter estimates of Equation (16). As can be seen from this figure, the late 1980s and 1990s were a boom/bust period for Southern California.
properties. This is consistent with the evolution of property values in that region. Between 1988 and the end of 1989 the per unit price of Los Angeles County apartment building increased by approximately 16 percent. Between the 1989 peak and November 1995 trough per unit prices fell by more than 48 percent. By the beginning of 2000 per unit prices are more than 40 percent above their low. Table 4 summarizes the parameter estimates based on the above analysis.

Table 4: Parameter estimates used as a base case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate, $r$</td>
<td>.05</td>
</tr>
<tr>
<td>Risk premium, $\theta \sigma_{cc}$</td>
<td>.03</td>
</tr>
<tr>
<td>Volatility of the operating profits, $\sigma$</td>
<td>.07</td>
</tr>
<tr>
<td>Initial investment, $I$</td>
<td>1</td>
</tr>
<tr>
<td>Depreciation rate, $\delta$</td>
<td>.014</td>
</tr>
</tbody>
</table>
Appendix C: First Passage Time for Maintenance

Let $\gamma$ denote the percent increase in profits required for resuming maintenance, $\gamma = \ln(\xi^*/X_0)$. Also, let $T$ denote the time until maintenance is first resumed, i.e., until the first time operating profits cross the critical maintenance boundary. Cox and Miller (1965, page 221) show that for a non-negative constant drift $\mu \geq 0$, the probability density function, $g(t)$, of the first passage time, $T$, when the operating profits first cross the critical maintenance boundary is given by:

$$g(t) = \frac{\gamma}{\sigma \sqrt{4\pi t}} \exp \left( -\frac{(\gamma - \mu t)^2}{2\sigma^2 t} \right)$$ (17)

In our case, however, the drift in the rate of operating profits is given by equation (2) and equals $\mu = -\delta - \frac{\sigma^2}{2}$, which is clearly negative. For a negative drift, equation (17) is no longer a probability density function as it does not integrate to 1 over the real line. In other words, there is no guarantee that with a negative drift the critical boundary, $\xi^*$, will be crossed at any time in the future. Nonetheless, we can still use equation (17) to compute the probability of first passage occurring before a given time in the future, $t$:

$$\Pr(T \leq t) = \int_0^t \frac{\gamma}{\sigma \sqrt{4\pi s^3}} \exp \left( -\frac{(\gamma + \delta + \sigma^2/2s)^2}{2\sigma^2 s} \right) ds$$ (18)

Note that this is not a probability distribution function as it does not necessarily integrate to 1 over the real line.


