The Effects of Maturity Choices on Loan Guarantee Portfolios

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Abstract
This paper analyses the effects of the maturities of credit-enhanced debt contracts on the value of an insurer’s loan guarantee portfolios. We propose a contingent-claims model, that not only includes important features such as coupon payments and absolute priority violations, but also allows for stochastic interest rate and stochastic assets volatility. This is a first attempt at modelling multiple maturities in the context of portfolios of loan guarantees. We consider loan guarantees for projects that run either in parallel or in sequence. Our results indicate that for a portfolio of two parallel projects with different specific risks, one high-risk and the other low-risk, the trade-off between maturities of the loans increases with the projects expected losses, hence the maturity choice decision is crucial for portfolios subject to high expected losses. For a portfolio of two sequential projects, we find that, regardless of the order of execution of the projects (high- or low risk), it is the maturity of the debt supporting the high-risk project which drives the risk exposure of the portfolio.

*JEL Classification code:* C15, G11, G12, G13, G22, G33.

*Key words:* Contingent claims analysis, Correlation risk, Credit risk, Financial guarantee, Insurance, Maturity choices, and Projects implementation.
1. Introduction

Economic integration and free trade are creating new markets and opportunities for investors worldwide. To exploit these promising economic opportunities, multinational corporations establish subsidiaries in emerging markets in compliance with indigenous laws and regulations. However, due to the embryonic nature of the domestic capital markets, the severity of political risk, and the fragility of the local economy, investing in these emerging countries is often risky. Development agencies that wish to encourage investment in such countries often must provide financial guarantees to secure private funds (e.g., Babbel [1996]). Effectively, over past decades, international financial institutions such as the Asian Development Bank and the World Bank have increased their participation in the fostering of private sector development in emerging economies (The World Bank [1995]). These institutions employ two main funding methods: direct loans to private corporations, and guarantees of loans between commercial banks and private investors. The latter of these funding methods, requiring at the outset less cash outlay, is less expensive and, therefore, more efficient.

Furthermore, financial guarantees would stimulate investment at lower cost than direct loans because the guarantor only insures risk it has a competitive advantage. For example, since it can exert pressure on governments of developing countries the World Bank gains a competitive edge by specializing in political risk insurance (The World Bank [2002]). Financial guarantees are also used as a tool in resolving financial crises and distresses. Recent examples include government defaults in Asia and Eastern Europe in 1998, failure of Long Term Capital Management in 1998, the California energy crisis

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1 Financial guarantees can be classified into two classes: explicit and implicit guarantees. An explicit guarantee is a contractual promise from a third party to make good on payments to the funds providers in case of default of the borrower. Meanwhile, subsidiaries may enjoy implicit guarantees provided by their parent companies, allowing them to obtain a better credit grade than otherwise. The capacity of private institution to provide guarantees is determined by the value of its assets. Thus, guarantor default is possible.

of 2000 and the airline industry financial problems after the September 11, 2001 terrorist attacks in the United States (see for instance Blair [2002], Wall Street Journal [2002]).

Even in the absence of sophisticated contracting features and imbedded options, the management of debt portfolios is a complex task.\(^3\) In particular, debt portfolios must be optimized with a view to avoiding the phenomena of crisis at maturity (i.e., most of the debts mature at the same time). Such a crisis occurred in 1998, in South Korea and other South Eastern Asian countries. Short-term debts representing a large portion of their portfolios matured at the same time, creating a liquidity problem and forcing some countries to default, (see, for instance, Chang and Velasco [2000], Diamond and Rajan [2001]). Although short-term borrowers face at least three kinds of risk: interest rate risk, refinancing risk, and liquidity risk, they benefit from reduced agency costs.\(^4\)

Viewing loan guarantees as contingent claims, Merton [1977] is the first to establish an isomorphism between a put option and a financial guarantee. A financial guarantee can be considered as a put option written by the guarantor and given to the bondholder. Since then, the so-called structural models based on options theory have been used to value financial insurance in most studies on public, or default-free loan guarantees. Some authors have studied private financial guarantees provided by a vulnerable guarantor (i.e., the guarantor is subject to default risk), e.g., Johnson and Stulz [1987], Lai [1992], Chen, Hung, and Mazumdar [1994], Lai and Gendron [1994], and

\(^3\) Ramaswamy [2002]: “Managing credit risk in a bond portfolio is usually more difficult than managing market risk. The reason is that credit risk is a low-probability default event that leads to highly skewed return distributions…To a certain extent, managing credit risk is similar to managing event risk when pricing an insurance premium.”

\(^4\) Myers [1977] and Barclay and Smith [1995] find that firms with more growth options in their investments opportunity set have less long-term debt than large firms and regulated firms. They also show that firms with higher credit ratings issue short-term debt because the refinancing risk is small, and firms with lower credit rating prefer long-term debt to reduce the refinancing risk. Stohs and Mauer [1996] find that larger, less risky firms with longer-term asset maturities use longer-term debt, and firms with high or very low bond ratings use shorter-term debt. Very risky firms must borrow short-term, because adverse selection prevents anyone from offering them long-term financing. Scherr and Hulburt [2001] find the same results using a sample of small firms. Emery [2001] uses an equilibrium framework where the demand is cyclical and the supplier needs to finance its operations by issuing debt. He finds that suppliers use short-term debt to match their assets’ and liabilities’ maturities and that their incentive to do so is stronger, the larger the term premium. Jun and Jen [2003] find that only firms with greater financial flexibility and financial strength can use proportionately more short-term debt, and they use proportionately more short term debt when the term premium is high.

Invariably, a guarantor will manage a portfolio of credit guarantees supporting investments with correlated outcomes. Such cases have been investigated in Chen and Mazumdar [1996] and Gendron, Lai, and Soumaré [2002], with no account for interest-rate or volatility risks. In an international context, Chen and Mazumdar [1996] study implicit guarantees provided by a parent company to its subsidiaries, and determine the optimal asset risk and debt levels chosen by all subsidiaries and influence on the parent’s optimal equity stake in each subsidiary. Gendron, Lai, and Soumaré [2002] use contingent-claims analysis to evaluate portfolios of vulnerable private loan guarantees and investigate their risk diversification properties. In all these studies, the authors assume the same maturity for the guarantees composing the portfolio. None has studied portfolios of guarantees with different maturities.

5 The structural approach to the analysis of credit risk requires that the assumption of a stochastic process for the value of the total assets of the firm to be financed. Bankruptcy is defined to occur when the value of the firm's total assets falls below a predefined fixed amount at or before maturity. Merton [1974] sets this predefined fixed amount to the face value of the debt at maturity, whereas Black and Cox [1976] used a fixed amount at any time before maturity. Many authors have extended the models established in these early studies. Shimko, Tejima, and Van Deventer [1993] allow the short-term interest rate to follow Vasicek [1977] interest rate process, and for non-zero correlation between the interest rate and the asset returns. Kim, Ramaswamy, and Sundaresan [1993] account for coupon payments and the possibility of firm default on coupon payments. In addition, they employ the Cox, Ingersoll, and Ross (CIR)[1985] interest rate process, and allow for non-zero correlation between the interest rate and the firm's asset returns. Longstaff and Schwartz [1995] use the Black and Cox [1976] framework but allow for the violation of the Absolute Priority Rule, their interest rate follows a Vasicek [1977] process, and they also permit non-zero correlation between the interest rate and firm's asset returns.

6 Interest-rate risk can play a critical role in cash flows discounting and the assets return processes. Shimko et al. [1993] and Longstaff and Schwartz [1995] find that correlation between asset returns and the interest rate plays a critical role in determining the properties of credit spreads. In contrast, Kim et al. [1993] find it to have less impact. In the first two papers, the authors use Vasicek [1977] term structure, whereas Kim et al [1993] use the CIR [1985] process. It seems that the impact of the correlation between the interest rate and the asset returns is dependent on the interest rate process chosen. We test for this effect by calculating results using both the CIR [1985] and Vasicek [1977] assumptions (see our Section 4.2 later).
Since the management of portfolios of guarantees is of significant importance to many organizations both domestically and internationally, the goal of this paper is to study the consequences of maturity choices for loan guarantee portfolios. Can a guarantor that insures several projects, effectively set different debt maturities to reduce its overall risk? For a given level of risk, what combination of maturity structures produces the least risk to the guarantor? We use as measure of credit insurance risk, the market value of the private guarantee, which accounts for projects’ and guarantor specific risks, correlations risks as well as financial leverages.

To evaluate the effects of different combinations of debt maturities on a guarantor’s portfolio risk, we consider a vulnerable guarantor insuring two projects’ debts with multiple maturities. The projects are run either in parallel or in sequence. The specific risk of the projects and the insurer’s asset volatility are time-varying under stochastic interest-rate regimes. The debts pay coupons before maturity, and when default occurs, the absolute priority rule is violated. We rely on optimization by Monte Carlo simulations for numerical results. For fixed levels of portfolio risk and insuring capacity of the insurer, the optimization program assists us in selecting the debt maturities combinations which produce the lowest net loss exposures to the insured debt lenders.

Our simulation results indicate that for a portfolio of two parallel projects with different specific risks, one high-risk and the other low-risk, the trade-off between maturities of the loans increases with the projects expected losses exposure; hence the maturity choice decision is crucial for portfolios subject to high expected losses. For a portfolio of two sequential projects, our results show that, regardless of the order of execution of the projects (high- or low risk), it is the maturity of debt financing the high-risk project which drives the risk exposure of the portfolio. These results are robust to the presence of non-nil correlations between values of assets of the project and the guarantor.\(^7\)

\(^7\) Childs, Ott, and Triantis [1998] use a real options framework to explore the effect of project interrelationships on investment decisions and project values. They find the choice of development policy depends on the relative values of the embedded options for each strategy. Sequential development is shown to be superior to parallel development when projects have highly correlated values, and when they require a large commitment of capital for development, are short term in nature, and have relatively low volatility. In contrast, we deal with guarantees for finance projects, and given a set of project loans and terms, the maturity choice is made by the insurer.
The structure of the remainder of the paper is organized as follows. Section 2 spells out the model’s assumptions. Section 3 presents the claims valuation formulas and the optimization program. Section 4 discusses the results from Monte Carlo simulation. The last Section 5 concludes.

2. Model set-up and assumptions

We make the standard assumptions of contingent-claims analysis (i.e., perfect, frictionless markets in which securities are traded in continuous time, with no tax, no transaction costs, and no asymmetric information, etc).\(^8\) We assume that each project is owned by a firm or sponsor and the project cash flows are used to pay the debt of the project. In this standard project finance framework or non- or limited recourse financing, lenders depend on the performance of the project itself for repayment rather than the credit of the sponsor. We assume a simple capital structure for the projects and the insurer, consisting of single debt and equity contracts.

A.1 Absolute Priority Rule: We assume the violation of the Absolute Priority Rule (APR) when bankruptcy occurs.\(^9\) Eberhart, Moore and Roenfeldt [1990] find the percentage \(\alpha\) of the project's value taken by shareholders to be 7.57 percent in case of bankruptcies, thus creditors recover 92.43 percent.

A.2 Projects' cash flows dynamics: We analyse a portfolio consisting of two projects: Projects 1 and 2. We assume that each project’s total cash flow follows geometric Brownian motion with constant mean return and time-varying stochastic volatilities. In terms of the risk-neutralized probability\(^10\), the process describing the projects cash flows’ return process is

\[
\frac{dV_{i,t}}{V_{i,t}} = (r_i - \delta_i)dt + \sigma_{i,t}dZ_{i,t}, \quad i = 1,2, \tag{1}
\]

\(^8\) See Merton [1974] for the complete standard contingent claims assumptions.

\(^9\) The APR is violated in more than 75% of bankruptcies. Franck and Torous [1989] find that the APR is violated in 78% of bankruptcies in their sample. Eberhart, Moore, and Roenfeldt [1990] report that in a sample of 24 firms, the APR was violated in 23 cases. Weiss [1990] examines 37 firms and reports violations of the APR in 27 cases.

\(^10\) Following Ingersoll [1987], we prefer to use the term “risk-neutralized” rather than “risk-neutral”.
\[ \text{corr}(dZ_{1,t}, dZ_{2,t}) = \rho_{12} dt, \]  
\[ \text{corr}(dZ_{1,t}, dZ_{2,t}) = \rho_{12} dt, \]

where \( V_{1,t} \) and \( V_{2,t} \) are Project 1 and Project 2's total cash flow values at time \( t \), \( r_t \) is the short-term interest rate; \( \sigma_{i,t} \) is the volatility of project \( i \)'s cash flow returns at time \( t \); \( \delta_i \) is the payout rate by project \( i \) and accounts for the payment made on the debts; and \( Z_{i,t} \) is the standard Wiener process. The correlation \( \rho_{12} \) between the two insured projects' returns is non-zero.

The instantaneous volatilities of the two projects are time-varying and follow the continuous stochastic processes

\[ d\sigma_{i,t}^2 = \nu_i (\beta_i - \sigma_{i,t}^2) dt + \omega_i \sigma_i \rho_{i\sigma} dZ_{i,t} + \omega_i \sigma_i \sqrt{1 - \rho_{i\sigma}^2} d\eta_{i,t}, \quad i = 1, 2, \]

where \( \nu_i \), \( \beta_i \) and \( \omega_i \) are respectively the speed of adjustment, the long-run mean and the variation of the diffusion volatility, and \( \rho_{i\sigma} \) represents the correlation between project \( i \)'s asset returns and its instantaneous volatility.\(^{11}\) \( Z_{i,t} \) and \( \eta_{i,t} \) are independent Brownian motions also called Wiener processes.

According to Equation (3), the specific risk of the project varies over time. We justify a mean-reverting stochastic volatility as a consequence of monitoring. We assume that due to the monitoring of the insurer, the project manager will be forced to keep its risk around its long-term mean. It is possible that projects’ managers have an incentive to change their risk taking behaviour after a guarantee contract has been signed; this is the asset substitution phenomenon. Adverse selection teaches us that if a project finances with long-term debt, investors will require interest rates that rise with project risk. Therefore, the manager of a project that can only secure debt at high rates will have incentive to choose very high-risk projects. To prevent this behaviour a guarantor needs to monitor a manager's decisions.\(^{12}\)

A.3 Guarantor's wealth dynamics: In Merton’s [1977] model, the guarantee is risk-
free, meaning that creditors always receive full payment. In our model, we allow the guarantor to default on its obligation. Thus, in the event of default by the project, the insurer may be unable to fully repay all obligations of the debt. This is known as a vulnerable guarantor.

The guarantor’s balance sheet can be written as $V_{G,t} = D_{G,t} + E_{G,t}$ where $V_{G,t}$, $D_{G,t}$ and $E_{G,t}$ are respectively the market values of the guarantor’s total assets, debt and equity at time $t$. The return process of the guarantor’s total assets is described in the risk-neutralized world by

$$
\frac{dV_{G,t}}{V_{G,t}} = (r_t - \delta_G)dt + \sigma_{G,t}dZ_{G,t},
$$

(4)

$$
corr(dZ_{G,t}, dZ_{i,t}) = \rho_{G,i}dt, \quad i = 1, 2,
$$

(5)

$$
d\sigma^2_{G,t} = \nu_G (\beta_G - \sigma^2_G)dt + \omega_G \sigma_G \rho_{G,i} dZ_{G,t} + \omega_G \sigma_G \sqrt{1-\rho^2_{G,i}} \eta_{G,i},
$$

(6)

where $r_t$ is the short-term interest rate, $\delta_G$ is the payout rate by the guarantor and accounts for the debt payments, $\sigma_{G,t}$ is the guarantor's asset returns instantaneous volatility, and $Z_{G,t}$ is the Brownian motion for the asset return process. Correlations between assets returns of the guarantor and the projects returns ($\rho_{1G}$ and $\rho_{2G}$) are non-zero. The instantaneous volatility of the guarantor's asset returns is time-varying. The parameters $\nu_G$, $\beta_G$ and $\omega_G$ are respectively the speed of adjustment, the long-run mean and the variation of the diffusion volatility. $\rho_{G,i}$ represents the correlation between the guarantor's asset returns and its instantaneous volatility, and $\eta_{G,i}$ is an independent Brownian motion.

In Gendron et al. [2002], the guarantor is assumed to have no debt. Thus, all assets of the guarantor are available to back a guarantee. In practice, and in our formulation, this is not necessarily the case, as the guarantor may carry its own debt. Accounting for the debt of the guarantor increases the risk of default on the guarantee.

We subtract the guarantor’s debt value from total asset value and use this net-asset value

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13 To adhere to regulatory pressures and to preserve its reputation and competitiveness, the insurer will maintain its volatility close to the long-term mean. From the insured projects standpoint, it is better to keep the insurer’s volatility as lower as possible, whereas for the insurer, it can be beneficial to increase its asset risk level.
as the available wealth for insurance. Thus, the guarantor’s ability to meet obligations is limited by its available wealth. Denote the wealth available to back a guarantee as $W_t$. Using the notation defined above and noting that $W_t$ must be bounded below by zero, we have

$$W_t = \max(V_{G,t} - D_{G,t}, 0).$$

Thus wealth is equivalent to current equity value. If the guarantor has a huge amount of debt, then its wealth will be very small. As with equity, a guarantor's wealth can be viewed as a call option on its total assets.

**A.4 Interest rate dynamics:** The stochastic interest rate process is given by

$$dr_t = \kappa(\theta - r_t)dt + \sigma_r r_t^\gamma dZ_{r,t},$$

$$\text{corr}(dZ_{r,t}, dZ_{i,t}) = \rho_{ir} dt, \quad i = 1, 2, G,$$

where $\kappa$ is the speed of adjustment, and $\theta$ is the long run mean. $\sigma_r$ is the volatility of the interest rate changes which is set to a constant, and $Z_{r,t}$ is a Wiener process. The CIR [1985] and Vasicek [1977] interest rate processes are particular cases of this process, with $\gamma$ equals to 0.5 for the CIR [1985] process and 0 for the Vasicek [1977] process. The correlations between the interest rate changes and the projects' returns, $\rho_{ir}$ and $\rho_{2r}$, and the insurer's asset returns, $\rho_{Gir}$, are non-zero.

**A.5 Bond covenants and restrictions:** We consider coupon bonds paying $C$ at each interval $\Delta$ of time (i.e., monthly, quarterly, semi-annually or annually), and $F$ at maturity date $T$. The projects can’t issue any new senior claims nor can they pay cash dividends or do share repurchase prior to the maturity of the debt. Nonetheless, $\delta$ in Equations (1) and (4) account for the payments made on the debts. In the event of default of one of the coupons, the future payments are also defaulted. The guarantor takes over the project at the first default date. If the project misses one of its coupons, then all future coupons and the principal payments will be stopped and the guarantor pays the current coupon and the

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14 The interest rate risk is rewarded, i.e., the risk factor earns a risk premium proportional to $r_t (\lambda_{r_t})$. This is implicitly reflected in equation (8) and adjusted through $\kappa$; or we can assume the risk premium of the interest rate risk ($\lambda_r$) to be zero.

15 We should point out that the model does not allow for recapitalization.
present value of all future payments at bankruptcy. The guarantor uses the liquidation value of the project’s assets along with its available wealth to fulfill all debt obligations. The payment is partial if there were insufficient funds to cover the remaining debt payments.

In the case of a zero-coupon bond case, default can only occur at maturity. If the project's asset value ends up higher than the debt face value $F$, creditors receive full payment. In absence of guarantee, if the project’s value is less than $F$, creditors take over the project's asset, and hence lose $F - (1-\alpha)\mathcal{V}_t$. If the loan has been guaranteed then, in case of default, the third party will pay the remaining amount, $F - (1-\alpha)\mathcal{V}_T$, fully or partially. Hence, the value of the default-free guarantee at maturity is $(F - (1-\alpha)\mathcal{V}_T)\times I_{\{V_T < F\}}$. Since there are no moderate payments until maturity of the debt, the payoff is exactly the same as the payoff of a European put option on $(1-\alpha)$ times the project’s asset value with exercise price $F$.

### 3. Claims valuation formulas

Let us denote $S_t = [V_{1,t}, V_{2,t}, W_t, r_t]$ and consider a security $Y_t$ depending on $S_t$ and $t$, paying at finite intervals of time. Then following Harrison and Kreps [1979] and Harrison and Pliska [1981], at time $t$ this default-free claim can be priced using the Equivalent Martingale Measure (EMM) $Q$.

#### 3.1 Guaranteed coupon-bearing bond pricing

As indicated earlier, the guarantor insures coupon-bearing bonds that promise to pay a constant amount $\$C$ at each interval date $\Delta$ and a final payment $\$F$ at maturity $T$. Default occurs if the project’s cash flows aren’t sufficient to make the required payment. The default-free claim can be priced using the EMM $Q$, and the price at time zero, $D_0$ is given by

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16 We assume the economy to be complete even with stochastic volatilities. However, our set-up still holds even if stochastic volatilities render the markets incomplete. If it were the case, there is no unique equivalent martingale measure or risk-neutralized probability for the underlying assets. For the purpose of computing contingent claims, one would invoke the concept of Minimal Equivalent Martingale Measure (for details, see Hofmann, Platen, and Schweizer [1992]).
\[ D_0 = E^Q_0 \left[ \sum_{k=1}^{l} \exp \left( - \int_0^{k\Delta} r_s \, ds \right) C + \exp \left( - \int_0^\tau r_s \, ds \right) F \right] , \]  

(10)

and, at time \( \tau \), by

\[ D_\tau = E^Q_0 \left[ \sum_{k=0}^{l-\tau} \exp \left( - \int_{\tau}^{\tau + k\Delta} r_s \, ds \right) C + \exp \left( - \int_{\tau}^\tau r_s \, ds \right) F \right] . \]  

(11)

This formula states that the default-free coupon bond price at time \( \tau \) is the expected present value of all future payments until maturity including the coupon payment at time \( \tau \). The first part of the expectation expression is the present value of all coupons and the second part is the present value of the final payment.

Project \( i \) defaults at time

\[ \tau = \inf \{ t : \text{ s.t. } V_{i,t} < C \text{ or } V_{i,t} < C+F \text{ if } t = T \}. \]  

(12)

\( \tau \) is a first exit time - the first time at which the cash flows value drops below the required payment.

The time zero price of the risky debt without guarantee \( D_{0,NG} \) is

\[ D_{0,NG} = E^Q_0 \left[ \sum_{k=1}^{l} \exp \left( - \int_0^{k\Delta} r_s \, ds \right) C + \exp \left( - \int_0^{\tau} r_s \, ds \right) (1-\alpha_i) V_{i,\tau} \right] . \]  

(13)

In the absence of a guarantee, upon bankruptcy at time \( \tau \), creditors take possession of project assets and sell them. Since we allow for violation of the APR, bondholders recover \((1-\alpha_i)\) of the sale value. Debt holders retain all previous coupons and recover \((1-\alpha_i)\) times the actual sale value of the project's total assets. All future payments are lost.

The risky debt price is the expectation (under the risk-neutralized probability \( Q \)) of the present value at time zero of all coupons received prior to default plus the present value of the project’s asset value at default adjusted for the APR violation. Here, the bankruptcy trigger date is random and can be any coupon date. For a zero coupon bond, the bankruptcy trigger date has no random component, corresponding exactly to the maturity date of the debt. If the average time to default, \( E^Q_0[\tau] \), is far below the debt
maturity \((T)\), then \(D_{0,NG}\) will be small. For high \(\alpha_i\), \(D_{0,NG}\) is low because creditors recover only \((1-\alpha_i)\) times the salvage value of the distressed project.

With a third private party guaranteeing the debt, the price at time zero of the guaranteed debt, \(D_{0,G}\), is

\[
D_{0,G} = D_{0,NG} + E_0^Q \left[ \exp \left( -\int_0^\tau r_s ds \right) \min \left( a_i W_{\tau}, D_{\tau} - (1-\alpha_i) W_{i,\tau} \right) \right],
\]

(14)

where \(a_i\) is the allocation percentage of the insurer's wealth to project \(i\). The price of the debt guaranteed by a third party is equal to the price of the risky debt without guarantee plus an extra amount representing the value of the guarantee. Upon default, if the guarantor's contribution combined with the project's asset value is sufficient to cover all remaining payments on the debt then the lender receives full payment. Otherwise, the guarantor can only provide partial payment. \(D_{0,G}\) is an increasing function of \(E_0^Q[r]\). Indeed, the longer the project survives, the more coupon payments are made, thereby alleviating the burden of the insurer. For a wealthier guarantor, \(\alpha_i\) will have less effect on \(D_{0,G}\) since the insurer has ample capacity to meet the shortfall even if a big portion of the distressed project's asset is lost during the course of bargaining between shareholders and debt holders. \(^{17}\) \(D_{0,G}\) increases with the percentage of insurer’s wealth allocated to the project \(a_i\).

The guaranteed coupon-paying bond can be viewed as a sequence of compound options where the state of the current option depends on the state of the previous one (i.e., if default has occurred or not). At each coupon date before maturity, the debt face value is \(C\) with an option to exercise the put option or not. At maturity, the face value of debt is \(C+F\). If default occurs at \(t\), then the put options for the coupons before \(t\) are worth zero. At \(t\), the put payoff is equal to the guarantor's wealth allocated to the payment of the remaining part of the coupon at that time. After \(t\), since the project is liquidated, the option value is equal to the amount of the guarantor's wealth allocated to the payment of

\(^{17}\) Indeed with a third party guaranteeing the loan, one could ask if absolute priority can still be violated given the monitoring of the insurer. Since delays in court dispute increase bankruptcy costs and encourage asset looting, debt holders and the guarantor still have to “bribe” shareholders to avoid lengthy costly legal bankruptcy proceedings. However, assuming otherwise by setting the percentage \(\alpha\) equals to zero doesn’t affect our main results.
the corresponding time coupon.

From Equations (13) and (14), \( D_{0,G} - D_{0,NG} \) represents the expected present value of the guarantee. We express it as the difference between two options. The loss amount at time \( \tau \) without guarantee is \( \max(D_\tau - (1 - \alpha_i)W_{i,\tau,0}) \), whereas the loss in presence of the guarantee is \( \max(D_\tau - (1 - \alpha_i)W_{i,\tau} - a_iW_{i,\tau,0}) \). The today value of the guarantee is the difference between these two quantities; i.e.,

\[
D_{0,G} - D_{0,NG} = E^Q_0 \left[ \exp \left( - \int_0^\tau r_s ds \right) \min(a_iW_{i,\tau} - (1 - \alpha_i)W_{i,\tau,0}, D_\tau - (1 - \alpha_i)W_{i,\tau,0}) \right].
\]

This is the difference between two exchange options, each with exercise price \( D_\tau \) when exercised at time \( \tau \). The guarantee is tantamount to holding a long position in the first option and being short the second one.

### 3.2 Expected Loss and Guarantee Value per dollar of debt

The Expected Loss per dollar of debt (ELPD) and Guarantee Value per dollar (GVPD) give, respectively, the expected loss and guarantee value today normalized by the proceeds from the loan. Working with per dollar amount best handles the scaling problem and also subsumes the problem of duration matching when comparing costs.

Again, if we denote the default-free bond price by \( D_0 \), the risky bond price without guarantee by \( D_{0,NG} \), and the risky bond price with guarantee by \( D_{0,G} \), the Expected Loss, EL, is equal to \( D_0 - D_{0,NG} \). The Expected Loss per dollar of loan proceeds (ELPD) is the ratio of the Expected Loss (EL) to the default-free bond price \( D_0 \),

\[
ELPD = \frac{EL}{D_0} = \frac{D_0 - D_{0,NG}}{D_0}.
\]

\(^{18}\) Since their uncertain exercises prices \( D_\tau \) depend on the random time variable \( \tau \), the options described here are different from the ordinary put options.
or equivalently, 

$$E^Q_0 \left[ \exp \left( -\int_0^{\tau} r_s \, ds \right) \max \{ D_{\tau} - (1 - \alpha_s) V_{i, \tau}, 0 \} \right]$$

or equivalently,

$$ELPD = \frac{E^Q_0 \left[ \exp \left( -\int_0^{\tau} r_s \, ds \right) \max \{ D_{\tau} - (1 - \alpha_s) V_{i, \tau}, 0 \} \right]}{D_0}.$$ 

The ELPD measures the relative expected loss for each dollar of the loan proceeds the project sponsor obtains today. Extending the concept of actuarially-fair deposit insurance premium (see for instance, Ronn and Verma [1986]), ELPD captures not only the leverage and specific risks of the insured projects, but also the interest-rate risk, the stochastic asset risks, and the correlation risks among these sources of uncertainty. For low values of $E^Q_0 [\tau]$, ELPD is big because of the high probability of default.

The Expected Guarantee Value, GV, is equal to the price of the risky bond with guarantee, $D_{0,G}$ minus the price of the risky bond without guarantee $D_{0,NG}$, i.e., $GV = D_{0,G} - D_{0,NG}$. The Expected Guarantee Value per dollar of debt proceeds, GVPD, is the Expected Guarantee Value divided by the default-free bond price,

$$GVPD = \frac{GV}{D_0} = \frac{D_{0,G} - D_{0,NG}}{D_0}, \quad (17)$$

or equivalently,

$$GVPD = \frac{E^Q_0 \left[ \exp \left( -\int_0^{\tau} r_s \, ds \right) \max \{ D_{\tau} - (1 - \alpha_s) V_{i, \tau}, 0 \} - \max \{ D_{\tau} - (1 - \alpha_s) V_{i, \tau} - a_s W_{\tau}, 0 \} \right]}{D_0}.$$

The GVPD is the present value of the amount the guarantor provides to creditors in case of default by the borrowing entity on one-dollar debt. GVPD increases with the allocation percentage, $a_i$, and with the wealth value of the insurer, $W$. For low values of $E^Q_0 [\tau]$, the price of the risky debt without guarantee, $D_{0,NG}$, is low, which implies that GVPD needs be high depending of course on $a_i$ and $W$.

The difference between ELPD and GVPD measures the expected shortfall of the guaranteed debt to insured lenders. The “coverage” is a full faith and full credit one if ELPD-GVPD equals 0, which is the case of a credit insurance provided by the government. ELPD-GVPD can be substantial with a vulnerable guarantor. We call
ELPD-GVPD the Non-Recoverable Expected Loss per dollar loaned (NRPD). NRPD gives the expected portion of the guaranteed loan which is not recovered by debt holders due to the insurer’s default risk or the vulnerability of the guarantor. The fraction of debt recovered by insured debt holders, (i.e., lenders), the Recovery Ratio, equals 1-NRPD.

3.3 Maturity decision program

Our debt maturity problem differs from those found in the corporate finance literature in two ways: first, we analyse the maturity structure of loan guarantees and not the one of the loans themselves, and second, in our set-up, the debt guarantee maturity choice is taken by the guarantor and not the project sponsors. For a given portfolio risk level proxied by the expected loss, the guarantor is to choose a pool of projects to optimize its insuring coverage. The guarantor decides which projects to insure based on projects’ characteristics and the financing terms of the project. A same expected loss can be obtained with different combinations of maturities. However, for a (given) combination of maturities, and corresponding to the same expected loss, only one maximum guarantee can be provided by a private guarantor. In other words, for a given expected loss from the project loans, what maturity structure yields the best insurance coverage? For the sake of our analysis, we consider a vulnerable guarantor insuring two projects’ debts with different maturities. The insurer then maximizes the GVPD provided subject to its capacity constraint (i.e., its initial wealth) and a maximum tolerable ELPD. This means that the guarantor maximizes the market value of its guarantee for a given risk level without having to mobilize additional assets for its insurance activities. The optimization problem can be stated as follows:

\[
\begin{align*}
\max_{T_1, T_2} & \quad GVPD \\
\text{s.t.} \quad & \text{ELPD} \leq \text{ELPD}^* \quad \text{and} \quad W_0 = \bar{W}
\end{align*}
\]

where again ELPD and GVPD are the Expected Loss and Guarantee Value per dollar of debt, and \(W_0\) the initial wealth of the insurer. \(T_1\) and \(T_2\) are the arguments from the optimization and represent the maturities of the two loans. \(ELPD^*\) is the tolerable

\footnote{We rule out strategic defaults by project sponsors.}
Expected Loss per dollar of debt and $\bar{W}$ the insurer’s wealth limit.\(^{20}\) As discussed earlier, by analogy to the fair deposit insurance value, ELPD is a proxy for the overall risk exposure of the portfolio.\(^{21}\) Equation (18) states that, given an ELPD, the insurer’s decision is to choose maturities of both projects’ debts which maximize the GVPD or reduce the NRPD. Figure 1 depicts the trade-off between losses and guarantees for the Scenario 2P case outlined later in Section 4.3.\(^{22}\) We note that, due to its vulnerability, the gap (or differential) between the portfolio loss and the guarantee indemnified by the guarantor widens with increases in the ELPD.

**Insert Figure 1 here.**

We examine two cases of decision regarding the loan guarantee maturities: simultaneous and sequential. The simultaneous case applies when all projects are running in parallel; and the sequential case when one project is run first followed by another one and so on.\(^{23}\) For the parallel case, the task is to choose projects based on specific risk levels, the interactions between stochastic terms and the projects’ financing terms. The insurer in some cases will try to adjust its net-asset duration to the loan portfolio duration. In the sequential case, the portfolio is assumed to include already one project and insuring terms must be decided for a second project. It is clear that the maturity of the loan and risk characteristics of the current project play a determining role in the choice of the subsequent project. The guarantee will depend on the projects’ loans maturities, the initial risk characteristics, their interrelationships, and also their risk variation over the life of the project.

\(^{20}\) From a social welfare point of view, the goal is to improve the economic gain since overall, the insurer improves the recovery rate to debt holders without having to utilize additional assets.

\(^{21}\) At optimum, the constraints on ELPD are binding, i.e., $\text{ELPD} = \bar{W}$. The first order condition for this optimization is $\frac{\partial \text{GVPD}}{\partial t_i} = \frac{\partial \text{ELPD}}{\partial t_i}$, or in words, the ratio of marginal guarantees provided for every pair of maturities is equal to the ratio of the associated marginal losses. We don’t examine the sensitivity of the results to variation of $\bar{W}$, this is left for future studies.

\(^{22}\) The shape of the curve is the same for the other scenarios of Section 4.3.

\(^{23}\) In both cases (parallel and sequential), the two projects may be owned by the same firm or by two separate entities. However, even if the two projects are owned by the same sponsor, these can still exhibit different risk levels.
4. Simulation results

We conduct optimization by Monte Carlo simulations to compute the Expected Losses and Guarantee Values per dollar of debt. Due to the curse of dimensionality, Monte Carlo simulations are most apt for tackling our problem.\textsuperscript{24}

4.1 Baseline parameter values

Projects’ and guarantor's initial values, the face value of projects’ debts and coupon rates, and allocation percentages are listed in Table 1. Projects' initial net cash flow values are 2000 and the guarantor's initial asset value is 4000. Hence, the total asset value of the portfolio is 4000. The insurer’s initial volatility is 0.20. All debt face values are fixed to 1000; the coupon rate of 10 percent is the annual rate payable each six months up to the maturity of the debts. The insurer’s debt matures in 20 years and has face value 1000 and 10 percent annual rate coupon payable semi-annually. Thus the initial wealth available for insurance is 2400. The portion $\alpha$ of the asset value that is lost in case of bankruptcy is 8 percent for each firm.\textsuperscript{25} The guarantor’s wealth is assigned in equal percentage to each project.

\textbf{Insert Table 1 here.}

Two main components of default risk are leverage and project risk. A project’s risk is determined by its volatility and the correlation of its returns with the other returns processes and the interest rate. The leverage is given by Merton’s \cite{1974} pseudo-debt ratio, which is the ratio of the risk-free debt price over the project initial value i.e., $D_0/V_{i,0}$.

Table 2 gives parameter values for the stochastic volatility models. Initial values for the volatilities are 0.20 for low-risk, 0.35 for moderate-risk, and 0.50 for high-risk. We choose $\nu$ equals 1.25, $\beta$ equals $0.20^2$ (low-risk), $0.35^2$ (moderate-risk) and $0.50^2$ (high-risk), $\omega$ equals 0.20. The correlation between asset returns and volatility changes, $\rho_{\sigma}$, is set to -0.50.

\textbf{Insert Table 2 here.}

We set long run variances, $\beta$, equal to the initial variances. Monitoring should

\textsuperscript{24} For the integration of simulation and optimization, see Fu \cite{1994} and Pflug \cite{1996}.

\textsuperscript{25} Previous works guide us in choosing this value of $\alpha$, e.g., Eberhart et al. \cite{1990} find $\alpha = 7.57\%$. 

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keep the volatilities near those observed at the signature of the guarantee contract. Why? One can argue that the pricing is done based on observed characteristics of the project, and any deviation from these values affects the fair price and will be countered by monitoring.

Parameters of the interest-rate process are given in Table 3. The initial interest rate, \( r_0 \), is 5 percent, the long run mean, \( \theta \), is 5 percent, the adjustment speed, \( \kappa \), is 0.25, and the interest rate volatility \( \sigma_r \) is 0.05. The value of \( \gamma \) is 0.5 for the CIR [1985] process and 0 for the Vasicek [1977] process.

**Insert Table 3 here.**

Table 4 presents the correlations between the projects' returns, the guarantor's returns, and the short-term interest rate. The correlation value between the two projects, \( \rho_{12} \), is chosen from the set \{-0.30, 0.30\}, the correlations between the projects and the insurer, \( \rho_{1G} \) and \( \rho_{2G} \), are chosen from the set \{-0.30, 0.30\}, and the correlations between the projects or the guarantor and the interest rate, \( \rho_{1r} \), \( \rho_{2r} \), and \( \rho_{Gr} \), are set equal to -0.20.\(^{27}\)

**Insert Table 4 here.**

For an insurer specializing in a specific sector, the insured projects would be highly positively correlated. In general, as they operate in the same industry, the correlations between a parent company and its subsidiaries are likely to be high. We choose as values for the correlation between the two projects (\( \rho_{12} \)) -0.30 and 0.30.

To analyze the effects of the maturity structures, we run Monte Carlo simulations using these baseline parameters values for different debt maturities varying from 1 to 20 years. We also vary some parameters values to gauge their effects on the ELPD and GVPD.

**4.2 Sensitivity analyses with respect to the APR and interest rates**

We study here the effects of taking into account the violation of the APR and the interest-rate risk on the ELPD and GVPD. For the sake of brevity, we do not present all


\(^{27}\) Despite the negative correlations values, the covariance matrix remains positive semi-definite due to the high positive values of the diagonal.
results, but only summarize some key findings.28

The sensitivity of the GVPD with respect to the bargaining power of the shareholders, proxied by $\alpha$, decreases with time to maturity. The higher the value of $\alpha$, the less of a project’s value is recovered at default, thereby reducing the value of the risky bond and increasing ELPD. Deviating more from the APR in favour of the shareholders decreases the risky bond value, because more wealth is transferred from debt holders to shareholders when default occurs. However, the expected value of this wealth transfer is lower for longer maturity debts because longer maturity debt have an expected default time that is farther in the future than short term debt.

The interest rate parameters have a larger impact on both ELPD and GVPD with the Vasicek [1977] process than with the CIR [1985] process. The greater is the correlation between the portfolio elements and the interest rate, the greater is the ELPD, because the portfolio is subject to both default and interest-rate risks. The ELPD is very sensitive to small variations of the interest-rate volatility when the Vasicek [1977] process is used. For the CIR [1985] process, the impact of volatility changes is small and is only significant for long maturity debts. As the correlation between the interest rate and the guarantor’s wealth increases, the GVPD decreases, especially for maturities more than 5 years. An increase in this correlation exposes the guarantor to more interest-rate risk, thereby increases the likelihood that it will not be able to meet its insurance obligations.

We next present simulation results pertaining to the main theme of our paper.

### 4.3 Maturity choices and policy implications

Let us consider three classes of projects: a low-risk project, a moderate-risk project, and a high-risk project. Low-risk corresponds to an initial volatility level of 0.20, moderate-risk to an initial volatility level of 0.35, and high-risk to an initial volatility level of 0.50 (see Table 2 for the time-varying volatility parameters used). We consider three scenarios involving these three classes of projects.29

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28 These simulation results can be obtained from the authors upon request.
29 Two envelope cases are left out, the combination of two low-risk projects, or two high-risk projects. From the standpoint of the insurer, the former one is the most favourable and the later is the most severe.
Scenario 1 combines two moderate-risk projects. Two cases are considered. One in which the two projects run in parallel (Scenario 1P) and one in which the projects run sequentially (Scenario 1S).

Scenario 2 combines a low-risk project and a high-risk project. As for Scenario 1, two cases are considered. One in which the two projects run in parallel (Scenario 2P) and one in which the projects run sequentially (Scenario 2S). When run sequentially, the low-risk project is run first.

Scenario 3 also combines a low-risk project and a high-risk project. Since for projects running in parallel, the order being irrelevant, Scenario 3P is the same as Scenario 2P. One single case is considered here in which the two projects are run in sequence (Scenario 3S). The order is reversed from that used in Scenario 2S, i.e., the high-risk project is implemented first followed by the low-risk one.

4.3.1 Parallel development

We consider here cases where projects are run simultaneously. Table 5 presents the ELPD and GVPD for different combinations of debt maturities for the two projects for each scenario. Consider the results for Scenario 2P. For positive correlations between the projects, and between the projects and the insurer (Panel a of Table 5), the maturities structure 1/10 (1 year maturity debt for the low-risk project and 10 years maturity debt for the high-risk project) or 20/5 for the two projects will generate an ELPD of 18 cents. The maturities structure 5/15 or 20/10 generates an ELPD of 23 cents. And finally, with the maturities structure of 10/20 or 20/15, we get 26 cents of ELPD. Based on the above optimization criteria that the insurer for the same expected loss (ELPD) maximizes the value of the private guaranty (GVPD) [Equation (18)], the longer is the low-risk project maturity, the better is the recovery rate to debt holders. The explanation is having a longer maturity for the less risky project debt, diminishes the horizon uncertainty. We perform this analysis for different correlations between the projects and between the projects and the insurer, and the results do not change qualitatively. When the correlations between the projects and between the projects and the insurer are negative however, these two cases are not interesting since their implications are obvious. Hence, the scenarios presented in the paper make our point without loss of generality. Results for the intermediate cases can be obtained from the authors upon request.
(Panel b of Table 5), the same pattern is observed. Indeed, in Panel b of Table 5, for 18 percent ELPD, we obtain the maturities structure of 1/10 or 20/5. With an ELPD of 23 cent, we have 5/15 or 20/10 as maturities structure. Finally, with 26 percent ELPD, we have the combination 10/20 or 20/15 of maturities.

**Insert Table 5 here.**

Figure 2 depicts graphically the optimization results. The lines in the graph are not symmetric with respect to the median. In the case of a low-risk project, increasing further its debt maturity doesn’t affect much the ELPD. ELPD is most sensitive to the debt maturity of the high-risk project, more so for high ELPD. The trade-off between maturities of the loans increases with the projects’ losses; hence, the maturity choice decision is crucial for portfolios subject to high expected losses. Higher GVPD can be offered for debts of longer maturity taken by the less risky project sponsor.

**Insert Figure 2 here.**

To assess visually the correlation effects, we plot in Figure 3 the ELPD, GVPD and NRDP for different correlation values between the projects and between the projects and the guarantor. The ELPD are higher, (and the GVPD are lower) for positive correlations (Exhibit a) than those for negative correlations (Exhibit b). Thus, the recovery value to debt holders is higher with negative correlations as a result from better portfolio risk diversification.

**Insert Figure 3 here.**

In Table 6, we quantify the risk-neutralized default probabilities (Panel a) and probabilities of full recovery (Panel b) for both low and high-risk type of projects having the same debt maturity. The default probability increases with debt maturity and remains high for high-risk project. For short debt maturities (say less than 3 years), the low-risk project does not need credit enhancement since the default probability is almost zero. The likelihood of achieving full recovery decreases with maturity. In effect, as uncertainty grows with increases in maturity, coupon payments may be missed. Therefore, the portfolio risk increases with maturity, and so does the cost of the guarantee. Almost full
recovery is expected for short-term insured debt.\textsuperscript{30} For a debt with maturity of 20 years, a low-risk project has an approximately 50 percent chance of defaulting. The same risk-neutralized probability of default is obtained for a high-risk project with debt of 5 years maturity. Therefore, insuring a low-risk project debt with 20 years maturity is equivalent to insuring a high-risk project debt of 5 years maturity. The probability of full recovery is highest with negative correlations between the projects and among the projects and the guarantor. This case results from diversification across sectors or activities.

\textbf{Insert Table 6 here.}

Distributions of ELPD, GVPD and NRPD for 10 years maturity debts are plotted in Figure 4. The median ELPD is 25 cents for the low-risk project versus 50 cents for high-risk project (Exhibit a). The median GVPD is 21 cents for low-risk project versus 40 cents for high-risk project (Exhibit b). High-risk project’s ELPD distribution skews further to the right than that for low-risk project. The median NRPD is 16 cents for low-risk project versus 28 cents for high-risk project (Exhibit c).

\textbf{Insert Figure 4 here.}

To sum up, it is best for the guarantor to insure the longer maturity debts related to the less risky projects and support the shorter maturity debts taken by the risky projects. The trade-off between the loan maturities increases with the projects loss exposure; hence, the maturity choice decision is crucial for portfolios subject to high expected losses (see Figure 2). At best, for very low-risk (i.e., safe) projects, a debt with sufficiently short maturities does not need guarantee at all. For the same loan tenor, the loss distribution is skewed to the right for the risky project (see Figure 4). Since short-term borrowers tend to default less and investors in these loans demand less of a term premium, sponsors of the more risky project will be more likely to buy credit insurance for the long-term debt rather than for the short-term debt. Nonetheless, the recoverable value to debt holders is improved when the correlations are negative because the portfolio benefits from risk diversification (see plots at the right-hand side of Figure 3).

\textsuperscript{30} Despite the increase of the default probability with the debt maturity, we should remember that short-term debt has associated refinancing and liquidity risk. The difference between the long-term and short-term premiums is the term premium and can be less than the refinancing cost for short-term effect.
4.3.2 Sequential development

The projects now run sequentially, i.e., the second project is executed upon completion of the first. The maturities of the debts are chosen in function on the insurer’s objective of providing the maximum GVPD for a given ELPD. Given the current project’s characteristics, what should be the next project’s attributes, knowing that the insurer has a limited wealth, i.e., finite insuring capacity? The ELPD and GVPD for the three scenarios are presented in Table 7. For debts maturities of more than 5 years for the first project, Scenario 2S has the lowest ELPD. But for short debt maturities of the first projects, the ELPD is higher in Scenario 2S because of the high-risk second project. When the first project is low-risk with debt maturity more than 5 years, the next project even of high-risk, doesn’t increase significantly the risk level relative to the case when the first project was high-risk. It is advantageous to have a longer debt maturity for the low-risk project.

Insert Table 7 here.

In Scenario 3S, when the first project is high-risk, the ELPD is very high, especially when its debt maturity is long. For an ELPD of 16 cents, 20 years debt maturity for the first project (low-risk) and 5 years debt maturity for the second project (high-risk) in Scenario 2S is equivalent of having 5 years debt maturity for the first project (high-risk) and 10 years debt maturity for the second one (low-risk) in Scenario 3S. For an ELPD of 19 cents, the debt maturities structure 20/10 in Scenario 2S is roughly equivalent to the debt maturities structure of 5/20 in Scenario 3S.

Figure 5 illustrates schematically these findings. The graphs show that conforming to basic intuition, regardless of the order of execution of the projects, it is better to have a long-term maturity debt for the low-risk project and keep the high-risk

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31 It is more likely that the decision to guarantee the next project loan is made only when the guarantee for the current project debt ends. However, given the information available at time zero, and the one in the future perfectly forecasted, we presume the future decision known today and use the term structure predictions to discount the project cash flows. The question is then, given the current portfolio characteristics and available information, what project should the guarantor insure in the future? Of course, at decision time in the future, a different choice can be made since more information will be available. In either case, managers forecast their future projects financing terms today. Also, today menu of guarantees may be dictated by projects to be implemented in the future, especially for projects developed in stages and owned by the same entity.
project debt maturity short. It is interesting to note that for the upper plots (Fig. 5-1/2), we obtain a downward sloping line for 16 percent ELPD and upward sloping lines for 19 and 24 percent ELPD. The reason is one can incur small ELPDs by increasing the loan maturity of the first (low-risk) project while decreasing the debt maturity of the second (high-risk) project. Whereas, to suffer big ELPD, since increasing the loan maturity of the first (low-risk) project is not sufficient to cause these same high losses, one has to increase the maturity of the second high-risk project. In the bottom graphs (Fig. 5-2/2), the lines slant leftward from the vertical line for small losses, and lean toward the straight vertical line for big ELPD. In effect, increasing the first (high-risk) project loan maturity increases dramatically the portfolio ELPD no matter what maturity we pick for the loan supporting the second (low-risk) project. Therefore, regardless of the order of execution of the projects (high- or low risk), it is the maturity of debt financing the high-risk project which drives the risk exposure of the portfolio.

Insert Figure 5 here.

Figures 6 and 7 plot the ELPD, GVPD and NRDP respectively for Scenario 2S and Scenario 3S. In both these scenarios, as in the parallel development case, negative correlations between the projects and among the projects and the insurer improve the recovery ratio to creditors. Therefore, portfolio risk reduction from diversification is still important even for sequential projects.

Insert Figure 6 here.

Insert Figure 7 here.

To summarize, if the guarantor currently has a portfolio of guarantees and is deciding how to structure credit insurance for a new project to be executed sequentially, the choice of maturity is not simple. Nevertheless, the insurer is invariably better off to start insuring on a long term basis the low-risk project debt, and thereafter chooses the debt maturity of the next project depending on the risk level it is able to cover. To secure a high recovery ratio within a tolerable portfolio risk level, if the first project is high-risk, the insurer should keep its debt maturity short and choose the next project as low-risk. Indeed, since starting with a low-risk project with low default probability, the insurer is in a strong position to cover the next risky project. On the other hand, if it begins with a
risky project, thereby straining its strength and weakening its ability to cover the next project creditors. By and large, regardless of the order of execution of the projects (high- or low risk), it is the maturity of debt financing the high-risk project which drives the risk exposure of the portfolio.

5. Conclusion

As the use of financial guarantees becomes so widespread and pervasive, for instance, in fostering international development and mitigating financial crises, the management of portfolios of loan guarantees has taken de facto paramount importance. In this paper, we analyse the effects of the maturities of credit-enhanced debt contracts on the value of an insurer’s loan guarantee portfolios. To achieve our goal, we propose a contingent-claims model, that not only includes important features such as coupon payments and absolute priority violations, but also allows for stochastic interest-rate and stochastic assets volatility. This is a first attempt at modelling multiple maturities in the context of portfolios of loan guarantees. We consider loan guarantees for projects that run either in parallel or in sequence. Using optimisation by Monte Carlo simulations, we compute the value of the guarantee portfolios and the net loss exposure to the insured debt lenders.

For a portfolio of two parallel projects with different specific risks, one high-risk and the other low-risk, the trade-off between maturities of the loans increases with the projects expected losses exposure, hence the maturity choice decision is crucial for portfolios subject to high expected losses. Also, portfolio risk diversification improves the recovery rate to creditors. For a portfolio of two sequential projects, the insurer is always better off insuring first, under long terms, the debt required for the low-risk project. However, if the first project must be of high risk, then the insurer should keep its debt maturity short, and choose a less risky second project. Stated differently, for a given expected loss, credit enhancement is most valuable to funds suppliers when the insurer backs low-risk projects debts on a long-term basis and supports high-risk projects loans by short-term contracts. Further, regardless of the order of implementation of the projects (high- or low risk), it is the maturity of debt supporting the high-risk project which drives
the risk exposure of the portfolio. The results are robust to the presence of non-zero correlations between values of assets of the projects and the guarantor.
References


Cox, J.C., J.E. Ingersoll, Jr., and S.A. Ross. “A theory of the term structure of interest


Table 1: Projects’ and guarantor’s assets dynamics parameters, debt contracts, and allocation percentages.

This table contains projects’ and guarantor’s initial values, the face value and coupon rate of each project’s debt, and the guarantor’s allocation percentages. The table includes payment intervals for coupons and the portion, $\alpha$, of the project’s asset lost by creditors in case of bankruptcy. The coupon rates are annualized.

<table>
<thead>
<tr>
<th></th>
<th>Project 1</th>
<th>Project 2</th>
<th>Guarantor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Asset value ($V_{i0}$)</strong></td>
<td>2000</td>
<td>2000</td>
<td>4000</td>
</tr>
<tr>
<td><strong>Debt face value ($F_i$)</strong></td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td><strong>Coupon rate (C)</strong></td>
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<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Coupon payment interval ($\Delta$)</strong></td>
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<td>6 months</td>
<td>6 months</td>
</tr>
<tr>
<td><strong>$\alpha$</strong></td>
<td>0.08</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td><strong>Allocation percentage ($a_i$)</strong></td>
<td>0.50</td>
<td>0.50</td>
<td></td>
</tr>
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</table>
Table 2: Parameter values for stochastic volatility processes.

This table presents the parameters values for mean-reverting stochastic volatility processes. \( \beta_i \) is the long run mean, \( \nu_i \) the speed of adjustment, \( \omega_i \) the variation coefficient of the diffusion volatility, \( \rho_{\sigma} \) the correlation between the project \( i \) (Project 1 and Project 2) or guarantor \( (G) \)'s asset return and volatility processes, and \( \sigma_{i,0} \) is the project or guarantor’s initial volatility. \( Z_{it} \) and \( \eta_{it} \) are independent Brownian motions. The volatilities are annualized.

\[
d\sigma_{ij}^2 = \nu_i (\beta_i - \sigma_{ij}^2) dt + \omega_i \sigma_{ij} \rho_{\sigma} dZ_{ij} + \omega_i \sigma_{ij} \sigma_{i,0} \sqrt{1 - \rho_{\sigma}^2} d\eta_{ij}, \quad i = 1,2 \, , G.
\]

<table>
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<tr>
<th></th>
<th>( \sigma_{i,0} )</th>
<th>( \nu_i )</th>
<th>( \beta_i )</th>
<th>( \omega_i )</th>
<th>( \rho_{\sigma} )</th>
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<td>1.25</td>
<td>0.35</td>
<td>0.20</td>
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<tr>
<td>High-risk</td>
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<td>1.25</td>
<td>0.50</td>
<td>0.20</td>
<td>-0.50</td>
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</tbody>
</table>
Table 3: Interest rate process parameters values.

This table contains parameter values for the interest rate \( r_t \) process including initial interest rate value, \( r_0 \). \( \kappa \) is the speed of adjustment, \( \theta \) the long run mean and \( \sigma_r \) the interest rate volatility. \( \gamma \) has two possible values depending on the process considered: 0 for the Vasicek [1977] process and 0.50 for the CIR [1985] process. \( Z_{r,t} \) is a standard Wiener process. All values are annualized.

\[
    dr_t = \kappa(\theta - r_t)dt + \sigma_r r_t^\gamma dZ_{r,t}.
\]

<table>
<thead>
<tr>
<th>Values</th>
<th>( r_0 )</th>
<th>( \kappa )</th>
<th>( \theta )</th>
<th>( \sigma_r )</th>
<th>( \gamma )</th>
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<tbody>
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<td></td>
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<td>0.05</td>
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<td>{0, 0.50}</td>
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Table 4: Correlations matrix values.
This table presents correlations between the Wiener processes in the model: projects’ returns, guarantor’s returns, and the interest rate. The values are annualized.

<table>
<thead>
<tr>
<th></th>
<th>Project 1</th>
<th>Project 2</th>
<th>Guarantor</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project 1</td>
<td>1.00</td>
<td>{-0.30, 0.30}</td>
<td>{-0.30, 0.30}</td>
<td>-0.20</td>
</tr>
<tr>
<td>Project 2</td>
<td></td>
<td>1.00</td>
<td>{-0.30, 0.30}</td>
<td>-0.20</td>
</tr>
<tr>
<td>Guarantor</td>
<td></td>
<td></td>
<td>1.00</td>
<td>-0.20</td>
</tr>
<tr>
<td>Interest rate</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 5: Matrix of Expected Loss and Guarantee Value per dollar of debt for the simultaneous projects (Table 5-1/3)

This table presents the Expected Loss and Guarantee Value per dollar of debt (ELPD and GVPD) when the loan maturities for the two simultaneously executed projects take values in the set \{1, 5, 10, 15, 20\} for two scenarios (Scenario 1P and Scenario 2P). Three classes of projects ranked by risk level are considered: low-risk project, moderate-risk project, and high-risk project. For the low-risk level, we choose 0.20 as initial volatility level, 0.35 for the moderate-risk initial volatility level and 0.50 for the high-risk initial volatility level. Scenario 1P combines two moderate-risk projects and Scenario 2P combines a low-risk project and a high-risk project. The correlation between the two projects’ returns ($\rho_{12}$), and between the projects’ returns and the insurer’s asset returns ($\rho_{1G}, \rho_{2G}$) are in the set \{-0.30, 0.30\}. We perform the simulations using the CIR [1985] term structure of interest rates.

Panel a: $\rho_{12} = \rho_{1G} = \rho_{2G} = 0.30$

<table>
<thead>
<tr>
<th>Scenario 1P</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moderate-risk project</td>
<td>Moderate-risk project</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0.01</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>0.07</td>
<td>0.12</td>
</tr>
<tr>
<td>10</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>15</td>
<td>0.16</td>
<td>0.20</td>
</tr>
<tr>
<td>20</td>
<td>0.18</td>
<td>0.22</td>
</tr>
</tbody>
</table>
### Table 5: Matrix of Expected Loss and Guarantee Value per dollar of debt for the simultaneous projects (Table 5-2/3)

<table>
<thead>
<tr>
<th>Scenario 2P</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High-risk project</td>
<td>High-risk project</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Low-risk project Debt Maturity</td>
<td>1</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Panel b: $\rho_{12} = \rho_{1G} = \rho_{2G} = -0.30$

<table>
<thead>
<tr>
<th>Scenario 1P</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moderate-risk project</td>
<td>Moderate-risk project</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Moderate-risk project Debt Maturity</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Table 5: Matrix of Expected Loss and Guarantee Value per dollar of debt for the simultaneous projects (Table 5-3/3)

<table>
<thead>
<tr>
<th>Scenario 2P</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High-risk project Debt Maturity</td>
<td>High-risk project Debt Maturity</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Low-risk project Debt Maturity</td>
<td>1</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Table 6: Risk-neutralized default probabilities and probabilities of full recovery for the two parallel projects.

This table presents risk-neutralized default probabilities (Panel a) and probabilities of full recovery (Panel b) for the two parallel projects, for debts maturities of 1, 5, 10, 15 and 20 years. The debts considered here have coupon rate of 10% payable semi-annually up to the maturity of the debt. Low-risk project’s returns volatility is 0.20 and high-risk project’s returns volatility is 0.50. The guarantor’s asset returns volatility is 0.20. The correlation between the two projects returns, $\rho_{12}$, and between the projects’ returns and the insurer’s asset returns, $\rho_{1G}$ and $\rho_{2G}$, are taken from the set \{-0.30, 0.30\}. Correlations between projects’ returns or the insurer’s asset returns and the interest rate are –0.20. The recovery ratio is given by $1 - (\text{ELPD} - \text{GVPD})$ where ELPD and GVPD are respectively the Expected Loss and Guarantee Value per dollar of debt. Full recovery is indicated by a recovery ratio of one. The simulations are performed assuming the CIR [1985] term structure of interest rate.

Panel a: Default probabilities

<table>
<thead>
<tr>
<th>Debt Maturity (years)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-risk project</td>
<td>0%</td>
<td>14%</td>
<td>32%</td>
<td>45%</td>
<td>51%</td>
<td>1%</td>
<td>14%</td>
<td>33%</td>
<td>45%</td>
<td>53%</td>
</tr>
<tr>
<td>High-risk project</td>
<td>15%</td>
<td>52%</td>
<td>72%</td>
<td>80%</td>
<td>84%</td>
<td>14%</td>
<td>51%</td>
<td>68%</td>
<td>78%</td>
<td>85%</td>
</tr>
</tbody>
</table>

Panel b: Probabilities of full recovery

<table>
<thead>
<tr>
<th>Debt Maturity (years)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-risk project</td>
<td>100%</td>
<td>96%</td>
<td>90%</td>
<td>86%</td>
<td>84%</td>
<td>100%</td>
<td>99%</td>
<td>96%</td>
<td>92%</td>
<td>87%</td>
</tr>
<tr>
<td>High-risk project</td>
<td>99%</td>
<td>84%</td>
<td>70%</td>
<td>63%</td>
<td>58%</td>
<td>100%</td>
<td>94%</td>
<td>81%</td>
<td>74%</td>
<td>67%</td>
</tr>
</tbody>
</table>
Table 7: Matrix of Expected Loss and Guarantee Value per dollar of debt for the sequential projects (Table 7-1/4)

This table presents the Expected Loss and Guarantee Value per dollar of debt (ELPD and GVPD) for the two sequential projects. Debt maturities take values in the set \{1, 5, 10, 15, 20\} for three scenarios (Scenario 1S, Scenario 2S and Scenario 3S). Three classes of projects are considered: low-risk project, moderate-risk project, and high-risk project. For the low-risk level, we choose 0.20 as initial volatility level, 0.35 for the moderate-risk initial volatility level and 0.50 for the high-risk initial volatility level. Scenario 1S chooses two moderate-risk projects that run sequentially. Scenario 2S consists of choosing the low-risk project to run first, and the high-risk project to be executed next. In Scenario 3S, the high-risk project is staging first followed by the low-risk project. The correlations between the projects (\(\rho_{12}\)), and between the projects and the insurer (\(\rho_{1G}, \rho_{2G}\)) are in the set \{-0.30, 0.30\}. We perform the simulations using the CIR [1985] term structure of interest rate.

Panel a: \(\rho_{12} = \rho_{1G} = \rho_{2G} = 0.30\)

<table>
<thead>
<tr>
<th>Stakeholders</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd project (Moderate-risk)</td>
<td>2nd project (Moderate-risk)</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td>Scenario 1S</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1st project</td>
<td>1</td>
<td>0.02</td>
</tr>
<tr>
<td>(Moderate-risk)</td>
<td>5</td>
<td>0.08</td>
</tr>
<tr>
<td>Debt Maturity</td>
<td>10</td>
<td>0.15</td>
</tr>
<tr>
<td>15</td>
<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td>20</td>
<td>0.25</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Table 7: Matrix of Expected Loss and Guarantee Value per dollar of debt for the sequential projects (Table 7-2/4)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd project (High-risk)</td>
<td>2nd project (High-risk)</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1st project (Low-risk) Debt Maturity</td>
<td>0.04</td>
<td>0.13</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>0.13</td>
</tr>
<tr>
<td>10</td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td>15</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>20</td>
<td>0.13</td>
<td>0.16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd project (Low-risk)</td>
<td>2nd project (Low-risk)</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1st project (High-risk) Debt Maturity</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>10</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>15</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>20</td>
<td>0.34</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Table 7: Matrix of Expected Loss and Guarantee Value per dollar of debt for the sequential projects (Table 7-3/4)

Panel b: $\rho_{12} = \rho_{1G} = \rho_{2G} = -0.30$

<table>
<thead>
<tr>
<th>Scenario 1S</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2\textsuperscript{nd} project (Moderate-risk)</td>
<td>2\textsuperscript{nd} project (Moderate-risk)</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>10</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>15</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>20</td>
<td>0.24</td>
<td>0.21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario 2S</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2\textsuperscript{nd} project (High-risk)</td>
<td>2\textsuperscript{nd} project (High-risk)</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td>1</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>0.05</td>
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<tr>
<td>10</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>15</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>20</td>
<td>0.13</td>
<td>0.11</td>
</tr>
</tbody>
</table>


Table 7: Matrix of Expected Loss and Guarantee Value per dollar of debt for the sequential projects (Table 7-4/4)

<table>
<thead>
<tr>
<th>Scenario 3S</th>
<th>Expected Loss per dollar of debt (ELPD)</th>
<th>Expected Guarantee Value per dollar of debt (GVPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2\textsuperscript{nd} project (Low-risk)</td>
<td>2\textsuperscript{nd} project (Low-risk)</td>
</tr>
<tr>
<td></td>
<td>Debt Maturity</td>
<td>Debt Maturity</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1st project (High-risk) Debt Maturity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>10</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>15</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>20</td>
<td>0.34</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Figure 1: Trade-off between Expected Loss and Optimal Guarantee provided.

This graph plots the expected optimal guarantee provided by the guarantor for different given levels of losses. The optimization program is formulated as follows:

$$
\max_{t_1, t_2} \text{ GVPD} \quad \text{with} \quad \text{ELPD} = \overline{\text{ELPD}} \quad \text{and} \quad W_0 = \overline{W}.
$$

It states that for a given Expected Loss per dollar of debt (ELPD), the guarantor selects the maturity combinations for which the Expected Guarantee Value per dollar of debt (GVPD) is highest. The initial wealth of the insurer is a constant $W$. Since the insuring capacity or strength of the insurer is bounded, the gap ELPD-GVPD [which represents the Non-Recoverable Loss per dollar of insured debt (NRPD)] increases with the loss level. This graph is constructed for Scenario 2P. Scenario 2P combines two projects running in parallel, a low-risk project and a high-risk project (see Section 4.3).
Figure 2: Isolines of Expected Loss per dollar of debt in the case of simultaneous projects.

This graph represents schematically the results of the optimization [Equation (18)] for Scenario 2P. Scenario 2P combines two projects running in parallel, a low-risk project and a high-risk project. The lines represent the debt maturity combinations for the same Expected Loss per dollar of debt (ELPD). The horizontal axis represents the debt maturity of the low-risk project and the vertical line the debt maturity of the high-risk project. Low-risk project has a volatility of 0.20 and high-risk project has volatility of 0.50. The graph also shows the expected optimal guarantee value provided (GVPD) for given fixed levels of ELPD. The ELPD is the same along the very line. The GVPD for each maturity combination under the same ELPD is not necessarily the same along the line. The difference between the ELPD and the GVPD gives the NRPD.
We consider two projects that run in parallel (Scenario 2P). The projects’ initial volatilities are 0.20 for low-risk project and 0.50 for high-risk project. The insurer’s initial volatility is 0.20. The correlations between the two projects, and between the projects the insurer are 0.30 in exhibit (a) and -0.30 in exhibit (b). We perform this comparative analysis using the CIR [1985] term structure of interest rate.

(a) $\rho_{12} = \rho_{1G} = \rho_{2G} = 0.30$

(b) $\rho_{12} = \rho_{1G} = \rho_{2G} = -0.30$
Figure 4: Distribution Expected Losses, Guarantee Values and Non-Recoverable Losses per dollar of project debts. (Fig. 4-1/2)

We consider two projects running in parallel (Scenario 2P). We choose the same coupon rate (10%) and debt maturity of 10 years for the two projects. The projects’ returns initial volatilities are 0.20 for low-risk project and 0.50 for high-risk project. The insurer’s returns initial volatility is 0.20. The correlations between the two projects’ returns, and between the projects’ returns and the insurer’s returns are 0.30. We perform this comparative analysis using the CIR [1985] term structure.

(a) Loss distribution

(b) Guarantee distribution
Figure 4: Distribution of the Expected Losses, Guarantee Values and Non-Recoverable Losses per dollar of project debts. (Fig. 4-2/2)

(c) Guarantee Exposure distribution

Low-risk project

High-risk Project
Figure 5: Isolines of Expected Loss per dollar of debt in the sequential projects case. (Fig. 5-1/2)

This graph depicts the results from the optimization [Equation (18)] for Scenarios 2S and 3S. The top graph represents the results for Scenario 2S and the bottom graph for Scenario 3S. The lines represent the debt maturity combinations for the same Expected Loss per dollar of debt (ELPD). The horizontal axis represents the debt maturity of the first project (low-risk in the top graph and high-risk in the bottom graph) and the vertical axis the debt maturity of the second project (high-risk in the top graph and low-risk in the bottom graph). The low-risk project has a volatility of 0.20 and the high-risk project has a volatility of 0.50. The graph also shows the expected optimal guarantee value provided (GVPD) for given fixed levels of ELPD. The ELPD is the same along the very line. The GVPD for each maturity combination under the same ELPD is not necessarily the same along the line. The difference between the ELPD and the GVPD gives the NRPD.
Figure 5: Isolines of Expected Loss per dollar of debt in the sequential projects case. (Fig. 5-2/2)
Figure 6: Total Expected Losses, Guarantee Values and Non-Recovered Values per dollar of debt for the sequential projects case.

We consider two projects running sequentially (Scenario 2S). The first project is a low-risk project and the second project is high-risk. The projects’ initial volatilities are 0.20 for low-risk project and 0.50 for high-risk project. The insurer’s initial volatility is 0.20. The correlation between the two projects, and between the projects and the insurer are 0.30 in exhibit (a) and -0.30 in exhibit (b). We perform this comparative analysis using the CIR [1985] term structure of interest rate.

(a) $\rho_{12} = \rho_{1G} = \rho_{2G} = 0.30$

(b) $\rho_{12} = \rho_{1G} = \rho_{2G} = -0.30$
Figure 7: Total Expected Losses, Guarantee Values and Non-Recoverable Losses per dollar of debt for the sequential projects case.

We choose two projects running sequentially (Scenario 3S). The first project is a high-risk project and the second project is low-risk. The projects’ initial volatilities are 0.20 for low-risk project and 0.50 for high-risk project. The insurer’s initial volatility is 0.20. The correlation between the two projects, and between the projects and the insurer are 0.30 in exhibit (a) and -0.30 in exhibit (b). We perform this comparative analysis using the CIR [1985] term structure of interest rate.

(a) $\rho_{12} = \rho_{1G} = \rho_{2G} = 0.30$

(b) $\rho_{12} = \rho_{1G} = \rho_{2G} = -0.30$