Strategic trading when some investors receive information before others

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Abstract

This paper examines trading and price behavior when some investors receive information before others. It shows that early-informed investors trade more intensely on the information they share with late-informed investors at first to exploit it before the latter can do so. They reverse their trading strategy in the second round. The paper also shows that price and price moves are positively correlated with information. More interestingly, it discovers that under some conditions, subsequent price changes are positively correlated. Finally, it shows that early-informed investors may behave like contrarians.

JEL classification: G00; G14; D00; D01; D53; D82
Keywords: Asymmetric information; Game theory; Market microstructure

Information plays a very important role in financial markets. Investors use their information to form their expectations about the payoff of an asset. These expectations affect investors’ trading behavior, and as a result, the asset price. In a perfect market, all investors receive information immediately and simultaneously. In reality, however, some investors, either due to the nature of their positions, e.g., corporate insiders and their favored analysts, or their superior skills, acquire more information and before others. By being first, an investor can exploit this information to great advantage. Investors who are uninformed or late-informed are aware of the fact that the actions of other investors are driven by their information. Therefore, they trade carefully, trying to infer the private information of the better and early-informed investors from their trading behavior. Better and early-informed investors, of course, realize that their trading is followed closely, so they try to use an optimal trading strategy to maximize the value of their information. In the following analysis, I will use the term “trader” and “investor” interchangeably.

The objective of the paper is to examine trading and price behavior in equilibrium when some investors receive information before others. Toward that end, I develop a theoretical framework to analyze investment choices of two risk neutral investors who investigate the long-term prospect of a firm. The high-ability investor uncovers the payoff-
relevant information early, while the low-ability investor uncovers less information and later. Instead of assuming that the rational expectations equilibrium is a competitive one, I analyze a Perfect Bayesian Nash Equilibrium (PBE). This equilibrium notion is studied because it captures the fact that informed traders are rational, forward-looking, and their actions have impacts on the market. That is, each informed trader takes into account that her demand will be used by other traders to update their beliefs concerning the fundamentals of the stock.

I show that, after getting information, the early-informed investor trades heavily on the information she anticipates the late-informed investor will uncover to exploit it first before the latter can do so. In addition, she trades less intensely, at first, on the information that will unlikely be released to the late-informed investor in order to save it for the last trading round. This strategy causes information to incorporate into the price gradually. As a result, price and price moves are positively correlated with the information of informed traders. More interestingly, the paper shows that under some condition successive price moves are positively correlated, i.e., market has momentum, as we sometimes observe in the market. Finally, using a numerical example, I demonstrate that if the early-informed trader’s private information significantly differs from the shared information, she may behave as if she were a contrarian, buying (selling) in period 1 and then selling (buying) in the next period.

This paper is built on the existing literature on market microstructure. The seminal paper of Kyle (1985) investigates an information monopolist model in which an informed trader chooses trading strategy to maximize the value of her information. The model characterizes how information is incorporated into price when trades go on. Jennings, Starks, and Fellingham (1981) and Jennings and Barry (1983) consider models in which informed traders do not obtain information simultaneously. However, in those models, public information such as prices and order flows, which are potentially important sources of information about the asset, is not used by uninformed traders in forming their trading strategy. Brown and Jennings (1989), Grundy and McNichols (1989), Kim and Verrecchia (1991), and Wang (1993) overcome this limitation by analyzing the dynamic trading behavior when uninformed investors condition their trades on all public information. However, in those models, all potentially informed investors receive information simultaneously. Hirshleifer, Subrahmanyam, and Titman (1994) examine trading behavior in a competitive equilibrium when some investors receive information before others. Even though the model provides important insights into herding effects in financial markets, the assumption that informed traders are too small to affect market price is unrealistic. In reality, informed investors (the ones with high research capability), such as institutional investors, are usually large. Thus, their actions would have effects on the price. To overcome this limitation, this paper analyzes a PBE where informed investors are strategic, i.e., they take into consideration the impacts of their actions on the market when they formulate their trading strategies.

In the literature on manipulation, Allen and Gale (1992) distinguish between trade-based, information-based and action-based stock price manipulation. In their trade-based manipulation model, there is a large trader who is either informed or uninformed. Other traders are price takers. Allen and Gale show that when the large trader is uninformed, it is optimal for her to manipulate the market by acting as if she received good news. The basic idea is that the large uninformed trader exploits the inability of the market to distinguish between informed and uninformed trades. Unlike Allen and Gale (1992), in this paper, investors are informed for sure and they try to maximize the value of their information. Brunnermeier (2001) shows that a trader who receives a signal before the public announcement can exploit this information twice: when she receives the signal and after the public announcement. The reason is that, unlike other traders, she can infer how much the announced information has been incorporated in the current price.

Pan, Zou, Liu, and Wu (2003) examine trading and price behavior in a PBE equilibrium when some investors receive less information and later than others. In their model, even though late-informed traders receive information, they do not strategically exploit it. Instead, Pan et al. assume that their trades follow a fixed normal distribution similar to the behavior of liquidity traders. Thus, the setting of the model is not different from Kyle’s (1985). Unlike Pan et al. (2003), this paper allows late-informed investors to trade strategically to exploit their information.

In order to combat the abuse of insider trading, the U.S. Congress enacts a law requiring insiders associated with a firm to report any equity transactions they make in the stock of that firm to the Securities and Exchange Commission. The reports are filed after the trade is completed, at which time they become publicly available. Huddart, Hughes, and Levine (2001) and Vo (2006) show that under this circumstance, informed insiders, in order to maintain their information superiority after disclosure, may add a random noise into their orders.

The remainder of the paper is organized as follows. Section 1 presents the model. Section 2 characterizes a linear equilibrium. Section 3 develops a numerical method to solve for equilibriums. Section 4 presents a numerical example to discuss the implications of the model. Section 5 concludes the paper.
1. The model

There are two assets in the economy: a risky stock and a risk-less bond. The interest rate of the bond is normalized to zero so that there is no need to discount future cash flows. Market participants include an early-informed trader, a late-informed trader, a market maker, and a number of liquidity traders. These traders buy or sell the stock in two periods. Both informed traders are assumed to be risk neutral. The fundamental value of the stock is a random variable $v$ which is normally distributed with mean zero and variance $\Sigma_0$. In this paper, a tilde is used to distinguish a random variable from its realization.

Before the first trading takes place, the early-informed trader, denoted by trader 1, learns the realization of the fundamental value of the stock, $v$, and receives a signal $s$ correlated to this value. The late-informed trader, denoted by trader 2, does not receive any information in period 1 but learns signal $s$ just before the market opens in period 2. Without loss of generality, this paper takes $s$ as the conditional expectation of the fundamental value of the asset, given the second trader’s information in period 2 (both public and private information). Signal $s$ is assumed to be drawn from a normal distribution with mean zero and variance $\Psi_0$. Thus, for trader 2,

$$v = s + \epsilon,$$

where the residual term $\epsilon$ is normally distributed with mean zero and variance $\Lambda_0$, and orthogonal with $s$. Taking variance in both sides of Eq. (1), we get:

$$\Sigma_0 = \Psi_0 + \Lambda_0.$$

Liquidity traders buy and sell shares for liquidity reasons which are exogenous to the model. The quantity traded by liquidity traders in period $t$, denoted by $\tilde{u}_t$, is normally distributed with mean zero and variance $\sigma_u^2$, serially uncorrelated, and independent of all other random variables in the model. After two trading rounds, the fundamental value of the stock is announced and stockholders are paid accordingly. This information structure is common knowledge.

The trading procedure involves two steps. First, traders submit orders to buy or sell stock to the market maker. Second, the market maker observes the aggregate order flow but does not know which orders come from which traders. She then sets price and trades the quantity necessary to clear the market. I follow the tradition in literature whereby the market maker is assumed to set efficient price at which her expected profit is zero. That is, the market maker sets price equal to the expected value of the asset, conditioned on the history of aggregate order flows received up to that time.

Denote the quantities traded in period $t$ by trader 1, trader 2, and liquidity traders by $x_t$, $y_t$, and $\tilde{u}_t$, respectively. The aggregate order flow in period $t$ is:

$$w_t = x_t + y_t + \tilde{u}_t.$$ 

The prices set by the market maker in the first and second trading rounds are:

$$p_1 = E(\tilde{v} \mid w_1) \quad \text{and} \quad p_2 = E(\tilde{v} \mid w_1, w_2).$$

This paper assumes that the portfolio of trader 2 is optimal before the first trading round. Since she receives no information in this round, she does not trade to rebalance her portfolio when the market opens in period 1. Thus, $y_1 = 0$ and the aggregate order flow in period 1 is

$$w_1 = x_1 + \tilde{u}_1.$$ 

To simplify the notation, I omit the subscript in trader 2’s order. Thus, trader 2’s order in period 2 is $y$ instead of $y_2$. The information structure of the model is summarized in Table 1. The timeline of the trading game is shown in Fig. 1.

In order to examine trading and price behavior in equilibrium, I will derive the resulting demands, $x_t$, $y_t$, and the prices $p_1$ and $p_2$. Toward this end, I represent the above economy as an extensive form game with imperfect information, and employ the notion of Perfect Bayesian Equilibrium (PBE). This equilibrium notion is studied because it captures the fact that informed traders are rational and forward-looking. That is, each informed trader takes into account that her demand will be used by others to update their beliefs concerning the fundamental value of the stock. The following definition defines formally a PBE of this trading game.
Definition. A Perfect Bayesian Equilibrium of the trading game is defined as a strategy profile \([x_1^*, x_2^*, 0, y^*]\) and a price system \([p_1^*, p_2^*]\) such that the following conditions hold:

1. Profit maximization
   \[x_2^* \in \text{argmax}_{x_2} E[x_2(\tilde{v} - p_2) | I_2^1],\]
   \[x_1^* \in \text{argmax}_{x_1} E[x_1(\tilde{v} - p_1) + x_2^*(\tilde{v} - p_2) | I_1^1],\]
   \[y^* \in \text{argmax}_y E[y(\tilde{v} - p_2) | I_2^2].\]

2. Market efficiency
   \[p_1^* = E(\tilde{v} | w_1^*),\]
   \[p_2^* = E(\tilde{v} | w_2^*, w_1^*).\]

Where \(I_1^1\) and \(I_2^1\) are the information sets of trader 1 in periods 1 and 2, respectively, and \(I_2^2\) is the information set of trader 2 in period 2. The conditional expectations are derived using Bayes’ rule to ensure that beliefs are consistent with the equilibrium strategy.

This equilibrium concept is based on dynamic programming argument. The strategy of trader 1 in period 2 is required to be optimal, not only when trader 1 plays her optimal strategy in period 1, but also when she plays any arbitrary strategy in period 1. However, there is no off-equilibrium observation of order flows by other market participants in the model (even when trader 1 deviates from her optimal strategy) as liquidity trades make every order flow possible. Consequently, we do not have to concern ourselves with the issue of how to assign off-equilibrium beliefs.

Next, I define variables to measure the stock of information available to traders to exploit:

\[\Sigma_1 = \text{var}(\tilde{v} | w_1) = \text{var}(\tilde{v} - p_1),\]
\[\Psi_1 = \text{var}(s | w_1) = \text{var}(s - p_1).\]

\(\Sigma_1\) is the variance of the fundamental value of the asset, given the market maker’s information after the first trading round. Thus, \(\Sigma_1\) measures the stock of information available to trader 1. A high value of \(\Sigma_1\) indicates that little information about the asset has been incorporated into the price and vice versa. \(\Psi_1\) is the variance of signal \(s\), given the market maker’s information after the first trading round. Thus, it measures the stock of information available to trader 2. The higher the value of \(\Psi_1\), the more information advantage trader 2 has over the market maker.

2. Characterization of equilibrium

2.1. Equilibrium

This paper explores the linear equilibrium of the trading game because, in addition to its appeal and tractability, given the normality assumption of all random variables, prices are linear functions of the history of aggregate order

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Information structure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Player</strong></td>
<td><strong>Period 1</strong></td>
</tr>
<tr>
<td>Trader 1</td>
<td>(v \text{ and } s)</td>
</tr>
<tr>
<td>Trader 2</td>
<td>None</td>
</tr>
<tr>
<td>Market maker</td>
<td>(w_1)</td>
</tr>
</tbody>
</table>
flows (by the projection theorem). Thus, prices fully reflect aggregate demand. To derive linear equilibriums, I begin by postulating that the price function has the following forms:

\[ p_1 = \lambda_1 w_1, \]
\[ p_2 = p_1 + \lambda_2 w_2. \]

In the ensuing analysis, I derive linear equilibriums in which the initial conjecture is confirmed to be correct. I employ backward induction because a PBE requires equilibrium strategies to be optimal for each information set under a given Bayesian rational belief system.

**Proposition 1.** A PBE in which all trading strategies and the market maker's pricing rule are of linear form is given by

\[ x_1 = \alpha_1 (v-s) + \beta_1 s, \]
\[ x_2 = \alpha_2 (v-s) + \beta_2 (s-p_1), \]
\[ y = \theta (s-p_1), \]
\[ p_1 = \lambda_1 w_1, \]
\[ p_2 = p_1 + \lambda_2 w_2, \]

\[ \alpha_1, \beta_1, \lambda_1, \alpha_2, \beta_2, \theta, \lambda_1 \text{ are given by:} \]

\[ \alpha_1 = \frac{3(3\lambda_2 - \lambda_1)}{2\lambda_1 (9\lambda_2 - \lambda_1)}, \]
\[ \beta_1 = \frac{9\lambda_2 - 2\lambda_1}{2\lambda_1 (9\lambda_2 - \lambda_1)}, \]
\[ \lambda_1 = \frac{\alpha_1 A_0 + \beta_1 \Psi_0}{\alpha_1^2 A_0 + \beta_1^2 \Psi_0 + \sigma^2}, \]
\[ \alpha_2 = \frac{1}{\lambda_2}, \]
\[ \beta_2 = \theta = \frac{1}{3\lambda_2}, \]
\[
\lambda_2 = \sqrt{\frac{9A_0 + 8\Psi}{36\sigma_u^2}},
\]
where
\[
\Psi_1 = \lambda_1^2 \sigma_1^2 A_0 + (1-\lambda_1 \beta_1)^2 \sigma_0^2 + \lambda_1 \sigma_u^2,
\]
if the second order conditions
\[
\lambda_1, \lambda_2 > 0,
\]
\[
9\lambda_2 - \lambda_1 > 0,
\]
are satisfied.

**Proof.** See Appendix.

### 2.2. Trading and price behaviors in equilibrium

#### 2.2.1. Trading behavior

In general, trading occurs for risk sharing purposes or for information reasons. Since all informed traders are risk neutral in this setting, their only motive to trade is to exploit their informational advantage. Eqs. (6) and (7) describe the trading strategy of trader 1. Her orders consist of two parts. The first one is based on the difference between the true value known only to her and the signal she shares with trader 2. The intensity at which she trades on this part is \(\alpha_1(t=1, 2)\). The second one is based on the difference between the signal she shares with trader 2 and the previous price (assuming the price is zero before the trading game starts). The intensity at which she trades on this part is \(\beta_1(t=1, 2)\).

An interesting feature of the model is that in the first trading round, trader 1 trades more intensively on the part of information that will be released to trader 2 and less intensively on the part of information that is known to her only (\(\beta_1 > \alpha_1\)). This is because she wants to exploit signal \(s\) first before it is revealed to trader 2. By doing so, she makes a significant part of \(s\) to be incorporated into the price. Thus, this strategy has a negative impact on the profit of trader 2 in the second trading round.

Eq. (8) outlines the order submission strategy of trader 2. Her order is based on the difference between signal \(s\) and the previous price. The intensity at which she trades is \(\theta\). If we compare the orders of both traders in period 2, we can see that trader 1 behaves as a monopolist on the part of information she possesses, and both of them behave as duopolists on their shared information. In the limit case when both receive the same information (\(v=s\)), they have the same order submission strategy in period 2 even though they get information at different time.

Since trader 1 trades on both signals \(v\) and \(s\) in both periods, this raises the possibility that if \(v\) and \(s\) are significantly different, trader 1 could have significantly different trading strategies in period 1 and period 2. For example, it could be the case that trader 1 will sell (buy) the asset in period 1 and buy (sell) it back in period 2.

I next examine the correlation between informed traders’ orders and their signals.

**Proposition 2.** The orders of informed traders are positively correlated with their signals. Specifically,

\[
\text{cov}(x_t, v) > 0,
\]
\[
\text{cov}(x_t, s) > 0,
\]
\[
\text{cov}(y_t, s) > 0.
\]

**Proof.** See Appendix.

Along with Proposition 1, Proposition 2 tells us that trader 1, in the process of maximizing her profit, tries to exploit both signals from the beginning. As a result, her order is positively correlated with her signals (\(v\) and \(s\)). By the same token, trader 2’s order is positively correlated with her signal \(s\). Since their orders constitute aggregate order flow \(w_t\), informed traders’ information is reflected in \(w_t\). Based on \(w_t\), the market maker can infer the information and uses it to adjust price.
2.2.2. Price behavior

This subsection examines some relevant relationships between price $p_t$, price moves (defined as the change in price: $p_t - p_{t-1}$), and information variables ($v$ and $s$).

**Proposition 3.** Assuming the price equals zero before the trading game starts, the price moves in periods 1 and 2 are always positively correlated with the signals of informed traders. Specifically,

$$\text{cov}(p_1, v)>0,$$
$$\text{cov}(p_2-p_1, v)>0,$$
$$\text{cov}(p_1, s)>0,$$
$$\text{cov}(p_2-p_1, s)>0.$$

**Proof.** See the Appendix.

The intuition is that in exploiting their information, informed investors submit orders correlated to their signals. Based on the aggregate order flows, the market maker can infer part of this information and uses it to adjust price accordingly. As a result, the price moves are positively correlated with the signals of informed traders. This result is consistent with the finding of Hirshleifer et al. (1994) in the context of competitive equilibrium.

**Proposition 4.** If liquidity trade is not very volatile, subsequent price changes are positively correlated. Specifically,

$$\text{cov}(p_2-p_1, p_1)>0,$$

if $\sigma_u^2$ is sufficiently small.

**Proof.** See the Appendix.

The intuition of Proposition 4 is as follows. When informed investors receive new information, they gradually trade on it to maximize profit. As a result, their orders reflect their information. The market maker infers new information based on aggregate order flows which encompass both informed and liquidity trades. Thus, if liquidity trade is stable, her inference is more precise and she is able to gradually incorporate new information into price. Specifically, if it is good (bad) news, she will gradually increase (decrease) price. This leads to a positive correlation between subsequent price moves. However, this might not be true if liquidity trade is too volatile because in this case the market maker’s inference is imprecise; as a result, noisy information is incorporated into price.

The implication of this result is that the market might have momentums. If the price increases (decreases) today, it is likely that it will increase (decrease) further tomorrow. This result is the outcome of trading strategies of informed investors who try to maximize their profit given their private information. In contrast, Hirshleifer et al. (1994) show that momentums do not exist in a competitive equilibrium because when traders behave competitively, new information is instantly incorporated into price.

### 3. A numerical method to compute equilibrium

This section discusses a numerical method to solve for equilibriums presented in Proposition 1. The initial information includes (i) $\Sigma_0$, the unconditional variance of the fundamental value of the stock, (ii) $\Psi_0$, the variance of signal $s$, (iii) $\sigma_u^2$, the variance of liquidity trade. I begin by choosing a starting value for $\Psi_1$. By definition, $\Psi_1$ is the stock of information available to trader 2. A restriction on $\Psi_1$ is:

$$0 \leq \Psi_1 = \text{var}(s|w_1) \leq \text{var}(s) = \Psi_0.$$  \hspace{1cm} (16)

In addition, the second order condition:

$$0 < \lambda_1 < 9\lambda_2$$  \hspace{1cm} (17)

is used to impose a restriction on $\lambda_1$. 

Fig. 2. Flowchart to solve for equilibriums.

\[
\Sigma_0, \Psi_0, \sigma_u^2
\]

Pick a \(\Psi_1\) in a fine grid \(0 \leq \Psi_1 \leq \Psi_0\)

\[
\lambda_2 = \sqrt{\frac{9\Lambda_0 + 8\Psi_1}{36\sigma_u^2}}
\]

\[
\alpha_2 = \frac{1}{\lambda_2}, \beta_2 = \theta = \frac{1}{3\lambda_2}
\]

Pick a \(\lambda_1\) in a fine grid \(0 < \lambda_1 < 9\lambda_2\)

\[
\alpha_1 = \frac{9\lambda_2 - 3\lambda_1}{2\lambda_2 (9\lambda_2 - \lambda_1)}, \quad \beta_1 = \frac{9\lambda_2 - 2\lambda_1}{2\lambda_1 (9\lambda_2 - \lambda_1)}
\]

\[
\lambda_1 = \frac{\alpha_1 \Lambda_0 + \beta_1 \Psi_0}{\alpha_1 \Lambda_0 + \beta_1 \Psi_0 + \sigma_u^2}
\]

Yes

\[
\Psi_1 = \lambda_1^2 \alpha_1^2 \Lambda_0 + \left(1 - \lambda_1 \beta_1\right)^2 \Psi_0 + \lambda_1^2 \sigma_u^2
\]

No

\((\alpha_1, \beta_1, \alpha_2, \beta_2, \theta, \lambda_1, \lambda_2)\) is a solution

Yes

No

Try all \(\lambda_1\) in the grid?

Yes

Try all \(\Psi_1\) in the grid?

Yes

Stop
For each value of $\Psi_1$ in the fine grid in the feasible range indicated by Eq. (16), Eq. (14) can be solved for $\lambda_2$ (by Eq. (2) $\Lambda_0 = \Sigma_0 - \Psi_0$). $\lambda_2$ is then used to solve Eqs. (12) and (13) for $\alpha_2, \beta_2, \theta$. With $\lambda_2$ just calculated, I follow the following steps:

1. in the fine grid in the feasible range of $\lambda_1$ indicated by Eq. (17), choose a $\lambda_1$,
2. calculate $\alpha_1, \beta_1$ using Eqs. (9) and (10),
3. recalculate $\lambda_1$ using Eq. (11).

If $\lambda_1$ calculated in step (3) is not the same as the one chosen in step (1), choose another $\lambda_1$ and repeat steps (2) and (3). If $\lambda_1$ calculated in step (3) is equal to the one chosen in step (1), use (16) to recalculate $\Psi_1$ and compare it with the value of $\Psi_1$ chosen before. If they are identical, then $\alpha_1, \beta_1, \lambda_1, \alpha_2, \beta_2, \lambda_2$ just calculated are equilibrium values. Otherwise, choose a new $\Psi_1$ and redo the whole process. The flow chart of this numerical method is shown in Fig. 2.

4. A numerical example

In this section, I solve numerically for equilibrium, using the method described in Section 3, in order to discuss the implications of the model. More specifically, I will examine the trading behavior of the early-informed trader under the presence of a late-informed trader, how the late-informed trader competes, and how their trading behavior affects the market price.

There are three parameters that fully describe the economic environment, namely, the variance $\Sigma_0$ of the fundamental value of the stock $\hat{v}$, the variance of liquidity trade $\sigma_u^2$, and the variance $\Psi_0$ of signal $s$. These values are chosen to be $\Sigma_0 = 2, \Psi_0 = 1, \sigma_u^2 = 1$.

The following is an equilibrium of the model:

\[ x_1 = 0.69(v-s) + 0.79s, \quad (18) \]
\[ x_2 = 0.77(v-s) + 0.51(s-p_1), \quad (19) \]
\[ y = 0.51s, \quad (20) \]
\[ p_1 = 0.57w_1, \quad (21) \]
\[ p_2 = p_1 + 0.65w_2. \quad (22) \]

We observe that trader 1's orders, $x_1$ and $x_2$, are functions of (i) her private information, which is the difference between the true value of stock, $v$, known only to her, and the signal $s$ she shares with trader 2 (the first term in Eqs. (18) and (19)), and, (ii) the shared information, which is the difference between the shared signal $s$ and the current market price (the second term in Eqs. (18) and (19)). In the first period, she trades more intensely on the shared information; however, this is reversed in the second period. As trader 1 trades more intensely on the shared information in period 1, more of this information is incorporated into the price $p_1$. This diminishes the information advantage of trader 2 in period 2. Thus, this trading strategy not only helps trader 1 retain more of her private information for the second trading round, but also helps her exploit the shared information first before trader 2 is able to do so.

Period 2 is the last period informed traders can exploit their information before it is known to the public, so they try to exploit their information as much as they can. For trader 1, she trades more intensely on her private information (the trading intensity on her private information in period 2 is 0.77 vs. 0.69 in period 1). We also observe that both informed traders have the same trading intensity on the shared information they share (0.51). This can be explained by the fact that trader 1 is a monopolist on his own information and both traders 1 and 2 are duopolists on the information they share. The covariance of the price moves in two successive periods in this case is 0.08.

Define marginal trading cost (MTC) in period $t$ as $\frac{\partial p_t}{\partial w_t}$. MTC measures the change in price when the aggregate order flow increases by 1. We observe that MTC is higher in period 2 than in period 1 (0.65 vs. 0.57). One explanation for this is that there are more informed traders in the market in period 2 than in period 1 and the market maker has to raise the
price more in order to protect herself. This result is in contrast with the case where investors receive information at the same time. Vo (2006) shows that if investors receive information at the same time and trade strategically to exploit it then the marginal trading cost declines over time.

Since trader 1 trades on both signals \( v \) and \( s \), there is a possibility that if \( v \) and \( s \) are significantly different, she may buy (sell) in one period and sell (buy) in the other. To illustrate this point, I solve for the expected orders of trader 1 when \( v = -0.5 \) and \( s = -5 \). The order of trader 1 in period 1 is \(-0.85\) while her expected order in period 2 is \(0.67\). (I calculate expected order of trader 1 in period 2 instead of the actual one because the latter depends on price \( p \) which, in turn, depends on liquidity trade). In this case, both traders believe that the stock is overvalued. At first, trader 1 sells to decrease price, then she buys back in the second period. Trader 2, on the other hand, sells in the second period (her expected order is \(-3.95\)). In contrast, in Kyle (1985) the information monopolist is expected to trade gradually on her information to exploit it and there is no expected contrarian trade.

5. Conclusions

This paper attempts to examine trading and price behavior when investors do not receive information at the same time. It shows that the early-informed investor, instead of trading only on the true value of the asset as in Kyle’s (1985) model, trades on both his private information and the information that she shares with the late-informed trader. She even trades more intensely on the information of the late-informed trader than her own information to exploit it before the late-informed trader can do so. The paper argues that the early-informed trader behaves like an information monopolist on the part of information only known to him. However, both informed traders behave like duopolists on the part of information shared between them. More interestingly, the paper discovers that under some conditions, subsequent price changes are positively correlated. Finally, given the fact that the early-informed investor trades on both signals, it could be the case that if they are significantly different, the early-informed investor may behave like a contrarian, buying (selling) in period 1 and selling (buying) in period 2.

Appendix A. Proof of proposition 1

In period 2, trader 2’s problem is to maximize her expected profit given her information.

\[
\text{Max } y \mathbb{E} [ y(v - p_2) | s, p_1 ]
\]

\[
\Rightarrow \text{Max } y \mathbb{E} [ y(v - p_1 - \lambda_1(x_2 + y + u_2)) | s, p_1 ]
\]

\[
\Rightarrow \text{Max } y [s - p_1 - \lambda_2(x_2^c + y)],
\]

where \( x_2^c \) is the trader 2’s forecast of \( x_2 \).

The first order condition is:

\[
s - p_1 - \lambda_2 x_2^c - 2\lambda_2 y = 0.
\]

\[
\Rightarrow y = x_2^c = \frac{s - p_1}{3\lambda_2}.
\]

if the second order condition \( \lambda_2 > 0 \) is satisfied. Compare Eqs. (26) and (8), we have

\[
\theta = \frac{1}{3\lambda_2}.
\]

Trader 1 solves the following optimization problem to determine her demand in period 2:

\[
\text{Max } x_2 \mathbb{E} [ v - p_2 | v, s, p_1 ]
\]

\[
\Rightarrow \text{Max } x_2 [v - p_1 - \lambda_2(x_2 + y)].
\]
The first order condition is:

\[ v - p_1 - 2 \dot{\lambda}_2 x_2 - \dot{\lambda}_2 v = 0, \]  

(29)

\[ \Rightarrow x_2 = \frac{v - p_1}{2 \dot{\lambda}_2} - \frac{v}{2}. \]  

(30)

Since trader 1 has more information than trader 2, she can infer exactly the optimal order of trader 2. Thus, her order is:

\[ x_2 = \frac{v - p_1}{2 \dot{\lambda}_2} - \frac{s - p_1}{6 \dot{\lambda}_2}, \]  

(31)

\[ \Rightarrow x_2 = \frac{1}{2 \dot{\lambda}_2} (v - s) + \frac{1}{3 \dot{\lambda}_2} (s - p_1). \]  

(32)

Compare Eqs. (7) and (32), we get:

\[ x_2 = \frac{1}{2 \dot{\lambda}_2}, \text{ and } \beta_2 = \frac{1}{3 \dot{\lambda}_2}. \]  

(33)

The second order condition is:

\[ \dot{\lambda}_2 > 0. \]  

(34)

Substitute Eq. (32) into the objective function Eq. (28), we get the value function of trader 1 as follows:

\[ V_1 = \frac{1}{\dot{\lambda}_2} \left( \frac{v - s}{2} + \frac{s - p_1}{3} \right)^2. \]  

(35)

In period 1, trader 1 chooses \( x_1 \) to solve the following problem

\[ \max_{x_1} E [x_1 (\bar{v} - p_1) + V_1 (p_1) | v, s], \]

\[ \Rightarrow \max_{x_1} \left[ x_1 (v - \dot{\lambda}_1 x_1) + \frac{1}{\dot{\lambda}_2} \left( \frac{v - s}{2} + \frac{s - \dot{\lambda}_1 x_1}{3} \right)^2 \right]. \]  

(36)

The first order condition is:

\[ v - 2 \dot{\lambda}_1 x_1 + \frac{2}{\dot{\lambda}_2} \left( \frac{v - s}{2} + \frac{s - \dot{\lambda}_1 x_1}{3} \right) \left( \frac{-\dot{\lambda}_1}{3} \right) = 0, \]

\[ \Rightarrow x_1 = \frac{9 \dot{\lambda}_2 - 3 \dot{\lambda}_1}{2 \dot{\lambda}_1 (9 \dot{\lambda}_2 - \dot{\lambda}_1)} (v - s) + \frac{9 \dot{\lambda}_2 - 2 \dot{\lambda}_1}{2 \dot{\lambda}_1 (9 \dot{\lambda}_2 - \dot{\lambda}_1)} s. \]  

(37)

Compare Eqs. (37) and (6), we get: \( \beta_1 = \frac{9 \dot{\lambda}_2 - 3 \dot{\lambda}_1}{2 \dot{\lambda}_1 (9 \dot{\lambda}_2 - \dot{\lambda}_1)} \) and \( \gamma_1 = \frac{9 \dot{\lambda}_2 - 2 \dot{\lambda}_1}{2 \dot{\lambda}_1 (9 \dot{\lambda}_2 - \dot{\lambda}_1)}. \)

The second order condition is

\[ \dot{\lambda}_1 \left( 1 - \frac{\dot{\lambda}_1}{9 \dot{\lambda}_2} \right) > 0. \]  

(38)
Eqs. (34) and (38) lead to:
\[ \lambda_1 > 0 \text{ and } 9\lambda_2 - \lambda_1 > 0. \]

In period 1, the pricing rule of the market maker is:
\[ p_1 = E(\tilde{v} | w_1) = \frac{\text{cov}(\tilde{v}, w_1)}{\text{var}(w_1)} w_1. \]

\[ \Rightarrow \lambda_1 = \frac{\text{cov}(\tilde{v}, w_1)}{\text{var}(w_1)}. \]

Since
\[ w_1 = x_1 + u_1 = x_1(v-s) + \beta_1 s + u_1, \]

\[ \Rightarrow \lambda_1 = \frac{\text{cov}[(v-s) + s, x_1(v-s) + \beta_1 s + u_1]}{\text{var}[x_1(v-s) + (\beta_1 + \theta) s + u_2]}, \]

In period 2, the pricing rule is:
\[ p_2 = E(v | w_1, w_2) = p_1 + \lambda_2 w_2, \]

\[ \Rightarrow \lambda_2 = \frac{\text{cov}(v-p_1, w_2)}{\text{var}(w_2)}. \]

\[ w_2 = x_2 + y + u_2 = x_2(v-s) + (\beta_1 + \theta) s + u_2, \]

\[ \Rightarrow \lambda_2 = \frac{\text{cov}[(v-s) + (s-p_1), x_1(v-s) + (\beta_2 + \theta) s + u_2]}{\text{var}[x_1(v-s) + (\beta_2 + \theta) s + u_2]}, \]

where \( \Psi_1 \) is given by:
\[ \Psi_1 = \text{var}(s-p_1) = \text{var}\{s-\lambda_1 [x_1(v-s) + \beta_1 s + u_1]\}, \]

\[ \Rightarrow \Psi_1 = \lambda_1^2 \beta_1^2 A_0 + (1-\lambda_1 \beta_1)^2 \Psi_0 + \lambda_1^2 \sigma_u^2. \]

Substituting Eqs. (27) and (33) into (42) yields:
\[ \lambda_2 = \frac{A_0}{2\sigma_2^2} + \left( \frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2} \right) \Psi_1 \]

\[ \Rightarrow \lambda_2 = 6\lambda_2 \cdot \frac{3A_0 + 4\Psi_1}{9A_0 + 16\Psi_1 + 36\lambda_2^2 \sigma_u^2}, \]

\[ \Rightarrow \lambda_2 = \sqrt{\frac{9A_0 + 8\Psi_1}{36\sigma_u^2}}. \]
Appendix B. Proof of proposition 2

\[ \text{cov}(x_1, v) = \text{cov}[x_1(v-s) + \beta_1 s, (v-s) + s], \]  
\[ \Rightarrow \text{cov}(x_1, v) = x_1 A_0 + \beta_1 \Psi_0 = \lambda_1(x_1^2 A_0 + \beta_1^2 \Psi_0 + \sigma_u^2), \]  
\[ \Rightarrow \text{cov}(x_1, v) > 0, \]  

because \( \lambda_1 > 0 \) (by the second order condition in Proposition 1.)

\[ \text{cov}(x_1, s) = \text{cov}[x_1(v-s) + \beta_1 s, s] = \beta_1 \Psi_0. \]  

To prove \( \beta_1 > 0 \), notice that \( \beta_1 > \lambda_1 \), therefore,

\[ A_0(\beta_1 - \lambda_1) > 0. \]  

Moreover, by Eq. (45), we have:

\[ x_1 A_0 + \beta_1 \Psi_0 > 0 \]  

Adding Eqs. (48) and (49) yields:

\[ \beta_1 (A_0 + \Psi_0) > 0, \]  

\[ \Rightarrow \beta_1 > 0. \]  

Thus,

\[ \text{cov}(x_1, s) = \beta_1 \Psi_0 > 0. \]  

\[ \text{cov}(x_2, v) = \text{cov}[x_2(v-s) + \beta_2 (s-p_1), (v-s) + (s-p_1) + p_1] \]  
\[ = x_2 A_0 + \beta_2 \Psi_1 > 0. \]  

The last inequality comes from the fact that \( A_0, \Psi_0 \) are variances, and \( x_2, \beta_2 \) are both positive.

\[ \text{cov}(v, s) = \text{cov}[\theta(s-p_1), (s-p_1) + p_1] = \theta \Psi_1 > 0. \]  

The last inequality comes from the fact that \( \Psi_1 \) is variance and \( \theta \) is positive.

Appendix C. Proof of proposition 3

\[ \text{cov}(p_1, v) = \text{cov}[\lambda_1 (x_1 + u_1), v] \]  
\[ = \lambda_1 \text{cov}(x_1, v). \]  

The second equality is because \( u_1 \) and \( v \) are independent. Since \( \text{cov}(x_1, v) > 0 \) by Proposition 2 and \( \lambda_1 > 0 \) by Proposition 1, we have \( \text{cov}(p_1, v) > 0 \).

To prove \( \text{cov}(p_2-p_1, v) > 0 \), notice that

\[ p_2-p_1 = \lambda_2 w_2 = \lambda_2 (x_2 + y + u_2) = \lambda_2 [x_2(v-s) + \beta_2 (s-p_1) + \theta(s-p_1) + u_2] \]  
\[ = \lambda_2 x_2(v-s) + \lambda_2 (\beta_2 + \theta)x_2 + \lambda_2 (s-\lambda_2) + \theta(p_1 + \lambda_2 u_2) \]  
\[ = \lambda_2 x_2(v-s) + \lambda_2 (\beta_2 + \theta)x_2 + \lambda_2 (s-\lambda_2) + \theta(p_1 + \lambda_2 u_2) \]  
\[ = \lambda_2 x_2(v-s) + \lambda_2 (\beta_2 + \theta)x_2 + \lambda_2 (s-\lambda_2) + \theta(p_1 + \lambda_2 u_2) \]  
\[ = (\lambda_2 x_2 - \lambda_2 \hat{\beta}_2 x_2) + \lambda_2 (\lambda_2 \beta_2 + \theta)(v-s) + \lambda_2 (\beta_2 + \theta)(1-\hat{\beta}_2)s + \lambda_2 u_2 - \lambda_2 (\beta_2 + \theta)u_1. \]
Substituting the values of $\alpha_2, \beta_2, \theta, \alpha_1, \beta_1$ given in Proposition 1 in to the last line yields:

$$p_2-p_1 = \frac{3\lambda_2 + \lambda_1}{2(9\lambda_2 - \lambda_1)} (v-s) + \frac{3\lambda_2}{9\lambda_2 - \lambda_1} s + \lambda_2 u_2 - \frac{2}{3} \lambda_1 u_1.$$ (55)

Therefore,

$$\text{cov}(p_2-p_1, v) = \text{cov}[p_2-p_1, (v-s) + s]$$

$$= \frac{3\lambda_2 + \lambda_1}{2(9\lambda_2 - \lambda_1)} A_0 + \frac{3\lambda_2}{9\lambda_2 - \lambda_1} \Psi_0.$$ (56)

Thus, $\text{cov}(p_2-p_1, v) > 0$ by the second order condition in Proposition 1.

I next prove $\text{cov}(p_1, s) > 0$ and $\text{cov}(p_2-p_1, s) > 0$.

$$\text{cov}(p_1-s) = \text{cov}(\lambda_1 w_1, s) = \lambda_1 \text{cov}[\alpha_1 (v-s) + \beta_1 s + u_1, s] = \lambda_1 \beta_1 A_0 > 0.$$ The last inequality comes from Eq. (51).

$$\text{cov}(p_2-p_1, s) = \text{cov}\left(\frac{3\lambda_2 + \lambda_1}{2(9\lambda_2 - \lambda_1)} (v-s) + \frac{3\lambda_2}{9\lambda_2 - \lambda_1} s + \lambda_2 u_2 - \frac{2}{3} \lambda_1 u_1, s\right)$$

$$\Rightarrow \text{cov}(p_2-p_1, s) = \frac{3\lambda_2}{9\lambda_2 - \lambda_1} A_0 > 0.$$ (55)

**Appendix D. Proof of Proposition 4**

Using Eq. (55), we get:

$$p_2-p_1 = \frac{3\lambda_2 + \lambda_1}{2(9\lambda_2 - \lambda_1)} (v-s) + \frac{3\lambda_2}{9\lambda_2 - \lambda_1} s + \lambda_2 u_2 - \frac{2}{3} \lambda_1 u_1.$$ In addition

$$p_1 = \alpha_1 (v-s) + \beta_1 s + u_1,$$

$$\Rightarrow \text{cov}(p_2-p_1, p_1) = \alpha_1 \beta_1 (3\lambda_2 + \lambda_1) A_0 + \beta_1 \frac{3\lambda_1 \lambda_2}{9\lambda_2 - \lambda_1} \Psi_0 - \frac{2}{3} \lambda_1^2 \sigma_u^2.$$ Notice that the first 2 terms are positive. Thus, if $\sigma_u^2$ is small enough then $\text{cov}(p_2-p_1, p_1) > 0$.

**References**


