Business networking for the purpose of becoming globally more competitive seems to form the very basis of strategic decisions in many companies today. The concept of "network company" has recently been the subject of many studies in the literature, perhaps mostly due to its worldwide practice among successful companies. Yet, there is no formal treatment of the concept per se leading to the development of formal frameworks or models that are instrumental in formulating networking strategies. This paper addresses itself to formalizing the concept of "network company" within the context of global competition. For this purpose, "network company" is positioned in the value chains of pertinent product-market systems and then its functioning is decomposed into a base set of minimal and basic components, which are termed elementary resources, methods, products and activities. The base set thus defined at the detailed level is used to analyze and evaluate "network companies" at any desired condensed level reflecting the needs of a project or a function for the purpose of competitive strategy formulation. The formal analytical framework developed is then discussed in association with three basic approaches to competitive strategy formulation: resource-based strategy, activity-based strategy, and strategy based on the economic theory of the firm. The usefulness of the proposed framework in association with these approaches is expressed in terms of formal propositions.

KEY WORDS: NETWORK COMPANY, VALUE CHAIN, STRATEGIC ALLIANCE, COMPETITIVE STRATEGY, GRAPH THEORY
1. INTRODUCTION

The success of companies is attributed to many factors, but mostly to the process of creating core competencies that are difficult to imitate or duplicate (Prahalad et Hamel, 1990; Porter, 1991, for instance). It is also claimed that, to meet the challenges of globalization, companies must constantly conceive and devise strategies to sustain and even improve their competitive positions. The teaming up of companies to form business networks seems to be a promising competitive strategy since it permits the partner companies to concentrate on those activities of the value chain they perform best and thus every company forming the network maximizes its own added values (Poulin, Montreuil, Gauvin, 1994, Oral and Kettani 1997). The conventional concept of competition that one company competes against the other companies is no longer valid in the world village of globalization. Instead, network against network is the name of the game and therefore the competitive advantage of a company is largely determined by the competitive advantage of the network the company belongs to. We are in fact observing a transition from an economy of mass to an economy of value. In this context, it is essential for companies to develop their missions around their core competence areas and establish their business networks accordingly.

There are basically three approaches to competitive strategy formulation in the literature: resource-based strategy, activity-based strategy and strategy based on the economic theory of the firm. Mahoney and Pandian (1992) convincingly argue that the resource-based approach incorporates concepts from the mainstream strategy view. They observe that distinctive competencies as defined by Andrews (1971), Ansoff (1965) and Selznick (1957) are a fundamental component of the resource-based view. They also claim that the resource-based approach can be considered a fifth branch of the organizational economics tree of knowledge.
along with positive agency theory (Eisenhardt, 1989), property rights (Alchian, 1984), transaction cost economics (Williamson, 1985) and evolutionary economics (Nelson and Winter, 1982). Collis (1991), on the other hand, critically examines the contribution of the resource-based view of the firm to global competition in particular and to strategic management in general. Through a detailed field-based case study of three firms in the worldwide bearings industry, he concludes that the resource-based view of the firm complements economic analysis and that both are essential to a complete understanding of global strategy.

The activity-based view of the firm is mostly due to Michael Porter (1980, 1985). According to Porter, the sources of competitive advantage mainly center around activities, because a firm is basically a collection of interrelated economic activities of different sorts. In essence, therefore, a firm’s strategy defines its configuration of activities and how they interrelate. The basic unit of competitive advantage is then an activity, and it is activities that determine relative cost, buyer value, and hence differentiation. Porter (1991) arrays the activities in a firm in what he terms the value chain and value system, where the term value refers to customer value, from which potential profit ultimately derives. In this context, a firm’s strategy is manifested in the way in which it configures and links the many activities in its value chain relative to competitors. Porter (1991) claims that the resource-based view of the firm cannot be an alternative theory of strategy, because strategy cannot be separated from the cross-sectional determinants of competitive advantage and from the conception of a firm as a collection of activities. Furthermore, he states that resources are not valuable by themselves, but because they allow firms to perform activities that create advantages in particular markets. For him, resources and activities are, in a sense, duals of each other. These statements by Porter (1991) suggest that it should be possible to establish an explicit link between resources and activities.
The economic theory of the firm is well established. An account of its basic results which fits naturally with the approach taken in this paper is found in Naylor and Vernon (1969). The concept of "network company" has recently been the subject of many studies in the literature, perhaps mostly due to its worldwide presence among successful companies (Poulin, Montreuil, Gauvin ; 1994, Miles and Snow, 1995 ; Biemans, 1996). Yet, there is no formal treatment of the concept per se leading to the development of formal frameworks or models that are instrumental in formulating networking strategies to improve and sustain competitive advantage. Biemans (1996) observes that the concept of a network, to the majority of firms, remains fuzzy and difficult to implement.

This paper addresses itself to formalizing the concept of "network company", through an analytical framework based on the concepts of value chain and value system of Porter (1985), within the context of global competition. For this purpose, "network company" is positioned in the value chains of pertinent product-market systems and then its functioning is decomposed into a base set of minimal and basic components, which are termed elementary resources, methods, products and activities. The base set thus defined at the detailed level is used to analyze and evaluate a "network company" at any desired condensed level reflecting the needs of a competitive strategy formulation project.

The organization of the paper is as follows: Section 2 presents definitions and notations that are necessary to develop the concept of network company in terms of its elementary components; namely, durable resource, method, activity and product. The concepts of product-market chains, condensed chain and resource attribute are also developed in this section. These definitions and notations are then used, in Section 3, to define single and multi-chain companies and business ventures. Employing the notations and formulations developed, Section 4 presents
certain propositions than link different approaches used in the competitive strategy area. Section 5 provides a formal definition of a network company. Section 6 concludes the paper with some remarks and suggestions for the future.

2. BASIC DEFINITIONS AND NOTATIONS

The primary objective of this section is to provide definitions and to develop notations that are necessary to formalize the network company concept and to provide instruments leading to the development of analytical models for strategic networking decisions. Our modeling constructs are based on the «comparative static’s» assumption which lies behind the neoclassical theory of the firm (Naylor and Vernon, 1969). That is, we assume that the enterprise examined is in a state of equilibrium so that it is sufficient to model it for a fixed time horizon (say a year). The limitations inherent to this assumption are discussed at the end of the paper.

**Elementary Level Concepts**

The term "product-market chain" is used to mean a system of technological and managerial requirements that are necessary to produce and deliver a consumer product to a market. It is, in a sense, an abstraction or a blue-print of technological and managerial know-how that are needed to transform raw materials into finished consumer products to satisfy the needs of certain groups of customers. If a company is active in the manufacture of more than one consumer product, then one needs to study all the related product-market chains. A product-market chain requires the definitions of four basic components: elementary durable resources, elementary methods, elementary activities and products (including initial inputs, intermediary products and final outputs).
Elementary Durable Resource

The resources that are used on a regular basis to transform inputs into outputs are called *durable* resources. A durable resource is called elementary durable resource if it is at the most detailed meaningful level that is required. According to this definition, although inputs and outputs are also resources, they are not included in the set of durable resources for they are in the production pipe-line on a temporary basis. Note, however, that some durable resources may, at times, also be inputs and outputs. For example, a specific machine tool is a durable resource when it is used to transform materials, but it is the input and the output of a maintenance process. There are many ways according to which durable resources can be classified: for instance, human versus non-human, tangible versus intangible, monetary versus non-monetary, internal versus external, and the like. For the purpose of conceptualizing network companies, we simply refer to them as durable resources and we attach a label to each one of them. The index used for durable resources is \( r = 1, 2, \ldots, R \), assuming that there are \( R \) distinct resources. The set of all distinct durable resources is denoted by \( \mathbf{R} \), the amount, under some appropriate metric, of durable resource \( r \) available for the horizon considered by \( x_r \), and the column vector of these amounts by \( \mathbf{x} \).

Elementary Method

An elementary method is a procedure or technique that is associated with a minimal set of durable resources in order to transform a certain group of inputs into a certain group of outputs. It is in a sense a part of technological and managerial know-how in use. Although an elementary method is not mentioned while discussing durable resources, it is in fact an intangible resource that is fundamental to the definition of product-market chain.
An elementary method associated with a set of durable resources is assumed to be unique and optimally chosen. The uniqueness and optimality of the chosen elementary method is quantity related: the quantities of inputs and durable resources used to produce a certain quantity of output can change only if the elementary method is changed. In other words, for a given combination of durable resources we may have several elementary methods with different resource quantity requirements. The index used for elementary methods is \( m = 1, 2, \ldots, M \), and the set of all distinct methods is denoted by \( M \). Each method \( m \in M \) is characterized by a vector valued function \( f_m(\ ) \) which relates output quantities to input and durable resource quantities. This function is conceptually similar to the production functions used in economics and the transfer functions used in system dynamics; that is \( \text{output} = f_m(\text{input}) \).

**Elementary Activity**

An elementary activity is a minimal set of durable resources, with an associated elementary method, used to convert a group of inputs into a group of outputs. The relationship between elementary method and elementary activity is one-to-one and unique. The index used for elementary activities is \( a = 1, 2, \ldots, A \), assuming that there are \( A \) distinct elementary activities. The set of all distinct elementary activities is denoted by \( A \). The set of the elementary durable resources associated with elementary activity \( a \) is denoted by \( R_a \subset R \), and its method by \( m_a \in M \). In Figure 1, for example, elementary activity \( a = 5 \) consists of Elementary Resource 7 and 8 and Elementary Method \( m_5 = 4 \). It uses Products 6, 7, 8 and 11 as inputs and produces Products 12, 13 and 14. An activity is realized when its durable resources use its method to convert inputs into outputs. The family\(^1\) \( \{ R_a, m_a \} \) of the durable resources and method

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\(^1\) In this text, we refer to a set of sets as a family.
performing an activity is referred to as an *actor*. In Figure 1, actor $a = 5$ is thus characterized by the family $\{ R_5, m_5 \} = \{ \{7, 8,\}, 4 \}$. Note that a given durable resource can be used in more than one activity. The set of all the elementary activities in which a durable resource $r$ is involved is denoted by $A_r \subset A$.

**FIGURE 1: PRODUCT-MARKET CHAIN**

The definition given involves an explicit association of actors (durable resources and method) to activities. When modeling a product-market chain, one must however define some convenient boundaries outside of which the actors performing the activities are either unknown or of no interest. These *external* activities may however be the source of an initial input or the destination of a final output of the product-market chain, and they need to be represented. In what follows, external activities are included in the set $A$ of activities, but their durable resource set $R_a$ and method $m_a$ are not specified. Also, when they are a source or a sink for the product-
market chain, they either have no inputs or no outputs. The set $A$ can thus be partitioned into the set $A^I$ of internal activities and the set $A^E$ of external activities. $A^E$ is the union of the external input and external output activity sets $A^i$ and $A^o$. The amount of durable resource $r$ used by an internal activity $a$ during the time horizon considered is denoted by $x_{ra}$, and the matrix of all the $x_{ra}$’s by $X$. Clearly, these amounts must satisfy the relation

$$\sum_{a \in A^I} x_{ra} \leq x_r, \quad \forall r \in R$$

(1)

**Product**

Products are the outputs of elementary activities. In a product-market chain, an elementary activity uses the products of the immediately preceding elementary activities as its inputs, and provides products for the immediately succeeding activities as their inputs. The last products on the product-market chain are the consumer products delivered to market, whereas the first ones are mostly raw materials and basic components. Note that products are not necessarily physical. They may be information, knowledge, service, etc. Note also that the products associated with some support activities may be elementary durable resources (the product of a training activity, for example) or even methods (the product of a design activity). To avoid any confusion, when a resource can be simultaneously an elementary durable resource (or a method) and a product, we give it a distinct label for each of its roles. Normally, primary activities associated to inbound logistics, production, outbound logistics, marketing, sales and service (Porter, 1985) do not have products which are simultaneously durable resources or methods.

The index used for products is $p = 1, 2, \ldots, P$, where it is assumed that there are $P$ products on the product-market chain of interest. The set of products on a given product-market
chain is denoted by $P$. To distinguish input products from output products, we associate two sets with an elementary activity $a$: the set of input products, $X_a$, and the set of output products, $G_a$. The quantity of the input product $p \in G_a$ used by activity $a$ in the time horizon considered is denoted by $y_{ap}$ and the quantity of the output product $p \in X_a$ by $y_{a,p}$. The set of all the products associated with activity $a$ is $P_a = P_a^i \cup P_a^o$ and, clearly, $P_a \subset P$. For example, in Figure 1, we have $P_5 = \{6, 7, 8, 11, 12, 13, 14\}$, $X_5 = \{6, 7, 8, 11\}$, $P_5 = \{12, 13, 14\}$. The set of all the activities in which product $p$ is either an input or an output is denoted by $A_p = A_p^i \cup A_p^o$, where $A_p^i$ is the set of the activities with $p$ as an input and $A_p^o$ is the set of the activities with $p$ as an output. For an elementary activity $a$, the family of resources formed by the set of its durable resources, the set of its products and its method is defined by $E_a = \{R_a, P_a, m_a\}$. In Figure 1, for example, we have $E_5 = \{R_5, P_5, m_5\} = \{7, 8, \}, \{6, 7, 8, 11, 12, 13, 14\}, 4\}$. The family of the three types of resources in a product-market chain is denoted by $E = \{R, P, M\}$.

**Modeling Constructs**

Based on the definitions and concepts developed in the previous section, formalizations of product-market chain, hierarchical condensation and resource attribute can now be provided.

**Product-Market Chain**

A complete product-market chain is illustrated in Figure 1. As can be observed from this figure, a product-market chain shows, in a sequential order, the elementary activities which are required to produce and market a given consumer product in a specific market. It starts with raw material procurements and ends with marketing of the consumer products. Let the triplet $(i, j, p)$ be an arc identifier in a product-market chain and let $y_{ijp}$ denote the amount of product $p \in P$
passed from node i to a node j during the relevant time horizon. When an activity \( a \in A^I \) is considered, its input-product quantity \( y_{ap} \) is simply the sum of the amount \( y_{iap} \) of product p provided to a by all its sources i \( (y_{ap} = \sum_i y_{iap}) \), and the vector of all the inputs \( y_{iap} \) of activity a is denoted by \( y^i_a \). In Figure 1, for example, we have \( y_{*,12,23} = y_{14,12,23} + y_{10,12,23} \) and the corresponding column vector \( y^1_{12} = [y_{14,12,23}, y_{10,12,23}] \). Similar relationships exist between output quantities \( y_{ajp} \) and the output column vector of activity \( a \) is \( y^o_a \).

Some remarks regarding product-market chains are now in order. An arc in a chain must not be interpreted as a move of a product from one activity to another. A move always involves an activity, such as material handling, transportation or electronic transmission, changing the position of a product in space and it must therefore be associated with a node. Arcs here simply model the fact that at some point in time a product resulting from an activity is taken charge of by another activity. Activities fall into one of the three main categories: (1) time activities, such as stores or files, which preserve products in time, (2) space activities, such as delivery or communication services, which move products in space, and (3) form activities, such as assembly operations, order processing or decision making, which change the nature or status of the input products. This leads to the eventual partitioning of \( A^I \) into three sub-sets: \( A^I_T \), the set of time-activities; \( A^I_S \), the set of space-activities and \( A^I_F \), the set of form-activities. The activities in \( A^I_F \) have different products on their input and output arcs but the activities in \( A^I_T \) and \( A^I_S \) usually have the same products on their input and on their output arcs, and the same input and output quantities (i.e. \( y^i_a = y^o_a \)). Each activity has its own method however, and these methods are characterized by the vector-valued functions

\[
f_{m_a}(y^i_a, x_a, y^o_a) = 0, \quad \forall \ a \in A^I
\]
where $x_a$ is the column vector of the durable resources that activity $a$ uses to transform the inputs $y_{a,i}$ into the outputs $y_{a,o}$ by employing methods $m_a$.

Product-market chains are not simply conceptual descriptions of technological and managerial know-how needs, but they can also reflect the physical deployment of activities. If a company uses several similar assembly lines at different locations, for example, although they may conceptually all be considered as a single activity, in a product-market chain, each of them could be represented explicitly and given different activity and method numbers. This can be seen by comparing the chains in Figures 1 and 2. In Figure 1, activity 5 is implemented in a single location. In Figure 2, this activity is represented twice with two different activity numbers (5 and 17) because it is physically implemented in two different locations. In both cases the same input products (6, 7 and 11) and durable resources (7 and 8) are used but the methods and the outputs are not exactly the same.

Hierarchical Condensation

The detailed view of a product-market chain as described above is most useful for operational and engineering decisions. A more aggregated view may be of more interest or use under other circumstances. For instance, formulating a competitive strategy might necessitate an aggregate view of the strategic activities of a company, rather than a detailed view. In what follows we generalize the definitions and their corresponding notations to permit modeling at an higher aggregation level and we show that, through a condensation process, one can transform a detailed model into a more synthetic representation. The degree of aggregation is rather arbitrary and flexible. It is determined by the level and nature of decision making.

At the highest aggregation level, a product-market chain could be considered as a single economic activity. In most applications, however, the level of aggregation is somewhere between
a single economic activity and the detailed level shown in Figures 1 and 2. The notations introduced are in fact adequate for any aggregation level and passing from one aggregation level to another can be done recursively in a relatively simple manner. As far as terminology is concerned, the aggregate concepts are the same as the elementary level concepts, but without the "elementary" qualifier, i.e. we talk simply of durable resources, methods and activities. The main idea behind this generalization is the fact that what is modeled as an actor and a durable resource depends entirely on the mandate and perceptions of the analyst in the context of a specific project, and the fact that when the model obtained at a given level is condensed into a higher level model, the actors in the lower level model become the durable resources in the higher level model.

Let $R = A^I_{elementary}$, i.e. define the set of durable resources for the condensed representation equal to the set of internal elementary activities. Now suppose that $A$ meaningful condensed activities are defined by integrating the activities of a number of elementary actors. For each condensed internal activity $a \in A^I$, this results in the definition of a new set of durable resources $R_a \subseteq R$, an integrative method $m_a$ and a new set of products $P_a$ composed only of the products which are inputs or outputs of the condensed activity. The condensation of the elementary chain is the new chain obtained by grouping all the condensed activities. Figure 3 presents a condensation of the product-market chain given in Figure 2, according to the shaded line groupings.
\[ R = \{1, ..., 24\}, \quad P = \{1, ..., 25\}, \quad M = \{1, ..., 12\}, \quad A^E = \{1, 13, 14, 15, 16\}, \quad A^I = \{2, 3, ..., 12, 17\} \]

**FIGURE 2: DEPLOYED PRODUCT-MARKET CHAIN**

\[ R = A^I_{\text{elementary}} = \{2, 3, ..., 12, 17\}, \quad P = \{1, ..., 11, 15, 16, 18, ..., 26\} \subset P_{\text{elementary}}, \]
\[ M = \{1,2, ...,6\}, \quad A^E = \{1, 8\}, \quad A^I = \{2, 3, ..., 7\}. \]

**FIGURE 3: CONDENSED PRODUCT-MARKET CHAIN**

Note that the integrative methods associated to the activities in the condensation implicitly incorporate the functionality of the elementary methods of the original activities as
well as the functionality previously described by the intra-activity arcs which have disappeared in the aggregation process. By induction, it should be clear from this discussion that, provided that the resources are renumbered from 1 to $R$ and the products from 1 to $A$, the concepts and notation introduced can be used at any aggregation level, and that links between levels can be formally established.

It should also be clear that several levels of condensation may be useful for analysis. For example, if one decides to group activities by facilities (plants, warehouses, stores, ...) the condensation obtained would be a product supply chain (Cavinato, 1992; Shapiro, 1993) of the type encountered in logistic studies. If one decides to group activities by companies, the condensation obtained would be a product-market industry map. These examples show that a product-market chain can be seen as a formation of industrial and business units from the viewpoint of a particular consumer product.

**Durable Resource and Product Attribute**

The concepts defined so far provide the skeleton necessary to study network companies but the notations introduced are only indices, sets of indices and quantity variables. To describe, evaluate, and design network companies one needs to work with costs, prices, characteristics, constraints and the like. This necessitates the introduction of what is called an attribute. The attribute of a durable resource, of an actor or of an intermediary or final consumer product is a set of observable and measurable characteristics that is essential in evaluating internal and external performance of network companies. A given durable resource, an actor or a product might have several value characteristics. For the purpose of analyzing network companies, we shall consider two sets of attributes: the first set is associated with the durable resources and the actors, and the second with the input and output products on the product-market chain. The set of
the attributes associated with durable resources and actors is called capability attribute set and it includes attributes such as costs, availability, quality and flexibility. As already explained, the amount of durable resource $r$, $x_r$, available at some point in time imposes capacity constraints, and the acquisition, leasing, renting or disposal of these resources may be limited by market conditions. The cost of using or owning $x_r$ units of resource $r$ during the time horizon considered can take various forms represented here by the cost function $c_r(x_r)$, the vector of these functions being denoted by $c(x)$. The amount of each resource used by an activity $a$ is specified by the vector $x_a$, and it is assumed that the price $u_a$ paid by activity $a$ for the use of these resources is based on a cost sharing mechanism described by the implicit composite vector valued function

$$C(c(x), X, u) = 0 \quad (3)$$

Attributes of resources (actors) such as technological capability, know-how, reliability and flexibility (capacity to customize a product, to adapt to volume fluctuations, etc.) shape the method $m_a$ of an activity $a$, and determine the form of its input-output function $f_{m_a}(y^i_a, x_a, y^o_a) = 0$. Over time, the usefulness and contributions of these attributes build the reputation of the activity which in itself may create value.

Attribute set characterizing products on a product-market chain, on the other hand, is called marketability attribute set and includes attributes appealing to customers such as product design, availability, price, quality (reliability and conformity with specifications), security, delivery time (duration and reliability) and after-sale service. These attributes are associated with the arcs $(i,j,p)$ of the product-market chain, and not to the products per se. This is due to the fact that most of the marketability attributes depend largely on what the clients $j$ want and on what the producers $i$ deliver. The quantities involved are denoted by the variables $y_{ijp}$, and the price paid for product $p$ depends on the extent to which producer $i$ is able to deliver the attributes client
j wants, as well as on the quantities involved. The resulting revenues are modeled by value functions $v_{ijp}(y_{ijp})$ and hence the average unit price paid for the product associated with arc $(i,j,p)$ is $v_{ijp}(y_{ijp})/y_{ijp}$. The function $v_{ijp}(y_{ijp})$ can take several forms depending on the nature of the source and the destination. If an arc starts with an external supply source or ends with an external demand destination, for example, its value function may increase up to a certain threshold after which it starts declining. The quantities on arcs involving external destination points may also be limited by market conditions. Although different, capability and marketability attribute sets are complementary to one another in the sense that the value of a product on an arc also depends on the attributes of the source activity, and in particular on its reputation.

Several activity performance measures such as productivity, efficiency and effectiveness may be derived from the concepts introduced. For our purpose in this paper, the main performance measure used is the value added by an activity. Let $v_a^i(y)$ and $v_a^o(y)$ be the value vectors for the input and output quantities $y_a^i$ and $y_a^o$, respectively, for activity $a$. Then the value added, $v_a$, by activity $a$ is given by the relation

$$v_a = e v_a^o(y) - e v_a^i(y) - u_a$$

where $e$ is a row vector of 1’s of the same dimension as the column vector by which it is multiplied so that, for example, $e v_a^o(y)$ gives the total value of all the outputs of activity $a$.

**Graph Theoretic Representation**

To provide a formal definition of a network company, we need some concepts from graph theory: directed multigraph\(^2\), network, sub-graph and bipartite graph. When durable

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\(^2\) A directed multigraph $G = (V, X)$ consist of a finite nonempty set $V = V(G)$ of nodes (or vertex) together with a prescribed bag $X = X(G)$ of ordered pairs $x = (v, w)$ of distinct nodes. A bag is a collection of objects which may
resources and methods are not taken into account, the product-market chains illustrated in Figures 1 to 3 can all be represented by a **directed multigraph** \( G \), with nodes defined by the set of activities and arcs defined by their input and output products. Associated with each of the arcs \((i, j, p)\) of \( G \), there is a quantity \( y_{ijp} \) and a value function \( v_{ijp}(y_{ijp}) \). Such graphs are usually referred to as **networks**.

Now, consider a directed multigraph \( G = (V, X) \) and the subset \( V_s \subset V \). Then \( G(V_s) = (V_s, X(V_s)) \) is a graph obtained by suppressing the nodes belonging to \( V_s \) and the arcs associated with them. The set of arcs of this graph is \( X(V_s) = \{(v, w) \in X : v, w \in V_s\} \) and evidently \( X(V_s) \subset X \). By definition, \( G(V_s) \) is the **sub-graph** prompted by the subset \( V_s \) of the nodes of \( G \).

The other concept needed is that of **bipartite directed graph**\(^3\). In fact, a product-market chain can be formally represented by the bipartite directed graph \( B = (E, A; L) \) as shown in Figure 4. Note that in this model, the node subset \( E \) is the family of the durable resources set \( R \), the products set \( P \) and the methods set \( M \). The specification of the set of arcs \( L \) follows directly from our definition of an activity. More specifically, the arcs incident to the node \( a \) associated with internal activity \( a \) are specified as follows: (i) an arc \((r, a)\) is defined for all \( r \in R_a \), (ii) an arc \((p, a)\) is defined for all \( p \in A^1_a \) and an arc \((a, p)\) for all \( a \in A^o_a \) and, (iii) an arc \((m, a)\) is defined for \( m = m_a \). The arcs incident to the nodes \( a \in A^E \) are specified by defining an arc \((p, a)\) for all \( p \in P^1_a \) and an arc \((a, p)\) for all \( p \in P^o_a \). Observe, that the sets \( A_r \) and \( A_p \) defined in the previous sections are the set of the activities at the other end of the arcs incident to a node \( r \in R \)

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\(^3\) A **bipartite directed graph** \( B = (V, W; L) \) is a graph whose node set \( V(B) \) can be partitioned into two subsets \( V \) and \( W \) such that every arc of \( B \) has its source in one subset and its sink in the other, i.e. such that for any arc \((v, w)\) in \( L \), if \( v \in V \) then \( w \in W \), and vice-versa.
or a node $p \in P$. Similarly, the family $E_a$, includes the set of the durable resources, products and methods, respectively, which are at the other end of the arcs incident to a node $a \in A$.

\[
\begin{align*}
\text{Resources} & \quad \text{Activities} \\
R_1 & \quad 1 \\
\vdots & \quad \vdots \\
R_r & \quad 2 \\
R & \quad \vdots \\
\end{align*}
\]

\[
\begin{align*}
\text{Products} & \quad \text{Methods} \\
P_1 & \quad 1 \\
\vdots & \quad \vdots \\
P_p & \quad 2 \\
P & \quad \vdots \\
\end{align*}
\]

\[
\begin{align*}
\text{Methods} & \quad \text{Resources} \\
M_1 & \quad R_{1} \\
\vdots & \quad \vdots \\
M_m & \quad R_r \\
M & \quad \vdots \\
\end{align*}
\]

\[
E = \{R, P, M\} \quad L \quad A
\]

**FIGURE 4: BIPARTITE GRAPH OF A PRODUCT-MARKET CHAIN**

The definition of sub-graph can be extended to the case of bipartite graphs. Consider the bipartite graph $B = (E, A ; L)$, where $E$ and $A$ are the sets of nodes forming the two parts of the graph $B$ and $L$ is the set of arcs between $E$ and $A$, as well as the subsets $E_s \subset E$ and $A_s \subset A$.

By definition, the bipartite sub-graph $B(E_s, A_s) = (E_s, A_s ; L(E_s, A_s))$ is obtained by suppressing the nodes belonging to $E_s$ and $A_s$ and the arcs $(e, a)$ which are their incidents. The arcs $L(E_s, A_s)$ of bipartite sub-graph $B(E_s, A_s)$ are then defined as: $L(E_s, A_s) = \{(e, a) \in L : e, a \in E_s \cup A_s\}$.
With these definitions and notations, we can now provide a formal characterization of a network company.

3. COMPANIES AND BUSINESS VENTURES

Single-Chain Company

A company is a *legally constituted* organizational system owning and managing a subset of resources, and performing a subset of activities, of one or several product-market chains. Figure 5 illustrates an example of a company and its activities on the particular product-market chain given in Figure 1. As can be observed from Figure 5, when compared to Figure 1, the company is performing only a subset of the activities on the product-market chain shown in Figure 1; namely, activities 5, 6, 7, 8, 9 and 10, and leaving activities 2, 3, 4, 11 and 12 outside the domain of its operations.

Note that the fact that an activity is performed by a company does not imply that all the resources involved in the activity are owned by the company. For example, suppose that the Elementary Resource 7 in Activity 5 is an expensive equipment required only from time to time by the company. Then this equipment could be rented when needed. In Figure 5, the symbols of the durable resources, products and methods that are not owned by the company are shown in shaded forms. Note also that some of the company resources may be used by other companies. In the example, activities 2 and 11 are not under the control of the company but they use resources 2 and 21 belonging to the company.
The definition of a single-chain company can be formalized as follows. Let $R_f$, $A_f$, $M_f$ and $A_f^I$ denote the sets of internal durable resources, methods, activities and products of the firm (company) $f$, and $A_f^E$ the set of activities in $A \setminus A_f^I$ which are adjacents to one of the activities in $A_f^I$, meaning they either supply to the company or purchase from the company. These activities are considered the external activities of firm $f$. Figure 5. illustrates these various notations. Assuming that the company is active only in product-market chain $B$, one can state that $E_f \subseteq E$ and $A_f \subseteq A$, where $E_f = \{ R_f, P_f, M_f \}$ and $A_f = A_f^I \cup A_f^E$. Then, under the single-chain assumption, company $f$ is defined as the bipartite sub-graph $B_f = B(E_f, A_f) = (E_f, A_f ; L(E_f, A_f))$ of $B$. 

**FIGURE 5: A COMPANY IN A PRODUCT-MARKET CHAIN**

$P_f = \{ 12, 13, \ldots, 21, 22 \} \subset P$; $M_f = \{ 5, 6, 7, 8, 9 \} \subset M$; $R_f = \{ 2, 8, 9, 11, 12, 13, \ldots, 18, 20, 21 \} \subset R$; $A_f^I = \{ 5, 6, 7, 8, 9 \}; A_f^E = \{ 16, 17, 18, 19 \} \subset A \setminus A_f^I$
Multi-Chain Company

Although we have concentrated on only one product-market chain, the approach can be repeated as many times as needed to cover the domain of the companies involved in several chains. To provide a formal definition of a multi-chain company we introduce the concept of *industrial trellis* formed by several product-market chains and/or companies. When an industrial trellis is considered, $A$ and $E = \{R, P, M\}$ is used to denote, respectively, the activities and the resources of all the product-market chains and companies of the trellis $B = (E, A ; L)$ of interest. Small modifications in the notations introduced are then needed to be able to refer to a specific chain or firm in the trellis. To deal with several product-market chains, it is sufficient to attach an index, say $c$, to the notations introduced earlier, thus denoting the activities, the durable resources, the products and the methods of product-market chain $c$ by $A_c \subset A$, $R_c \subset R$, $P_c \subset P$ and $M_c \subset M$. Then product-market chain $c$ is represented by the bipartite graph $B_c = B(E_c, A_c) = (E_c, A_c; L(E_c, A_c))$, where $E_c = \{R_c, P_c, M_c\}$.

Let $C_f \subset C = \{1, 2, \ldots, c_{\text{max}}\}$ be the set of product-market chains in which firm $f$ is active, and let the subsets $R_f \subset R$, $P_f \subset P$, $M_f \subset M$, and $A_f \subset A$ represent, respectively, all the durable resources, methods, activities and products of the firm. It should be noted here that these sets include resources and activities, such as strategic management activities, which might not belong to any of the chains. When all these resources and activities are taken into consideration, a multi-chain company is defined by the bipartite graph $B_f = B(E_f, A_f) = (E_f, A_f; L(E_f, A_f))$. The bipartite graph $B_f$, like the bipartite graph $B_c$, has the characteristics illustrated in Figure 4, but it is defined in association with the totality of the industrial trellis spanned by the firm under study.

Given these definitions, one can identify the resources, activities, methods, of a particular firm, say $f$, belonging to the product-market chain $c \in C_f$, as:
This part or division of the firm is represented by the bipartite graph $B_{fc} = B(E_{fc}, A_{fc}) = (E_{fc}, A_{fc}; L(E_{fc}, A_{fc}))$. It should be noted that the bipartite graphs $B_{fc}, c \in C_f$, are not necessarily mutually exclusive. Given that the company might have resources and activities outside of the chain, one should also observe:

$$R_{fc} = R_f \cap R_c; \quad P_{fc} = P_f \cap P_c; \quad M_{fc} = M_f \cap M_c; \quad A_{fc} = A_f \cap A_c; \quad E_{fc} = \{R_{fc}, P_{fc}, M_{fc}\}.$$  

**Business Ventures**

Companies can be considered as formal business ventures defined over an indefinite time horizon. Several other forms of formal, or informal, business ventures, defined over shorter and more specific time horizons, are found in practice. These include gentleman agreements between firms, contract between parties over specific projects, legally defined alliances to share strategic activities, etc. The business venture can take the form of a subcontracting relationship or of a more involved joint venture defined over a number of product-market chains and/or a number of strategic activities. The concepts introduced for a company can be generalized to model any kind of business venture. Since a given firm $f$ could be involved in, or consider its participation in, several business ventures, we use the index $b$ to represent business ventures. When we consider a business venture $b$ involving a set of product-market chains $C_b \subset C$, the relations defined in the previous section are still valid when the index $f$ is replaced by $b$. Also, if company $f$ is involved in business venture $b$, for each chain $c \in C_b$, the following relations hold:

$$R_{fc} \subset R_{bc} \subset R_c, \quad P_{fc} \subset P_{bc} \subset P_c, \quad M_{fc} \subset M_{bc} \subset M_c, \quad \text{and} \quad A_{fc} \subset A_{bc} \subset A_c.$$
In addition, the durable resources, methods, activities, and products of company \( f \) which are engaged in business venture \( b \) are respectively denoted by

\[
R_{fb} = R_f \cap R_b; \quad P_{fb} = P_f \cap P_b; \quad M_{fb} = M_f \cap M_b; \quad A_{fb} = A_f \cap A_b; \quad E_{fb} = \{ R_{fb}, P_{fb}, M_{fb} \}.
\]

For our purposes, a business venture is any kind of agreement defined over a given time horizon which specifies value functions \( v_{ijp}(y_{ijp}) \) for all the arcs of the business venture network, as well as a durable resources cost sharing function \( C_b(c(x), X, u) = 0 \) which covers all the durable resources and activities in the bipartite graph \( B_b = B(E_b, A_b) \) of the business venture. Business ventures are the end result of business networking activities and their analysis is central to the conceptual framework presented here.

4. BASIC PROPOSITIONS AND RELATIONS

As we have seen, graph theory provides a natural language to model product-market chains, companies, industrial trellis and business ventures. In this section, the analytical power of graph theory is used to show how formal links can be established between bodies of literature which have been developed separately from one another and which are often considered as competing paradigms, such as resource-based, activity-based and economic theories of the firm. These competing views of the firm, are related to each other through formal propositions. For this, however, we need to introduce the concept of reducing a bipartite graph to a simple graph.
a) Bipartite Graph $B = (E, A; L)$

b) Directed reduction of $B$ with respect to $A$

c) Directed reduction of $B$ with respect to $E$

FIGURE 6: DIRECTED REDUCTION EXAMPLE
Consider an arbitrary bipartite graph $B = (E, A ; L)$ such as the graph shown in Figure 6a. (in this example, the set of resources $E$ includes only products (i.e. $E \equiv P$)). For each node $a \in A$, one can define subsets of nodes in $E$ which have direct links with $a$. Taking arc orientations into account, this leads to the following set definitions:

$$E_a^i = \{ e \in E : (e, a) \in L \}; \quad E_a^o = \{ e \in E : (a, e) \in L \}; \quad E_a = E_a^i \cup E_a^o.$$  

Similarly, for each $e \in E$, we have:

$$A_e^i = \{ a \in A : (a, e) \in L \}; \quad A_e^o = \{ a \in A : (e, a) \in L \}; \quad A_e = A_e^i \cup A_e^o.$$  

The sets $E_a, a \in A$, are subsets of $E$ and their family can be denoted by $E$. Similarly, the sets $A_e, e \in E$, by $A$. We note that the pair $H_E = (E, E)$ and the pair $H_A = (A, A)$ form hypergraphs (Berge, 1983) and that $H_A$ is the dual of $H_E$.

The directed reduction of $B$ with respect to $A$ is obtained by defining a directed multigraph that has $A$ as its set of nodes and, for each $e \in E_a^o \cap E_j^i, i \neq j \in A$, a directed arc between the nodes $i$ and $j$. This is illustrated in Figure 6b. To distinguish the multiple arcs between a given pair of nodes $(i, j)$ we attach a specific label $e \in E_a^o \cap E_j^i$ to each arc, so that the triplets $(i, j, e)$ provide a unique arc identifier. The set of arcs $(i, j, e)$ thus defined is denoted by $X_E$. In what follows, the directed reduction of $B = (E, A ; L)$ with respect to $A$ is denoted by $\tilde{G}_A[B] = (A, X_E)$ and that with respect to $E$ by $\tilde{G}_E[B] = (E, X_A)$. A directed reduction with respect to $E$ is shown in Figure 6c. Again, $\tilde{G}_A[B]$ and $\tilde{G}_E[B]$ can be considered as duals of one another. Note that in the example, Figure 6b is an activity network and Figure 6c a bill of materials. The nature of directed reductions in the context of network companies is made more explicit below.
A reduction of \( B \), on the other hand, with respect to \( E \) is done through a definition of a simple graph that has \( E \) as its set of nodes and that has an undirected arc between each pair of nodes \((i, j), i \neq j \in E\), such that \( A_i \cap A_j \neq \emptyset \). The set of arcs thus defined is denoted by \( Y_A \). In what follows, the reduction of \( B = (E, A; L) \) with respect to \( E \) will be denoted by \( G_E[B] = (E, Y_A) \) and that with respect to \( A \) by \( G_A[B] = (A, Y_E) \). Note that \( G_E[B] \) and \( G_A[B] \) can be considered as simple graph representatives of the hypergraphs \( H_E \) and \( H_A \), respectively, and that, again, they are the dual of one another.

With the above concepts and notations, one can formulate several propositions. However, we shall concentrate on four of them to illustrate the usefulness of the framework developed in the paper. More specifically, the propositions considered here demonstrate that: (1) when sub-graphs of \( \tilde{G}_A[B] \) or \( H_A \) are opted for, an activity-based view of the firm is chosen, (2) when sub-graphs of \( \tilde{G}_E[B] \) or \( H_E \) are chosen, a resource-based view of the firm is preferred, and (3) these views are directly related to the economic theory of the firm. Now follows the propositions.

**Proposition 1:** The value chain of Porter (1985) for company \( f \) can be formalized by a network of activities through associating the value functions \( v_{ijp}(y_{ijp}) \) with the arcs \((i, j, p)\) of the directed multigraph

\[
G_f = \tilde{G}_A[B(P_f, A_f)] = (A_f, X_P).
\]

**Proposition 2:** The value system of Porter (1985) for a company can be formalized by a network of activities through associating the value functions \( v_{ijo}(y_{ijp}) \) with the arcs \((i, j, p)\) of the directed multigraph

\[
\]
where $B$ is the industrial trellis spanned by the company.

These propositions simply mean that the value chain and system of the firm are directed reductions, with respect to activities, of the subgraph of a company or industrial trellis obtained by neglecting durable resources and methods. The directed multigraphs $G$ and $G_f$ correspond to the systems represented respectively in Figures 1 and 5, when the resources and methods are not taken into account. In that sense, it can be said that value chains and systems are reductive conceptual frameworks. It should be clear, however, that no serious analysis of the strategic activities of the firm can be done without considering their durable resources and methods. Proposition 1 could naturally be extended to the case of any business ventures.

**Proposition 3:** Let $R^H \subset R$ be the set of human resources of the firm or of the business venture $B$. Then the structural graph,

$$S_h = G_{R^H \cap} [B(R^H, A^1)] = (R^H, Y_{A^1})$$

is a social network induced by the work relations in the firm or in the business venture.

This is a proposition that establishes the social structure in an organization in terms of the links between human resources induced by the activities of a company or business venture, i.e. by the role played by each human resource (Mackenzie, 1986). Proposition 3 can be extended to other types of resources, such as materials, information, machinery and equipment, etc. It shows that the resource-based view of the firm can be considered as a reduction of the model proposed in this paper for companies and that, in a sense, it is the dual of the activity-based view characterized in proposition 1. The example given in Figure 6c is an illustration of such a resource-based view with respect to materials (products).
Now, consider the network $G_f$ defined in proposition 1, and partition its arc set $X_{p_f}$ into three subsets, $X^1_{p_f}$, the arcs with an external origin, $X^0_{A_f}$, the internal arcs and $X^o_{A_f}$, the arcs with an external destination. Then, looking at company $f$ from the point of view of the theory of the firm leads to the following proposition.

**Proposition 4:** In order to maximize its profit, a firm $f$ would solve the following mathematical program:

$$\text{Max } \sum_{a \in A_f^1} v^a \equiv \sum_{i(a,j,p) \in X^1_{p_f}} v_{iap}(y_{iap}) - \sum_{(i,a,p) \in X^0_{A_f}} v_{iap}(y_{iap}) - \sum_{a \in A_f^0} u_a$$

subject to

1. $\sum_{a \in A_f^1} x_{ra} \leq x_r$, $r \in R_f^1$
2. $f_{m_a}(y^i_a, x_a, y^o_a) = 0$, $a \in A_f^1$
3. $C_f(c(x), X, u) = 0$
4. $y^i_a \geq 0, x_a \geq 0, y^o_a \geq 0, u_a \geq 0$, $a \in A_f^1$

When the firm $f$ is considered as a single activity, this value-added-maximization formulation reduces to the classical mathematical programming model of a multiproduct, multifactor firm (Naylor and Vernon, 1969). Under this paradigm, input products are variable factors and durable resources fixed factors. Proposition 4 generalizes this classical model to the case of a network company. The interested reader is referred to Lakhal, Martel, Oral and Kettani (1997) for a detailed discussion of the strategic management implications of this model.

These are just four propositions to illustrate how different schools of strategy can be handled using the concepts and notations developed here. For other and more detailed treatment, the reader is referred to Martel, Lakhal and Oral (1998).
It should be clear from our previous discussion that, to a certain extent, all companies are network companies and that, to develop a sustainable competitive advantage, a company must engage in a series of strategic positioning maneuvers within its industrial trellis. The additional notions of internal trellis, external trellis, vertically integrated company and virtual company will help to clarify the issue.

Consider a company \( f \), the set of chains \( C_f \) in which it is involved and the industrial trellis \( B \) formed by the company and its chains. Then the internal trellis of company \( f \) is defined by the subgraph \( B(E_f, A^1_f) \) and its external trellis by the complement of \( B(E_f, A^1_f) \) with respect to \( B \). A virtual company is a firm with a minimal internal trellis and a maximal external trellis. These companies have mainly a coordination role and few of their resources are engaged in the primary activities of their product-market chains, with the result that \( B_f \cap B_c = \emptyset \), for all \( c \in C_f \). At the other extreme, a vertically integrated company is a firm with a maximal internal trellis and a minimal external trellis, i.e. a company such that \( B_f \approx B_c \), for at least one \( c \in C_f \).

Most companies are located somewhere between these two extremes. The expression network company is a metaphor used to designate companies which have a non-negligible external trellis and which are trying to achieve a sustainable competitive advantage by continuously seeking the best possible balance between internalized and externalized activities and resources. This search often leads to participation in strategic business ventures and it involves a number of strategic decisions which have not been studied formally in the literature. Two important strategic issues for network companies are: (1) Given the actual state of the company and of its trellis and a set of possible future threats and opportunities, which business activities should be externalized and which ones should be developed internally to maximize its
economic value creation or, more generally, to achieve a sustainable competitive advantage? (2) 

What is the best way to externalize activities or, more specifically, how can one determine the value of a given business venture for a network company, or how can a number of possible business ventures be compared? A general discussion of the network company and of several of the strategic decisions it faces is found in Poulin, Montreuil and Gauvin (1994). Starting from Proposition 4, Lakhal, Martel, Kettani and Oral (1998) discuss the specific form of the value functions \( v_{ijp}(y_{ijp}) \), the input-output functions \( f_{m}(y_{i}, x_{a}, y_{a}) \) and the cost sharing functions \( C(c(x), X, u) \) and they propose a model and a solution procedure, which can be used to address question 1) above.

6. CONCLUDING REMARKS

This paper presents an analytical framework, based on graph theory concepts, which establish a link between three apparently distinct competitive strategy paradigms, namely the resource-based approach, the activity-based approach and the economic theory of the firm. It also provides a formal characterization of the network company and a foundation on which competitive networking decisions can be based. This is just an initial step, however, in the direction of building a formal theory of the network company. Several additional contributions are required before a truly useful theory is at end.

One important limitation of our framework is that it is static. The dynamics of industrial trellis needs to be taken into consideration while investigating the questions raised in the previous section. Industrial trellis are living organisms: the resources used and the activities performed by companies change in time as a result of the decisions made to compete better in their product market chains. This also raises the necessity of explicitly considering all the
companies competing in a given industrial trellis, as well as the impact of learning within the network company (Montreuil et al, 1996). We are currently working on several of these issues.

REFERENCES


