Asset Prices under Habit Formation and Reference-Dependent Preferences

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ABSTRACT

This paper proposes a consumption-based asset pricing model with low large-scale risk aversion that explains the key empirical facts, namely the low level and volatility of the real interest rate, and the high level and countercyclical variation of the equity premium. Investors are averse to losses in consumption relative to time-varying habit, and consequently require a high premium for holding stocks. The model's conditional moment restrictions are tested on consumption and asset returns data. The empirical estimate of large-scale risk aversion is low, while the estimate of loss aversion agrees with prior experimental evidence.

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The consumption-based asset pricing model (CCAPM) with power utility fails to explain important facts about stock returns, including the high equity premium, the high volatility of returns, and the countercyclical variation in the equity premium.\textsuperscript{1} In response to these failures, financial economists have considered alternative models of household preferences. One prominent approach is habit formation, in which utility depends on consumption relative to a reference level of consumption.\textsuperscript{2} Although habit-based asset pricing models are able to quantitatively match the key empirical facts, these models must ultimately appeal to high risk aversion to explain the high equity premium (see Campbell and Cochrane (1999, p. 243)). A problem with high risk aversion is that it has unappealing implications for large-scale risk (Kandel and Stambaugh 1991, Rabin 2000).

This paper proposes a habit-based asset pricing model with low large-scale risk aversion that explains the key empirical facts, namely the low level and volatility of the real interest rate, and the high level and countercyclical variation of the equity premium. The model is a standard identical-agent economy with external habit formation (e.g., Abel (1990) and Campbell and Cochrane (1999)). The point of departure from previous work is a new utility function for evaluating consumption relative to habit. Specifically, I use the reference-dependent model of K˝oszegi and Rabin (2004), which ties together neoclassical consumption utility with the gain-loss utility of Kahneman and Tversky (1979). The model offers a parsimonious framework to think about both large-scale risk aversion and loss aversion. Risk aversion refers to the curvature of consumption utility, which determines the household’s behavior for large gambles. Loss aversion refers to the magnitude of marginal utility for losses relative to gains, which determines the household’s behavior for small gambles.

Previous work, notably Barberis, Huang, and Santos (2001) and Benartzi and Thaler (1995), has shown that loss aversion can explain asset pricing puzzles. The model in this paper differs from previous models in that households care about gains and losses in con-

\textsuperscript{1}See Grossman and Shiller (1981), Kandel and Stambaugh (1990), Mehra and Prescott (1985), and Shiller (1982).

\textsuperscript{2}See Abel (1990), Campbell and Cochrane (1999), Constantinides (1990), Ferson and Constantinides (1991), and Sundaresan (1989).
sumption relative to habit, rather than gains and losses in wealth. One can debate which of the two approaches model household preferences in a more realistic way, but more importantly, the two models offer different answers to the key economic questions. Are fluctuations in consumption risky even though its volatility is low relative to stock returns? And why is the equity premium so high? In Barberis, Huang, and Santos (2001), fluctuations in consumption are safe since households have power utility with low risk aversion. The high equity premium is explained by the fact that investors care about fluctuations in wealth, which capture “feelings unrelated to consumption” (p. 6). This paper offers an alternative world view that fluctuations in consumption are risky since households are averse to losses in consumption relative to habit. The high equity premium is the reward that investors require for holding stocks, which delivers low returns during recessions when consumption falls relative to habit.

In related work, Garcia, Renault, and Semenov (2002) also propose a habit-based asset pricing model using preferences that exhibit loss aversion. This paper differs from their work in several ways. First, their model is based on a utility function that does not belong in the class of reference-dependent preferences with behavioral foundations (Kőszegi and Rabin 2004). The advantage of using preferences with explicit behavioral foundations is that the parameters can be interpreted in light of experimental evidence from psychology. Second, the preferences used in this paper are sufficiently simple that asset prices can be calculated in closed form, which gives insight into the relative contributions of habit formation and loss aversion in explaining asset prices. Loss aversion is important for explaining the level of the equity premium, while habit formation is important for explaining the low volatility of the riskfree rate and the time variation in the equity premium. Finally, Garcia, Renault, and Semenov find evidence for habit formation, but not for loss aversion, whereas I find that experimentally confirmed levels of loss aversion are consistent with asset prices.

The rest of the paper is organized as follows. Section I introduces a general class of reference-dependent preferences based on the work of Kőszegi and Rabin (2004). A method-
ological contribution of this section is to link together previously proposed functional forms of reference dependence in a unifying framework. Both the ratio model (e.g., Abel (1990)) and the difference model (e.g., Constantinides (1990)) are derived from standard gain-loss functions. In Section II, I calculate equilibrium asset returns under habit formation and reference-dependent preferences. I then calibrate the model to historical data on aggregate consumption and asset returns. In Section III, I estimate the model through its conditional moment restrictions. The empirical estimate of risk aversion is consistent with what economists believe are reasonable predictions for large gambles, based on the thought experiments of Kandel and Stambaugh (1991). The empirical estimate of loss aversion is consistent with the experimental estimate by Tversky and Kahneman (1992). Section IV concludes. The appendices contain descriptions of the data and derivations omitted in the main text.

I. A General Class of Reference-Dependent Preferences

Let $v(C)$ be a neoclassical utility function that is continuously differentiable, strictly increasing, and concave for all $C > 0$. A prominent example is the power utility function

$$v(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad (\gamma \geq 0),$$

where the special case $\gamma = 1$ is understood to be log utility $v(C) = \log C$.

Let $W(z)$ be a member of the class of gain-loss functions proposed by Kahneman and Tversky (1979):

1. $W(z)$ is continuous and strictly increasing for all $z \in \mathbb{R}$, where $W(0) = 0$.

2. $W(z)$ is twice differentiable for all $z \neq 0$, $W''(z) \leq 0$ for all $z > 0$, and $W''(z) \geq 0$ for all $z < 0$. 


3. $W(y) + W(-y) < W(z) + W(-z)$ for all $y > z > 0$, and $\lim_{z \downarrow 0} W'(-z)/W'(z) = \lambda > 1$. Property 1 is monotonicity, that utility is strictly increasing in the magnitude of gain. Property 2 is diminishing sensitivity, that the marginal effect of a gain or a loss diminishes with its magnitude. Property 3 is loss aversion, that the impact of a loss is greater than that of an equally sized gain. That the impact of an arbitrary small loss is greater than that of an arbitrarily small gain gives rise to a kink in the gain-loss function at $z = 0$ (Bowman, Minehart, and Rabin 1999).

Following K˝oszegi and Rabin (2004), consider a general class of reference-dependent preferences given by

$$u(C, X) = \alpha v(C) + (1 - \alpha)W(v(C) - v(X)),$$

(2)

where $\alpha \in [0, 1]$. Reference-dependent utility (2) is a weighted sum of two parts. The first part $v(C)$ is consumption utility, that is neoclassical utility derived from consumption $C$. The second part $W(v(C) - v(X))$ is gain-loss utility, that is utility derived from the deviation of consumption utility $v(C)$ from its reference level $v(X)$. The variable $X$ denotes the reference level of consumption. The household derives positive (negative) gain-loss utility when $C$ exceeds (is exceeded by) $X$.

Let subscripts denote partial derivatives. Marginal utility with respect to consumption and its reference level are given by

$$u_C = v'(C)[\alpha + (1 - \alpha)W'(v(C) - v(X))] > 0,$$

$$u_X = -(1 - \alpha)v'(X)W'(v(C) - v(X)) \leq 0,$$

whenever $C \neq X$. In words, utility is strictly increasing in consumption and decreasing in the reference level. Marginal utility is not well defined at $C = X$ due to the kink in the gain-loss function arising from loss aversion.

Suppose the gain-loss function satisfies a slightly stronger version of diminishing sensi-
tivity (Property 2):

2'. $W(z)$ is twice differentiable for all $z \neq 0$, $W''(z) < 0$ for all $z > 0$, $W''(z) > 0$ for all $z < 0$, and $\lim_{z \to \pm \infty} W'(z) = 0$.

Then $\lim_{C - X \to \pm \infty} u_C = \alpha v'(C)$. That is, for large deviations in consumption from the reference level, the behavior of the household is the same as that with neoclassical consumption utility. This large-risk behavior of reference-dependent utility accords well with the common view that neoclassical utility is adequate for describing aversion to large risks, but not for small risks (see Rabin (2000)).

Preferences that depend on a reference level of consumption have psychological foundations in hedonic adaptation (see Frederick and Loewenstein (1999)), and they have consequently been adopted in the finance literature. Early contributions include Abel (1990), Constantinides (1990), and Sundaresan (1989). Reference-dependent utility (2) is a useful framework for linking together previously proposed functional forms of reference dependence. To show the connections, I first parameterize consumption utility through the power utility function (1). I then consider two classes of parametric gain-loss functions, exponential and power, that satisfy the Kahneman-Tversky properties. The ratio model (e.g., Abel (1990)) is a special case under exponential gain-loss utility, and the difference model (e.g., Constantinides (1990)) is a special case under power gain-loss utility.

A. Exponential Gain-Loss Utility

The exponential gain-loss function (Köbberling and Wakker 2003) is given by

$$W_E(z) = \begin{cases} \frac{1 - \exp\{-\theta z\}}{\theta} & \text{for } z \geq 0 \\ \lambda \frac{\exp(\theta z) - 1}{\theta} & \text{for } z < 0 \end{cases} \quad (\theta \geq 0, \lambda > 1).$$

The parameter $\theta$ determines the degree of diminishing sensitivity, and the parameter $\lambda$ determines the degree of loss aversion. When $\theta > 0$, the exponential gain-loss function
satisfies the strong version of diminishing sensitivity (Property 2'). The special case \( \theta = 0 \) is understood to be the linear gain-loss function

\[
W_L(z) = \begin{cases} 
  z & \text{for } z \geq 0 \\
  \lambda z & \text{for } z < 0 
\end{cases} \quad (\lambda > 1).
\]  

(4)

**Exponential reference-dependent utility** is defined as a special case of reference-dependent utility with power consumption utility (1) and exponential gain-loss utility (3). The marginal utility of consumption in this case is

\[
u_C = \begin{cases} 
  C^{-\gamma} \left[ \alpha + (1 - \alpha) \exp \left\{ -\theta \left( \frac{C^{1-\gamma}}{C} - \frac{X^{1-\gamma}}{X} \right) \right\} \right] & \text{for } C > X \\
  C^{-\gamma} \left[ \alpha + (1 - \alpha) \lambda \exp \left\{ \theta \left( \frac{C^{1-\gamma}}{C} - \frac{X^{1-\gamma}}{X} \right) \right\} \right] & \text{for } C < X
\end{cases}.
\]  

(5)

When consumption is close to its reference level, marginal utility can be approximated as

\[
u_C \approx \begin{cases} 
  C^{-\gamma} & \text{for } C > X \\
  \alpha C^{-\gamma} & \text{for } C < X
\end{cases},
\]  

(6)

where \( \lambda = \alpha + (1 - \alpha)\lambda \). In this approximation, the marginal utility of consumption is the same as that of standard power utility when \( C > X \). When \( C < X \), however, marginal utility is higher than that of power utility due to loss aversion (parameterized as \( \lambda > 1 \)). The higher is the degree of loss aversion, the higher is marginal utility when consumption is below its reference level.

To motivate exponential reference-dependent utility, consider the special case when consumption utility takes the log utility form (i.e., \( \gamma = 1 \)). In this case, the utility function simplifies to

\[
u(C, X) = \begin{cases} 
  \alpha \log C - (1 - \alpha) \frac{1}{\theta} \left( \frac{C}{X} \right)^{-\theta} & \text{for } C \geq X \\
  \alpha \log C + (1 - \alpha) \lambda \frac{1}{\theta} \left( \frac{C}{X} \right)^{\theta} & \text{for } C < X
\end{cases},
\]  

(7)

up to an additive constant. Household utility in this case is a weighted sum of log consump-
tion utility and ratio reference-dependent utility (Abel 1990).

B. Power Gain-Loss Utility

The power gain-loss function (Tversky and Kahneman 1992) is given by

\[ W_p(z) = \begin{cases} 
\frac{z^{1-\theta}}{1-\theta} & \text{for } z \geq 0 \\
-\lambda\frac{|z|^{1-\theta}}{1-\theta} & \text{for } z < 0 
\end{cases} \quad (\theta \in [0, 1), \lambda > 1). \]  

(8)

The parameter \( \theta \) determines the degree of diminishing sensitivity, and the parameter \( \lambda \) determines the degree of loss aversion. When \( \theta > 0 \), the power gain-loss function satisfies the strong version of diminishing sensitivity (Property 2'). The special case \( \theta = 0 \) corresponds to the linear gain-loss function (4). Using experimental data, Tversky and Kahneman (1992) obtained the parameter estimates \( \theta = 0.12 \) and \( \lambda = 2.25 \).

**Power reference-dependent utility** is defined as a special case of reference-dependent utility with power consumption utility (1) and power gain-loss utility (8). The marginal utility of consumption in this case is

\[ u_C = \begin{cases} 
C^{-\gamma} \left[ \alpha + (1 - \alpha) \left( \frac{C^{1-\gamma}}{1-\gamma} - \frac{X^{1-\gamma}}{1-\gamma} \right)^{-\theta} \right] & \text{for } C > X \\
C^{-\gamma} \left[ \alpha + (1 - \alpha)\lambda \left| \frac{C^{1-\gamma}}{1-\gamma} - \frac{X^{1-\gamma}}{1-\gamma} \right|^{-\theta} \right] & \text{for } C < X 
\end{cases} \]  

(9)

When consumption is close to its reference level, marginal utility can be approximated as

\[ u_C \approx \begin{cases} 
(1 - \alpha)C^{-\gamma} \left( \frac{C^{1-\gamma}}{1-\gamma} - \frac{X^{1-\gamma}}{1-\gamma} \right)^{-\theta} & \text{for } C > X \\
(1 - \alpha)\lambda C^{-\gamma} \left| \frac{C^{1-\gamma}}{1-\gamma} - \frac{X^{1-\gamma}}{1-\gamma} \right|^{-\theta} & \text{for } C < X 
\end{cases} \]  

(10)

provided that \( \alpha \neq 1 \). In this approximation, the marginal utility of consumption when \( C > X \) is higher than that when \( C < X \) by a factor \( \lambda \). The greater is the degree of loss aversion, the higher is the difference in marginal utility between these two states of the world.

To motivate power reference-dependent utility, consider the special case when consump-
tion utility takes the linear utility form (i.e., $\gamma = 0$). In this case, the utility function simplifies to

$$u(C, X) = \begin{cases} 
\alpha C + (1 - \alpha) \frac{(C - X)^{1-\theta}}{1-\theta} & \text{for } C \geq X \\
\alpha C - (1 - \alpha) \lambda \frac{|C - X|^{1-\theta}}{1-\theta} & \text{for } C < X 
\end{cases}$$

Household utility in this case is a weighted sum of linear consumption utility and difference reference-dependent utility. Utility function (11) differs from the conventional specification of difference reference-dependent utility (e.g., Constantinides (1990)) in two important ways. First, the parameter $\theta \in [0, 1)$ has the interpretation of diminishing sensitivity, rather than risk aversion. Second, utility is well defined even when consumption falls below its reference level. The modeling convention that consumption never falls below its reference level can be thought of as optimal behavior for a household that is highly loss averse (i.e., $\lambda \gg 1$).

**C. Implications of Power Reference-Dependent Utility for Small and Large Gambles**

Kandel and Stambaugh (1991) noted that power utility, used to model household preferences in the canonical CCAPM, has difficulty explaining the household’s behavior for both small and large gambles. This section shows that reference-dependent utility is able to overcome this problem. In this section and throughout the rest of the paper, I focus on power (rather than exponential) reference-dependent utility since the power gain-loss function has been experimentally confirmed (Tversky and Kahneman 1992).

Table I reports the outcome of a simple thought experiment following Kandel and Stambaugh (1991). The household’s initial level of wealth is $75,000, and its reference level of consumption is also $X = 75,000$. In Panel A, the household faces a “small” gamble of $\pm 375 (0.5\% \text{ of wealth})$ with equal probabilities. In Panel B, the household faces a “large” gamble of $\pm 25,000 (33\% \text{ of wealth})$ with equal probabilities. The table reports the amount of a sure loss in wealth that makes the household indifferent to facing the gamble. In other words, the household is willing to pay the amount reported in the table to avoid the gamble. The
thought experiment is conducted at various parameter values for power reference-dependent utility: $\alpha = \{1, 0.5, 0\}$, $\theta = \{0, 0.12\}$, and $\gamma \in [0, 30]$. The degree of loss aversion is held fixed at $\lambda = 2.25$.

The column labeled $\alpha = 1$ corresponds to the power utility benchmark, also reported in Kandel and Stambaugh (1991). Power utility has difficulty explaining the household’s behavior for both small and large gambles. When $\gamma = 30$, the household is willing to pay $28.03 to avoid the small gamble, and $23,791 to avoid the large gamble. The amount that the household is willing to pay to avoid the large gamble seems implausibly large. When $\gamma = 2$, the household is willing to pay $1.88 to avoid the small gamble, and $8,333 to avoid the large gamble. The amount that the household is willing to pay to avoid the large gamble is more reasonable, but the amount paid to avoid the small gamble seems implausibly small. This tension between small- and large-risk behavior arises from the fact that any concave utility function implies approximate risk neutrality for sufficiently small gambles (Rabin 2000).

When $\alpha < 1$, the household’s utility depends not only on consumption utility, but also on gain-loss utility. In contrast to power utility, power reference-dependent utility is able to explain the household’s behavior for both small and large gambles with the same set of preference parameters. For instance, when $\alpha = 0.5$, $\theta = 0.12$, and $\gamma = 1$, the household is willing to pay $73.75 to avoid the small gamble, and $7,303 to avoid the large gamble. In general, the willingness to pay is decreasing in $\theta$ due to the convexity of gain-loss utility for losses, and increasing in $\gamma$ due to the concavity of consumption utility.

For small gambles in Panel A, the household’s willingness to pay does not vary much in $\gamma$. In other words, gain-loss utility (parameterized by $\theta$ and $\lambda$) plays a more prominent role in explaining the household’s behavior for small gambles. For large gambles in Panel B, the household’s willingness to pay varies significantly in $\gamma$. In other words, consumption utility (parameterized by $\gamma$) plays a more prominent role in describing the household’s behavior for large gambles. For this reason, I will refer to the parameter $\gamma$ as large-scale risk aversion, to
give it a name that is appropriate for its economic role. When \( \gamma \geq 10 \), the amount that the household is willing to pay to avoid the large gamble seems implausibly large.

The fact that, for small gambles, observed behavior does not vary much in \( \gamma \) presents problems for the identification of the parameter. In experimental studies, test subjects can only be subject to small gambles for ethical reasons. Therefore, measurements of \( \gamma \) will be confounded by the effect of loss aversion, which dominates behavior in the realm of small gambles. In estimating preference parameters from macroeconomic data, Kandel and Stambaugh (1991) emphasizes the fact that identification occurs in the domain of small gambles since aggregate consumption has low volatility. This is not to say that macroeconomic risks faced by households are small, but rather that observed consumption is an equilibrium outcome of an optimizing household that smooths consumption in the presence of risk. It is therefore important to have a model that accurately describes household behavior not only for large gambles, but also for small gambles.

II. Asset Prices under Reference-Dependent Preferences

A. An Economy with External Habit Formation

To study the asset pricing implications of reference-dependent preferences, I consider a simple endowment economy with external habit formation, following Abel (1990) and Campbell and Cochrane (1999). The economy is composed of identical households, indexed by \( h \), that maximize the expected discounted sum of future utility flows

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(C_{ht}, X_t).
\]

(12)

The parameter \( \beta > 0 \) is the household’s subjective discount factor, and \( C_{ht} \) is its consumption in period \( t \).
Each household’s utility depends on external habit $X_t$, which is a common reference level of consumption. External habit has an economic interpretation as the “subsistence level” or “standard of living”. Specifically, external habit has the dynamics

$$X_{t+1} = \exp\{\delta\}X_t^\phi C_t^{1-\phi},$$

(13)

where $C_t$ is per capita consumption in period $t$ and $\phi \in [0, 1)$. Habit is a geometric average of past consumption, rather than an arithmetic average as in Constantinides (1990). The model of habit in Ferson and Constantinides (1991) is a special case in which $\phi = 0$.

Let $G_{t+1} = C_{t+1}/C_t$ denote consumption growth, and let $Y_t = C_t/X_t$ denote the consumption-habit ratio. Also let lowercase letters denote the log of the corresponding uppercase variables. Then the log consumption-habit ratio can be expressed as

$$y_{t+1} = -\delta + \phi y_t + g_{t+1}.$$

(14)

In words, the consumption-habit ratio is an AR(1) with consumption growth as its innovation. Consumption and habit are cointegrated in this model, with $\phi$ capturing the degree of persistence in the deviation of consumption from habit. Model (14) is simpler than the model of habit in Campbell and Cochrane (1999), but has the same economic mechanisms. Additional complications are unnecessary because the reference-dependent preferences in this paper are well defined even when consumption falls below habit (i.e., $y_t < 0$).

Since households in this economy are identical, $C_{ht} = C_t$ in equilibrium. I will therefore drop the subscript $h$ to simplify notation. Under power reference-dependent utility (9), the intertemporal marginal rate of substitution (IMRS) takes the form

$$M_{t+1} = \beta G_{t+1} - \gamma \frac{w(Y_{t+1})}{w(Y_t)},$$

(15)
where
\[
    w(Y_t) = \begin{cases} 
    \alpha + (1 - \alpha) \left( \frac{c^{1-\gamma}_{t-1} - x^{1-\gamma}_{t-1}}{1-\gamma} \right)^{-\theta} & \text{for } Y_t > 1, \\
    \alpha + (1 - \alpha)\lambda \left| \frac{c^{1-\gamma}_{t-1} - x^{1-\gamma}_{t-1}}{1-\gamma} \right|^{-\theta} & \text{for } Y_t < 1.
\end{cases}
\] (16)

Under linear reference-dependent utility, which is the special case \( \theta = 0 \), this simplifies to
\[
    w_L(Y_t) = \begin{cases} 
    1 & \text{for } Y_t > 1, \\
    \lambda \alpha & \text{for } Y_t < 1,
\end{cases}
\] (17)

where \( \lambda \alpha = \alpha + (1 - \alpha)\lambda \).

The household’s first-order conditions and the envelope theorem imply the Euler equation
\[
    E_t[M_{t+1}R_{i,t+1}] = 1,
\] (18)
for any asset \( i \) with the gross return \( R_{i,t+1} \) from period \( t \) to \( t+1 \). The rest of the paper examines the asset pricing implications of reference-dependent preferences through this equation.

### B. Asset Prices under Linear Reference-Dependent Utility

Suppose households have linear reference-dependent utility, and consumption is conditionally log-normal. That is, \( g_{t+1} \sim N(\mu_t, \sigma^2_t) \), where I drop the subscript \( t \) in the moments of consumption growth to simplify notation. Then equilibrium asset returns (specifically, the riskfree rate and the maximum Sharpe ratio) can be calculated explicitly as a function of the preference parameters and the moments of consumption growth. The calculations provide essential intuition for the effect of loss aversion on asset prices.
B.1. Riskfree Rate

Let \( R_{ft} \) be the return on a conditionally riskfree asset in period \( t \), and let \( F(z) = \Pr(Z < z) \) denote the cumulative distribution function of a standard normal random variable \( Z \). The following proposition is proved in Appendix A.

**Proposition 1.** Suppose \( g_{t+1} \sim \mathcal{N}(\mu, \sigma^2) \), and the IMRS is given by equations (15) and (17). Then a conditionally riskfree asset has the return

\[
R_{f,t+1} = \left[ \beta \exp \left\{ -\gamma \mu + \frac{\gamma^2 \sigma^2}{2} \right\} A_t(\gamma, \lambda) \right]^{-1},
\]

where

\[
A_t(\gamma, \lambda) = \begin{cases} 
1 + (\lambda - 1)F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma) & \text{for } y_t > 0 \\
1/\lambda + (1 - 1/\lambda)F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma) & \text{for } y_t < 0
\end{cases}.
\]

The function \( A_t(\gamma, \lambda) \) has the following properties.

1. \( \partial A_t(\gamma, \lambda)/\partial \lambda \geq 0 \) if \( y_t \geq 0 \).
2. \( \partial A_t(\gamma, \lambda)/\partial y_t < 0 \) if \( \phi > 0 \).

When \( \alpha = 1 \), which corresponds to the power utility model, \( \lambda = 1 \) and \( A_t(\gamma, 1) = 1 \). The riskfree rate then simplifies to the familiar expression

\[
R_{f,t+1} = \beta^{-1} \exp \left\{ \gamma \mu - \frac{\gamma^2 \sigma^2}{2} \right\}.
\]

The first term inside the exponential represents intertemporal substitution, and the second term represents precautionary savings. The higher is risk aversion \( \gamma \), stronger is the intertemporal motive to borrow, and stronger is the precautionary motive to save.

To understand the effect of loss aversion on the riskfree rate, it is helpful to first consider the special case \( \phi = 0 \). Compared to the power utility benchmark, the riskfree rate (19)
is lower in a “boom” (i.e., $y_t > 0$) and higher in a “recession” (i.e., $y_t < 0$). Intuitively, marginal utility is low in a boom, so the household is motivated to save the marginal dollar for the possibility of a recession tomorrow, driving down the equilibrium interest rate. On the other hand, marginal utility is high in a recession, so the household is motivated to borrow, driving up the equilibrium interest rate. This precautionary savings motive induced by loss aversion is proportional to $\sigma$ (since $F(\gamma \sigma + (\delta - \mu)/\sigma)$ is of order $\sigma$). This is in contrast to the precautionary savings motive induced by risk aversion, which is proportional to $\sigma^2$.

In that sense, linear reference-dependent utility exhibits first-order risk aversion (Segal and Spivak 1990).

For a sufficiently high degree of loss aversion, the riskfree rate can be excessively counter-cyclical in the special case $\phi = 0$. This is where persistence in habit, parameterized as $\phi > 0$, plays a key economic role in inducing the right amount of precautionary motive to save. Intuitively, marginal utility is low in a boom, but the household is unmotivated to save the marginal dollar since consumption is expected to remain high relative to habit tomorrow. On the other hand, marginal utility is high in a recession, but the household is unmotivated to borrow since consumption is expected to remain low relative to habit tomorrow.

### B.2. Maximum Sharpe Ratio

Let $R_{et} = R_{it} - R_{jt}$ ($i \neq j$) be a generic excess return in period $t$. The following proposition is proved in Appendix A.

**Proposition 2.** Suppose $g_{t+1} \sim N(\mu, \sigma^2)$, and the IMRS is given by equations (15) and (17). Then the Sharpe ratio for any excess return has the bound

$$\frac{\mathbb{E}_t[R_{e,t+1}]}{\sigma_t(R_{e,t+1})} \leq \left[\exp\{\gamma^2 \sigma^2\} B_t(\gamma; \lambda_\alpha) - 1\right]^{1/2},$$

(22)

where

$$B_t(\gamma; \lambda_\alpha) = \frac{1 + (\lambda_\alpha^2 - 1)F(2\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)}{[1 + (\lambda_\alpha - 1)F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)]^2}.\quad (23)$$
The function $B_t(\gamma, \lambda, \alpha)$ has the following properties.

1. $B_t(\gamma, \lambda, \alpha) \geq 1$ and $\partial B_t(\gamma, \lambda, \alpha)/\partial \lambda > 0$.

2. $\partial B_t(0, \lambda, \alpha)/\partial y_t < 0$ if $\phi > 0$ and 

$$y_t > \frac{\sigma}{\phi} \left[ \frac{\delta - \mu}{\sigma} - F^{-1}(1/(\lambda + 1)) \right].$$

(24)

When $\alpha = 1$, which corresponds to the power utility model, $\lambda = 1$ and $B_t(\gamma, 1) = 1$. The maximum Sharpe ratio then simplifies to the familiar expression

$$\frac{E_t[R_{e,t+1}]}{\sigma_t(R_{e,t+1})} \leq [\exp\{\gamma^2 \sigma^2\} - 1]^{1/2} \approx \gamma \sigma.$$  

(25)

The higher is risk aversion $\gamma$, higher is the premium for holding risky assets.

Compared to the power utility benchmark, the maximum Sharpe ratio (22) is strictly greater and monotonically increasing in $\lambda$ (and hence $\lambda$). Simply put, higher is the degree of loss aversion, the greater is the reward that households demand for bearing risk. Due to loss aversion, the Sharpe ratio is proportional to $\sqrt{\sigma}$ (since $F(2\gamma \sigma + (\delta - \mu)/\sigma)$ is of order $\sigma$). This is in contrast to the Sharpe ratio based on risk aversion alone, which is proportional to $\sigma$. In that sense, linear reference-dependent utility exhibits first-order risk aversion, which can explain the high historical equity premium (Bekaert, Hodrick, and Marshall 1997, Epstein and Zin 1990).

When $\phi > 0$, the consumption-habit ratio $y_t$ is a state variable that induces time variation in the maximum Sharpe ratio. When $\gamma = 0$, Proposition 2 shows that the Sharpe ratio falls in the consumption-habit ratio at sufficiently high levels of $y_t$. While the quantitative implications of this relationship depends on the preference parameters and the moments of consumption growth, the Sharpe falls in $y_t$ for empirically relevant parameter values. This is made more precise in the next section, where I calibrate the model.
C. Calibration to Consumption and Asset Returns Data

Table II reports descriptive statistics for consumption and asset returns in the annual sample 1929–2001. (See Appendix B for a complete description of the data.) Log real consumption growth has mean 1.88% and standard deviation 2.27%. The real (ex-post) T-bill rate has mean 1.06% and standard deviation 4.47%. The table reports descriptive statistics for real excess returns on three portfolios: the value-weighted market portfolio, the SMB (Small Minus Big stock) portfolio, and the HML (High Minus Low book-to-market) portfolio. The equity, size, and value premia in this sample are 7.26%, 3.19%, and 4.99%, respectively. As is well known, the high Sharpe ratio of 0.36 for excess stock returns is difficult to reconcile with the low volatility of consumption growth in standard asset pricing models (Mehra and Prescott 1985).

To illustrate this well known failure, suppose log consumption growth is normal with $\mu = 1.88\%$ and $\sigma = 2.27\%$. Assume that households have power utility with $\beta = 0.99$ and $\gamma = 2$. Then the riskfree rate implied by equation (21) is 4.77%, which is somewhat higher but comparable to the historical mean of the T-bill rate. However, the Sharpe ratio implied by equation (25) is merely 0.05, which is an order of magnitude smaller than the historical Sharpe ratio for equity 0.36. A higher risk aversion of $\gamma = 20$ raises the Sharpe ratio to 0.48, but at the cost of raising the riskfree rate to 32.71%. Therefore, a “resolution” of the equity premium puzzle through higher risk aversion results in a riskfree rate puzzle (Weil 1989). Aside from this problem, high risk aversion in itself is problematic because of its implications for large-scale risk (see Table I).

Now consider linear reference-dependent utility with the parameters $\alpha = 0$, $\beta = 0.99$, $\gamma = 2$, and $\lambda = 2.25$. Assume for now that $\delta = \phi = 0$, so that habit has the simple dynamics $X_{t+1} = C_t$. Then the average riskfree rate implied by equation (19) is 3.42%, which is comparable to that implied by the power utility model. The Sharpe ratio implied by equation (22) is 0.45, which is of the same order of magnitude as the historical Sharpe ratio for excess stock returns. Therefore, linear reference-dependent utility can simultaneously
explain the high equity premium and the low average T-bill rate. However, the model fails in an important way, predicting an excessively volatile riskfree rate. The riskfree rate implied by equation (19) is -17.57% in a boom and 85.46% in a recession, so the standard deviation of the riskfree rate in the model is an order of magnitude higher than the standard deviation of the T-bill rate.

I now calibrate the linear reference-dependent model using the parameters reported in the first column of Table III. The preference parameters are the same as before, but I allow for persistence in the dynamics of habit. In order to have the calibration speak to the historical data, I feed actual consumption data through equation (14) to generate a historical realization of habit. I set the parameters $\delta = 0$ and $\phi = 0.51$, which are calibrated so that the riskfree rate implied by the model matches the historical mean and variance of the T-bill rate. The initial level of habit is set to $y_0 = -0.1$ since the data start during the Great Depression when consumption growth was persistently low. The results are not sensitive to reasonable variation in this initial level of habit.

Figure 1 reports the results of the calibration. Panel A is a plot of the realized consumption-habit ratio in the period 1931–2001. Consumption initially starts below habit during the Great Depression, but remains above habit for the rest of the sample. Panel B is the plot of the riskfree rate implied by equation (19), feeding in the consumption-habit ratio from Panel A. While there is some procyclical variation in the riskfree rate, its volatility is low, matching the historical standard deviation of the T-bill rate. Panel C is a plot of the Sharpe ratio implied by equation (22), feeding in the consumption-habit ratio from Panel A. There are two interesting facts to note. First, the Sharpe ratio is high on average, even though consumption remains above habit throughout most of the sample. It is not actual losses per se, but the fear of losses that induces a high equity premium. Second, the Sharpe ratio is strongly countercyclical, which is generated by the procyclical variation in the consumption-habit ratio. For instance, the Sharpe ratio peaks during the four most recent recessions in 1973–75, 1980–82, 1990–91, and 2001.
III. Estimation of the Reference-Dependent Model

The calibration in the last section shows that the reference-dependent model is able to explain the main empirical facts about asset returns. In this section, I turn to an empirical test of the model using a richer set of moments to further examine the model’s ability to explain the joint dynamics of consumption and asset returns.

A. Empirical Methodology

The Euler equation (18) implies the moment restriction

$$E[(M_{t+1}R_{t,t+1} - 1)z_t] = 0,$$

where $z_t$ is a vector of instrumental variables known at time $t$. As shown by Hansen and Singleton (1982), an asset pricing model can be tested, and its structural parameters can be estimated through this moment restriction. In this section, I estimate the power reference-dependent model with the IMRS given by equations (15) and (16).

In estimating the model, I fix the parameter $\alpha = 0$ for two reasons. First, $\alpha = 0$ corresponds to the case where the household has only gain-loss utility, which is similar in spirit to earlier studies on habit formation. Second, the parameter is nearly unidentified in the region $\alpha < 1$. Intuitively, identification of the preference parameters occurs in the domain of small gambles since the volatility of aggregate consumption is low. For small gambles, gain-loss utility is much more important than direct consumption utility, implying that the observed household behavior is similar for all $\alpha < 1$. More formally, the difficulty of identifying $\alpha$ can be seen by applying approximation (10) for the marginal utility of consumption to the IMRS (15).

The test assets used in estimation of moment restriction (26) are the three-month T-bill, value-weighted market portfolio, SMB portfolio, and HML portfolio. The instruments are consumption growth, dividend-price ratio, size spread, value spread, long-short yield spread,
and a constant. These assets and instruments capture economically important variation in expected returns, both in the cross-section and the time series. (See Appendix B for further details on the data.) I report results using instruments that are lagged two years to avoid problems with time aggregation in consumption data (Hall 1988), but the results are similar using once lagged instruments.

Estimation is by continuous updating generalized method of moments (GMM) (Hansen, Heaton, and Yaron 1996). Newey and Smith (2004) show that this estimator has desirable higher-order asymptotic properties, implying better performance than two-step GMM (Hansen 1982) in finite samples.

B. Estimates of the Preference Parameters

Table III reports estimates of the power reference-dependent model. The estimate of $\gamma$ is 0.65 with a standard error of 9.47. This estimate of large-scale risk aversion has reasonable predictions for large gambles, as reported in Table I. The estimates of the gain-loss utility parameters are $\theta = 0.08$ and $\lambda = 3.21$ with standard errors of 0.20 and 0.95, respectively. These estimates are consistent with prior experimental estimates of $\theta = 0.12$ and $\lambda = 2.25$ (Tversky and Kahneman 1992). The estimate of the subjective discount factor is $\beta = 0.90$, which implies that the model is able to fit the average T-bill rate. The $J$-test has a $p$-value of 3%, so the model is rejected at the conventional 5% level.

The large standard error for $\gamma$ can be explained by weak identification. When there is weak identification, conventional inference based on point estimates and standard errors can be invalid. (See Appendix C for a more complete discussion of weak identification.) There is economic reason to expect weak identification in the power reference-dependent model. As discussed in Section I, the parameter $\gamma$ has a strong effect on the household’s aversion for large gambles, but only a weak effect for small gambles. Therefore, $\gamma$ is difficult to identify from the small observed variation in consumption.

Figure 2 is a plot of the GMM objective function, concentrated in the parameters $\gamma$ and
The objective function is flat in a large region of the parameter space, which is evidence for weak identification. Since the concentrated objective function has an asymptotic $\chi^2$ distribution under the null, even when the parameters are weakly identified, its shape is useful for inferring economically relevant values of the parameters (see Appendix C). For a given value of $\gamma$, the objective is at the highest point when $\lambda = 1$, and flattens when $\lambda$ is between two and three. This suggests that loss aversion is necessary for explaining asset returns.

**IV. Conclusion**

The idea that fluctuations in consumption, rather than wealth, is the relevant measure of risk has a long tradition in economics, rooted in the permanent income hypothesis. The CCAPM has therefore been the canonical economic model of risk and return, despite its many empirical failures. More recent work has shown that habit formation can explain many features of asset prices (e.g., Abel (1999) and Campbell and Cochrane (1999)). In order to explain the high equity premium, however, habit-based asset pricing models must ultimately appeal to high risk aversion, which has unappealing implications for large-scale risk.

In contrast, the behavioral approach has focused on fluctuations in wealth, rather than consumption, as the relevant measure of risk (e.g., Barberis, Huang, and Santos (2001) and Benartzi and Thaler (1995)). Empirically, consumption and wealth are cointegrated, and the variance of wealth falls to that of consumption in the long run (see Cochrane (1994) and Lettau and Ludvigson (2001)). Therefore, a measure of risk based on wealth requires that investors care about *transitory* shocks to wealth above and beyond *permanent* shocks to consumption. Although the view that investors care about gains and losses in wealth may ultimately be right, it is incompatible with fundamental notions and measures of economic risk. Yet the advantage of the behavioral approach is clear. By using preferences with
realistic predictions in the domain of small gambles, the behavioral approach can explain the equity premium with reasonable levels of large-scale risk aversion.

Relative to this literature, the contribution of this paper is to show that the high equity premium can be explained without appealing to high risk aversion or having preferences over wealth. By doing so, this paper introduces an alternative world view that fluctuations in consumption are risky, even though large-scale risk aversion is low. Essentially, the model developed in this paper relies on consumption as the relevant measure of risk, but uses behaviorally realistic preferences. Investors are averse to losses in consumption relative to time-varying habit, and the fear of losses generates the high level and countercyclical variation of the equity premium.
Appendix A. Proofs of Propositions

I. Proof of Proposition 1

The proof is essentially an application of the following lemma.

Lemma 1. If \( g \sim N(\mu, \sigma^2) \),

\[
E[e^g | g > \bar{x}] = \exp \left\{ \frac{\mu + \sigma^2}{2} \right\} \frac{F(-\bar{x} - \mu - \sigma^2/\sigma)}{F(-\bar{x} - \mu/\sigma)},
\]

\[
E[e^g | g < \bar{x}] = \exp \left\{ \frac{\mu + \sigma^2}{2} \right\} \frac{F((\bar{x} - \mu - \sigma^2)/\sigma)}{F((\bar{x} - \mu)/\sigma)},
\]

where \( F(\cdot) \) is the cumulative distribution function of the standard normal.

Let \( \bar{x}_{t+1} = x_{t+1} - c_t = \delta - \phi y_t \). The IMRS can be written as

\[
M_{t+1} = \begin{cases} 
\beta \exp(-\gamma g_{t+1}) w_L(y_t) & \text{for } g_{t+1} > \bar{x}_{t+1} \\
\lambda_\alpha \beta \exp(-\gamma g_{t+1}) w_L(y_t) & \text{for } g_{t+1} < \bar{x}_{t+1} 
\end{cases},
\]

where

\[
w_L(y_t) = \begin{cases} 
1 & \text{for } y_t > 0 \\
\lambda_\alpha & \text{for } y_t < 0 
\end{cases}.
\]

For any \( n > 0 \),

\[
E_t[M_{t+1}^n] = \left( \frac{\beta}{w_L(y_t)} \right)^n \{ F(-\bar{x}_{t+1} - \mu)/\sigma)E_t[e^{-\gamma g_{t+1}} | g_{t+1} > \bar{x}_{t+1}] \\
+ \lambda_\alpha^n F((\bar{x}_{t+1} - \mu)/\sigma)E_t[e^{-\gamma g_{t+1}} | g_{t+1} < \bar{x}_{t+1}] \}.
\]
By Lemma 1,

\[ E_t[e^{-n\gamma g_{t+1}|g_{t+1} > \bar{g}_{t+1}}] = \exp \left\{ -n\gamma \mu + \frac{(n\gamma \sigma)^2}{2} \right\} \frac{F(-n\gamma \sigma - (\bar{g}_{t+1} - \mu)/\sigma)}{F((-\bar{g}_{t+1} - \mu)/\sigma)}, \]

\[ E_t[e^{-n\gamma g_{t+1}|g_{t+1} < \bar{g}_{t+1}}] = \exp \left\{ -n\gamma \mu + \frac{(n\gamma \sigma)^2}{2} \right\} \frac{F(n\gamma \sigma + (\bar{g}_{t+1} - \mu)/\sigma)}{F((\bar{g}_{t+1} - \mu)/\sigma)}. \]

Therefore,

\[ E_t[M_{t+1}^n] = \left( \frac{\beta}{w_L(y_t)} \right)^n \exp \left\{ -n\gamma \mu + \frac{(n\gamma \sigma)^2}{2} \right\} [1 + (\lambda^n - 1)F(n\gamma \sigma + (\bar{g}_{t+1} - \mu)/\sigma)]. \] (27)

For a conditionally riskfree asset, the Euler equation (18) can be written as

\[ R_{f,t+1} = E_t[M_{t+1}]^{-1}. \] (28)

This equation, together with equation (27) for \( n = 1 \), implies equation (19).

II. Proof of Proposition 2

The Euler equation for an excess return is \( E_t[M_{t+1} R_{e,t+1}] = 0. \) As shown by Hansen and Jagannathan (1991) and Shiller (1982),

\[ \frac{E_t[R_{e,t+1}]}{\sigma_t(R_{e,t+1})} \leq \frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]} = \left( \frac{E_t[M_{t+1}^2]}{E_t[M_{t+1}]^2 - 1} \right)^{1/2}. \] (29)

This equation, together with equation (27) for \( n = 1, 2 \), implies equation (22).

By differentiation of equation (23),

\[ \frac{\partial B_t(\gamma, \lambda_\alpha)}{\partial \lambda_\alpha} = \frac{2C_t(\gamma, \lambda_\alpha)}{[1 + (\lambda_\alpha - 1)F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)]^3}, \]
where

\[ C_t(\gamma, \lambda, \alpha) = \lambda \alpha F(2\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma) \left[1 - F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)\right] \]
\[ - F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma) \left[1 - F(2\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)\right] \]

If \( C_t(\gamma, \lambda, \alpha) > 0 \), \( \partial B_t(\gamma, \lambda, \alpha)/\partial \lambda > 0 \). Property 1 therefore follows from the fact that

\[ C_t(\gamma, \lambda, \alpha) > (\lambda - 1) F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma) \left[1 - F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)\right] > 0. \]

By differentiation of equation (23),

\[ \frac{\partial B_t(\gamma, \lambda, \alpha)}{\partial y_t} = - \frac{\phi(\lambda - 1) D_t(\gamma, \lambda, \alpha)}{\sigma [1 + (\lambda - 1) F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)]^2}, \]

where

\[ D_t(\gamma, \lambda, \alpha) = F'(2\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma) \left[1 + \lambda + (\lambda - 1) F(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)\right] \]
\[ - 2F'(\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma) \left[1 + (\lambda - 1) F(2\gamma \sigma + (\delta - \mu - \phi y_t)/\sigma)\right]. \]

If \( D_t(0, \lambda, \alpha) > 0 \), \( \partial B_t(0, \lambda, \alpha)/\partial y_t < 0 \). Property 2 therefore follows from the fact that

\[ D_t(0, \lambda, \alpha) = (\lambda - 1) F'(\delta - \mu - \phi y_t)/\sigma) \left[1 - (\lambda - 1) F(\delta - \mu - \phi y_t)/\sigma)\right] > 0 \]

if inequality (24) holds.
Appendix B. Consumption and Asset Returns Data

I. Consumption

Annual consumption data for the sample period 1929–2001 is from the U.S. national accounts. Following convention, consumption is measured as the (chain-weighted) sum of real personal consumption expenditures (PCE) on nondurable goods and services, divided by the population. In matching consumption to returns data, I use “beginning of the period” timing convention, following Campbell (2003). In other words, the reported consumption for each year $t$ is assumed to be the flow on the first (rather than the last) day of year $t$.

II. Asset Returns

Excess returns on the market portfolio, returns on the SMB portfolio, and returns on the HML portfolio are from Kenneth French’s webpage. The excess market return is the return on a value-weighted portfolio of NYSE, AMEX, and Nasdaq stocks minus the one-month T-bill rate. The SMB and HML portfolios are based on the six Fama-French benchmark portfolios sorted by size (breakpoint at the median) and book-to-market equity (breakpoints at the 30th and 70th percentiles). The SMB return is the difference in average returns between the three small and three big stock portfolios. The HML return is the difference in average returns between the two high and two low book-to-market portfolios. See Fama and French (1993) for further details.

The three-month T-bill rate is from the Center for Research in Security Prices (CRSP) Indices database. All nominal returns are deflated by the price index for PCE on nondurable goods and services.

III. Instruments

The dividend-price ratio is constructed as the dividend over the past year divided by the current price for the CRSP NYSE-AMEX value-weighted portfolio. The dividend-price ratio
is related, by a present-value relationship, to the expectation of future returns and dividend
growth and therefore predicts returns (Campbell and Shiller 1988).

Annual book equity and monthly market equity data for the six Fama-French benchmark
portfolios is from Kenneth French’s webpage. Following Cohen, Polk, and Vuolteenaho
(2003), the book-to-market equity for each of the six portfolios is computed as the book
equity in December of \( t - 1 \) divided by the market equity in December of \( t \).

The value spread is the difference in average book-to-market equity between the two
high and two low book-to-market portfolios. The value spread is related, by a present-value
relationship, to the expectation of future returns and profitability and therefore predicts
HML returns (Cohen, Polk, and Vuolteenaho 2003). The size spread is the difference in the
average book-to-market equity between the three small and three big stock portfolios.

Following Fama and French (1989), the long yield used in computing the yield spread is
Moody’s Seasoned Aaa corporate bond yield. The short rate used is the one-month T-bill
rate from the CRSP Fama Risk Free Rates database. The yield spread is counter-cyclical
and predicts excess returns on stocks and bonds (Fama and French 1989).

**Appendix C. GMM Tests Robust to Weak Identification**

Let \( \theta \) be an \( N \)-dimensional parameter vector in the interior of a compact parameter space
\( \Theta \). The true parameter \( \theta_0 \) is assumed to satisfy \( M \) conditional moment restrictions

\[
E_{t-1}[h(y_t, \theta_0)] = 0. \tag{30}
\]

Let \( z_{t-1} \) be a vector of \( I \) instrumental variables known at \( t - 1 \), and define the moment
function

\[
g_t(\theta) = h(y_t, \theta) \otimes z_{t-1}. \tag{31}
\]
The continuous updating GMM estimator minimizes the objective function

\[ S(\theta) = T g(\theta)' \Omega(\theta)^{-1} g(\theta), \]  

(32)

where

\[ g(\theta) = \frac{1}{T} \sum_{t=1}^{T} g_t(\theta), \]

\[ \Omega(\theta) = \frac{1}{T} \sum_{t=1}^{T} g_t(\theta) g_t(\theta)' . \]

Weak identification occurs when the population objective function \( \mathbb{E}[g_t(\theta)] \) is close to zero for a large set of \( \theta \neq \theta_0 \). When \( g_t(\theta) \) is linear in \( \theta \) (i.e., linear instrumental variables regression model), weak identification is more commonly referred to as “weak instruments”. When there is weak identification, conventional GMM tests may be invalid, that is reject the null hypothesis too frequently, even in large samples. For a survey of weak identification in GMM, see Stock, Wright, and Yogo (2002). For its empirical relevance in estimating asset pricing moment restrictions, see Neely, Roy, and Whiteman (2001), Stock and Wright (2000), and Yogo (2004).

Following Stock and Wright (2000), partition the parameter vector as \( \theta = (\theta_W', \theta_S)' \). \( \theta_W \) is a \( N_W \)-dimensional subvector of weakly identified parameters, and \( \theta_S \) is a \( N_S \)-dimensional subvector of strongly identified parameters. Therefore, \( \theta_W \) denotes the dimensions of \( \theta \) for which the population objective function is close zero for a large set of \( \theta_W \neq \theta_{W0} \).

Stock and Wright (2000) propose a test for \( \theta_W \), based on the continuous updating GMM objective function, that is valid even when there is weak identification. For a given \( \theta_W \), let

\[ \hat{\theta}_S(\theta_W) = \arg \min_{\theta_S \in \Theta_S} S(\theta) \]  

(33)

be the estimate of \( \theta_S \) that minimizes the objective function. Let \( \hat{\theta}_W = (\theta_W', \hat{\theta}_S(\theta_W)')' \). Under
the null \( \theta_W = \theta_{W0} \), the statistic \( S(\hat{\theta}_W) \) has the asymptotic distribution \( \chi^2_{M-I-N_S} \) (Stock and Wright 2000, Theorem 3).

Guggenberger and Smith (2003) propose a test for \( \theta_W \), based on the gradient of the objective function, that is valid even when there is weak identification. Let \( G_{Wt} = \partial g_t(\theta)/\partial \theta_W \) and \( G_{St} = \partial g_t(\theta)/\partial \theta_S \). Define the statistic

\[
LM(\hat{\theta}_W) = T g(\hat{\theta}_W)'\Omega(\hat{\theta}_W)^{-1}D(\hat{\theta}_W)[D(\hat{\theta}_W)'M(\hat{\theta}_W)D(\hat{\theta}_W)]^{-1}D(\hat{\theta}_W)'\Omega(\hat{\theta}_W)^{-1}g(\hat{\theta}_W),
\]

where

\[
D(\hat{\theta}_W) = -\frac{1}{T} \sum_{t=1}^{T} G_{Wt}(\hat{\theta}_W)[1 - g_t(\hat{\theta}_W)'\Omega(\hat{\theta}_W)^{-1}g(\hat{\theta}_W)],
\]

\[
M(\hat{\theta}_W) = \Omega(\hat{\theta}_W)^{-1} - \Omega(\hat{\theta}_W)^{-1}G_S(\hat{\theta}_W)[G_S(\hat{\theta}_W)'\Omega(\hat{\theta}_W)^{-1}G_S(\hat{\theta}_W)]^{-1}G_S(\hat{\theta}_W)'\Omega(\hat{\theta}_W)^{-1}.
\]

Under the null \( \theta_W = \theta_{W0} \), the statistic \( LM(\hat{\theta}_W) \) has the asymptotic distribution \( \chi^2_{N_W} \).
REFERENCES


Table I
Certainty Equivalent of Small and Large Gambles under Power Reference-Dependent Utility

Suppose the household’s initial wealth and its reference level of consumption are $75,000. Panel A (B) reports sure losses in consumption that equate utility to an equi-probable gamble of $375 ($25,000) in wealth. Various parameter combinations ($\alpha, \gamma, \theta$) for power reference-dependent utility are considered, including standard power utility ($\alpha = 1$) and linear reference-dependent utility ($\theta = 0$). The degree of loss aversion is fixed at $\lambda = 2.25$.

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<th>$\alpha = 0.5$</th>
<th>$\alpha = 0$</th>
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<td>$\theta = 0$</td>
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Panel A: Small Gambles ($\pm$375)

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</table>

Panel B: Large Gambles ($\pm$25,000)
Table II
Descriptive Statistics for Consumption and Asset Returns
The table reports the mean and standard deviation of log consumption growth, T-bill rate, excess market return, SMB return, and HML return. The Sharpe ratio is the mean excess return divided by the standard deviation. All returns are deflated by the price index for consumption, and the sample period is annual 1929–2001.

<table>
<thead>
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<th>Variable</th>
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<th>S.D. (%)</th>
<th>Sharpe Ratio</th>
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<tbody>
<tr>
<td>Consumption Growth</td>
<td>1.88</td>
<td>2.27</td>
<td></td>
</tr>
<tr>
<td>T-bill Rate</td>
<td>1.06</td>
<td>4.47</td>
<td></td>
</tr>
<tr>
<td>Market Return</td>
<td>7.26</td>
<td>20.40</td>
<td>0.36</td>
</tr>
<tr>
<td>SMB Return</td>
<td>3.19</td>
<td>14.20</td>
<td>0.22</td>
</tr>
<tr>
<td>HML Return</td>
<td>4.99</td>
<td>14.07</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Table III  
Parameters of the Reference-Dependent Model  
The first column reports the parameters used in calibration of the linear reference-dependent model. The second column reports estimates of the parameters for the power reference-dependent model. The test assets are the three-month T-bill, CRSP value-weighted portfolio, SMB portfolio, and HML portfolio. The instruments are lagged consumption growth, dividend-price ratio, size spread, value spread, yield spread, and a constant. Estimation is by continuous updating GMM. Standard errors and $p$-values for the $J$-test (test of overidentifying restrictions) in parentheses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibration</th>
<th>GMM Estimation</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>0.90</td>
<td>(0.20)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.00</td>
<td>0.65</td>
<td>(9.47)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.00</td>
<td>0.08</td>
<td>(0.20)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>2.25</td>
<td>3.21</td>
<td>(0.95)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.00</td>
<td>0.02</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.51</td>
<td>0.33</td>
<td>(0.39)</td>
</tr>
<tr>
<td>$J$-test</td>
<td></td>
<td>30.90</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>
Figure 1
Calibration of the Linear Reference-Dependent Model

The linear-reference dependent model is calibrated using parameters reported in Table III and annual consumption data in the sample period 1929–2001. Panel A is a plot of the log consumption-habit ratio. Panel B (Panel C) is a plot of the riskfree rate (maximum Sharpe ratio) implied by the model.
Figure 2
GMM Objective Function for the Power Reference-Dependent Model

The figure is a plot of the GMM objective function for the power reference-dependent model, concentrated in the parameters $\gamma$ and $\lambda$. 