DOCUMENT DE TRAVAIL 2002-003

ROUGH APPROXIMATION OF A PREFERENCE RELATION BY A MULTI-ATTRIBUTE DOMINANCE FOR DETERMINISTIC, STOCHASTIC AND FUZZY EVALUATION PROBLEMS

Kazimierz Zaras
Rough Approximation of a Preference Relation by a Multi-Attribute Dominance for Deterministic, Stochastic and Fuzzy Evaluation Problems

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Abstract

Let $A$ be a set of alternatives evaluated by a set of attributes. Three kinds of evaluations will be considered in this paper: deterministic, stochastic or fuzzy in relation to each attribute. The mixed-data multi-attribute dominance for a reduced number of attributes ($\text{MMD}_r$) will be suggested to model the preferences in this kind of problem. The case of mixed data, where we have the attributes of different nature is not well known in the literature, although it is essential from a practical point of view. To apply the $\text{MMD}_r$, the subset $R$ of attributes from which approximation of the global preference is valid should be known. The theory of Rough Sets gives us an answer to this issue. In order to represent preferential information we shall use a pairwise comparison table. This table is built for subset $B \subseteq A$ described by Dominance relations for particular attributes and a total order for the decision attribute given by the decision maker (DM). Using a Rough Set approach to the analysis of the subset of preference relations which includes the subset of $\text{MMD}_r$, a set of decision rules is obtained, and these are applied to a set $A \setminus B$ of potential alternatives.

Keywords: Preferences; Multiple attributes; Rough Sets; Mixed-data dominance; Mixed-data multi-attribute dominance for mixed data; Decision making
1. Introduction

Usually, the qualitative and quantitative risk assessment provides a conceptual framework for the evaluation of health risks. The first is based on the experts’ knowledge, the second is based on the statistical estimations. The nature of the qualitative risk assessment is rather fuzzy and the quantitative risk assessment is probabilistic. How can we put them together in terms of impacts to do global risk analysis?

The other example can be shown from the decision making process of the National Aerospace Defence. The Decision Support System has been built to support the Fighters Group Operation Center (FGOC). The purpose of this DSS is to aid the commander in choosing the best course of action which will stop Aerospace violations (interception, supervision, put pressure on enemy to land and so on). To make this kind of decision the commander has to consider several attributes of different natures: determinist (cost of equipment), probabilistic (risk of loss of pilots in battle or the equipment break down) or fuzzy (risk of bad timing in resource deployment). How can we do this kind of analysis? The answer to this question is the issue of this paper. The case of mixed data, where we have the attributes of different natures is not well known in literature.

However, Munda et al. (1995) suggested a multicriteria model whose data matrix may include either crisp, stochastic or fuzzy evaluations of the performance of an alternative $a_i$ with respect to a criterion $g_k$. In this model the standardisation of the
various performance evaluations is done by means of the notion of fuzzy relations and linguistic quantifiers. In the aggregation process, particular attention is paid to the problem of diversity of the single evaluations, while the entropy concept is used as a measure of the associated fuzziness.

In our approach, the standardisation is made by the extension of the dominance notion to all three types of evaluations (deterministic, probabilistic or fuzzy). In the aggregation process, we try to identify the vagueness and uncertainty of the overall preference by using the Rough Sets approach suggested by Greco, Matarazzo and Slowinski (1999). The residual part of the preference aggregation allows us to deduce a set of “if ... then “ rules which represent the model of preference of the DM described by the multi-attribute dominances MMD disrespect.

The other multicriteria model which allows us to consider the mixed data is called PAMSSEM (Procédure d’Agrégation du type Surclassement de Synthèse pour évaluations mixtes) and it was suggested by Martel, Kiss and Rousseau (1997). In this model the authors integrate the fuzzy language and the probabilistic language by using a fuzzy outranking relation such as suggested in ELECTRE III method.

This paper is structured as follows. The problem is formulated in section 2. Section 3 presents the results emanating from the Rough Sets methodology. In section 4, these results are applied to solve a modified example of choosing between ten computer development projects from (Martel and Zaras (1995)).
2. Formulation of the problem

The Muli-Attribute problem, which is considered in this paper, belongs to the class of problems which can be represented by the AXZ model where:

A – is a finite set of potential alternatives \( a_i \), \( i=1,2, ..., n \);

X – is a finite set of attributes \( k \) of different natures, deterministic, probabilistic or fuzzy, \( k=1,2, ..., m \);

Z – is the set of deterministic, probabilistic or fuzzy evaluations expressed by the function \( f_{ik}(x_k) \) which associates the performance of the alternative \( a_i \) with respect to the attribute \( X_k \).

These attributes are defined such that a larger value is preferred to a smaller value (“more is better”) and that the final form of the function \( f_{ik}(x_k) \) depends on the input information. If the nature of this information is probabilistic, we can have the continuous or discrete case. In the continuous case, the function \( f_{ik}(x_k) \) is a density function, which verifies the equation as follows:

\[
\int f_{ik}(x_k) \, d(x_k) = 1
\]  

(1)

In the fuzzy case, let \( \eta_{ik}(x_k) \) be a membership function of the evaluation of the alternative \( a_i \) with respect to the attribute \( X_k \). Then the function is obtained by rescaling the ordinates (Munda et al, 1995) of \( \eta_{ik}(x_k) \) such that:

\[
f_{ik}(x_k) = b \, \eta_{ik}(x_k)
\]  

(2)
where b is coefficient such that \( \int f_{ik}(x_k) \, d(x_k) = 1 \) like in the probabilistic case.

In the probabilistic and discrete case, we use Dirac’s pseudo-function \( \delta_{ik}(x_k) \) to define \( f_{ik}(x_k) \) like in (Czolgala, 1990) such that:

\[
f_{ik}(x_k) = \sum w_{kz} \delta_{ik}(x_k - x_{ks})
\]  

(3)

where \( w_{kz} \) denote respective probabilities which fulfil the equation that

\[
\sum w_{ks} = 1 \text{ and } s = 1, 2, ..., t
\]

By definition, the cumulated distribution function is as follows:

\[
F_{ik}(x_k) = \sum f_{ik}(x_{ks}) \text{ for all } z \text{ such that } x_{ks} < x_k
\]  

(4)

This formulation of the \( f_{ik}(x_k) \) allows us to consider the determinist evaluation \( x_k \) as a particular case where \( w_{ks} = 1 \) and the cumulated distribution \( F_{ik}(x_k) \) can be represented by a unit step function such that:

\[
F_{ik}(x_k) = 0 \text{ if } x_k < x_{ks};
F_{ik}(x_k) = 1 \text{ if } x_k \geq x_{ks}.
\]  

(5)

The comparison between two alternatives, \( a_i, a_j \in A \) leads to the comparison of two vectors of evaluations \( \{ f_{i1}(x_1), ..., f_{im}(x_m) \} \) and \( \{ f_{j1}(x_1), ..., f_{jm}(x_m) \} \). On each attribute \( k \) and for each pair \( (a_i, a_j) \in A \times A \), we must compare two evaluations (deterministic, probabilistic or fuzzy) to conclude if “\( a_i \) is at least as good as \( a_j \)”. 
This multi-attribute problem for deterministic, probabilistic or fuzzy evaluations is approached by using results of stochastic dominance conditions extended to mixed data to compare the alternatives, two by two, on each attribute considered individually. Three types of dominances for mixed data are used:

1) **First degree mixed-data dominance (FMD);**

   \[ a_i \text{ FMD } a_j \text{ if only if } F_{ik}(x_k) \neq F_{jk}(x_k) \text{ and } \]
   \[ H_1(x_k) = F_{ik}(x_k) - F_{jk}(x_k) \leq 0, \forall x_k \in \mathbb{R} \]  
   \[ (6) \]

2) **Second degree mixed-data dominance (SMD);**

   \[ a_i \text{ SMD } a_j \text{ if only if } F_{ik}(x_k) \neq F_{jk}(x_k) \text{ and } \]
   \[ H_2(x_k) = \int H_1(y_k) \, dy_k \leq 0, \forall x_k \in \mathbb{R}. \]
   \[ (7) \]

3) **Third degree mixed-data dominance (TMD);**

   \[ a_i \text{ TMD } a_j \text{ if and only if } F_{ik}(x_k) \neq F_{jk}(x_k) \text{ and } \]
   \[ H_3(x_k) = \int H_2(y_k) \, dy_k \leq 0 \text{ and } \mu(F_{ik}(x_k)) \geq \mu(F_{jk}(x_k)), \forall x_k \in \mathbb{R}. \]
   \[ (8) \]

In the probabilistic case, it can be proven (Hadar and Russel, (1969) and Whitmore, (1970)) that if any of these three types of dominance (FMD, SMD or TMD) prevails on attribute k, then \( a_i \) is preferred to \( a_j \) on this attribute for all utility functions which belong to Decreasing Absolute Risk Averse (DARA) family functions. The class of such utility functions denoted by \( U_4 \) can be defined as follows:
\[ U_4 = \{ \frac{U_k(x_k)}{U_k'(x_k)} > 0, U_k''(x_k) \leq 0, U_k'''(x_k) \geq 0; \]
\[ r_k'(x_k) = \left( -\frac{U_k''(x_k)}{U_k'(x_k)} \right) \leq 0, \forall x_k \in \mathbb{R} \} \] (9)

where \( r_k'(x_k) \) is a measure of absolute risk aversion (Pratt, 1964).

In the deterministic case where the evaluations can be expressed by unit step functions the preferences can be modelled based only on the FMD. The FMD is proven for a class of increasing utility functions (Hadar and Russel, (1969)). If two cumulated distributions or membership functions intersect at least one the FMD is not verified. The SMD is applied to model the preferences for probabilistic or fuzzy evaluations. It can also be used if these evaluations are compared in relation to a deterministic level like in the goal programming. The SMD is proven for a concave class of utility functions (Hadar and Russel, (1969)), which corresponds to a decision maker who is a risk averse. If the SMD is not verified we can try to check the TMD to model the preferences.

Generally speaking, for each attribute, if the mixed-data dominance (MD) is verified it can be proven that for an appropriate class of utility functions, the expected utility of the distribution \( f_{ik} = f_{ik}(x_k) \) is greater or equal to the expected utility of the distribution \( f_{jk} = f_{ik}(x_k) \) as follows:

\[ f_{ik} \, MD_k \, f_{jk} \rightarrow E[U_k(f_{ik})] \geq E[U_k(f_{jk})] \] (10)

The suggested approach consists of modelling global preferences which can be based on the multi-attribute dominance (MMD). The rule of this dominance, in the probabilistic case, was given by Huang et al. (1978), and can be expressed in the following manner “the alternative \( a_i \) is at least as great as the alternative \( a_j \)” in
the sense of the MMD, if and only if the alternative \( a_i \) dominates \( a_j \) by one of the dominances MD in relation to each attribute.

In practice, this rule is very rarely verified. The essential characteristic of a multi-attribute problem is that there are several conflicts between attributes and the MMD risks become too poor and useless to aid the DM. Based on the psychological observation that people tend to simplify the multi-attribute complex problem and to make their decision from a reduced subset of the most important attributes, we suggest the mixed-data multi-attribute dominance for a reduced number of attributes MMD\(_r\). This dominance can be defined in the same way as the MMD, but should be verified in relation to the subset \( R \) of \( X \) attributes (see Zaras et al (1994)).

Given \( a_i, a_j \in A \) and \( R \subseteq X \), this kind of the MD noted MMD\(_r\) can be defined as follows:

**Definition 1**

\[
a_i \text{ MMD}_r a_j \text{ if and only if } f_{ik} \text{ MD}_k f_{jk} \text{ for all } X_k \in R \text{ where } r = |R| \text{ and } r \geq 1,
\]

The problem is, what are the best subsets of attributes to approximate global preferences? To answer to this question we can use the idea of the approximation which was taken from the Rough Sets theory, developed by Polish researcher Pawlak (1991) as it was described for the probabilistic case in Zaras (1999). The rough approximations in Zaras (1999) was defined using an indiscernibility relation on the pairwise comparison table as in the original theory of Rough Sets.

From this point the suggested approach is different from Zaras (1999) and follows
Greco et al. (1999) where an indiscernibility relation was substituted by a dominance relation.

3. Rough Approximation of a Preference Relation

3.1. Decision Table

In order to represent the preferential information provided by the DM in the decision table, we will use the pairwise comparison of some alternatives as objects.

Let B be a finite subset of alternatives, considered by the DM as the basis for exemplary pairwise comparisons. Also let C ⊆ X be the set of attributes (condition attributes) describing the actions, and D the decision attribute. The decision table is defined as 4-tuple:

\[ T = \langle H, C \cup D, V_C \cup V_D, g \rangle, \] where H ⊆ B x B is a finite set of pairs of alternatives, C ∪ D are two subsets of attributes, called condition and decision attributes, V_C ∪ V_D are domains of these attributes respectively, and

\[ g : H \times (C \cup D) \rightarrow V_C \cup V_D \] is a total function where \( V_C = \bigcup V_k \).

This function is such that

1) \( g[(a_i, a_j), k] = 1 \), if \( f_{ik} = f_{jk} \) is verified for \( \forall X_k \in C \),
   and \( \forall (a_i, a_j) \in H \);

2) \( g[(a_i, a_j), k] = 0 \), if \( f_{ik} \) not \( f_{jk} \) is not verified \( \forall X_k \in C \),
   and \( \forall (a_i, a_j) \in H \).

and \( g[(a_i, a_j), k] \in V_k \ \forall X_k \in C \), and \( \forall (a_i, a_j) \in H \), and \( g[(a_i, a_j), D] \in V_D \),
∀(a_i, a_j)∈ H.

In our decision table \( g[(a_i, a_j), D] \) can also have two values on \( H \subseteq B \times B \):

1) \( g[(a_i, a_j), D] = P \), if \( a_i \) is preferred to \( a_j \);

2) \( g[(a_i, a_j), D] = N \), if \( a_i \) is not preferred to \( a_j \). \( (13) \)

In general the decision table can be presented as follows:

<table>
<thead>
<tr>
<th></th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>\ldots</th>
<th>( X_m )</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_p )</td>
<td></td>
<td></td>
<td>( g[(a_i, a_j), 1] )</td>
<td>( g[(a_i, a_j), 2] )</td>
<td>\ldots</td>
</tr>
<tr>
<td>( H_N )</td>
<td></td>
<td></td>
<td>( g[(a_i, a_j), 1] )</td>
<td>( g[(a_i, a_j), 2] )</td>
<td>\ldots</td>
</tr>
</tbody>
</table>

where \( H_p \) is the subset of the pairs which express the preferences and \( H_N \) is the subset of the pairs which express non-preferences.

3.2. Rough set analysis of a preference relation

Let \( R \subseteq C \) be a subset of condition attributes. For each pair of alternatives \( (a_i, a_j) \in H \) in the decision table we can identify multi-attribute dominance \( \text{MMD}_r \) and complementary multi-attribute non-dominance \( \text{not} \text{MMD}_c^{-r} \) where \( c^{-r} = |C - R| \). These dominances are like the P-positive dominance and P-negative dominance suggested by Greco et al (1999) for ordinal data.

According to them these dominances and non-dominances satisfy the following property:
Property 1

If \((a_i, a_j) \in MMD_q\) (not \(MMD_q\)) then \((a_i, a_j) \in MMD_r\) (not \(MMD_r\))

for each \(R \subseteq Q\) where \(q = |Q|\). \((14)\)

In this approach we propose to approximate the \(P\)-global preference relation by the \(MMD_r\) relation. Usually this approximation in the Rough Sets methodology is done by \(Q_\ast(P)\)-lower and \(Q^\ast(P)\)-upper approximations. According to Greco et al (1999) these approximations can be defined as follows:

\[Q_\ast(P) = \{(MMD_r \cap H) \subseteq P\},\]
\[Q^\ast(P) = \{(MMD_r \cap H) \supseteq P\}.\] \((15)\)

The \(Q\)-boundary (doubtful region) of a set of preferences \(P\) is defined as follows:

\[BN_Q(P) = Q^\ast(P) - Q_\ast(P)\] \((16)\)

The set \(BN_Q(P)\) contains the \(MMD_r\) which introduces uncertainty in the deduction of the decision rules using the subset of attributes \(Q\). Taking into account property 1, we obtain a certain number of the \(MMD_r\) which verify the condition of the lower approximation. We are looking for the \(MMD_r\) dominances which have the largest intersection with the set of pairs of alternatives. These dominances will be used to deduce the determinist decision rules because they assume the highest quality of approximation.

Analogously, we can approximate non-preference denoted by the letter \(N\) in the decision table by the Multi-Attribute non-dominance for a reduced number of attributes, not \(MMD_r\).
Q^*(N) = \{ (not MMD_{r} \cap H) \subseteq N \}, and
\[ Q^*(N) = \{ (not MMD_{r} \cap H) \supseteq N \} \tag{17} \]

We can deduce a generalised description of the preferential information contained in a given decision table in terms of decision rules. We will consider the following four kinds of decision rules:

1. if  \( a_{i} MMD_{r} a_{j} \rightarrow a_{i} P a_{j} \), which is noted as a \( D_{++} \) decision rule;
2. if  \( a_{i} not MMD_{r} a_{j} \rightarrow a_{i} N a_{j} \), which is noted as a \( D_{+} \) decision rule; \tag{18}
3. if  \( a_{i} not MMD_{r} a_{j} \rightarrow a_{i} P a_{j} \), which is noted as a \( D_{-}^{+} \) decision rule;
4. if  \( a_{i} MMD_{r} a_{j} \rightarrow a_{i} N a_{j} \), which is noted as a \( D_{-}^{-} \) decision rule;

The final set of decision rules is the set of minimal decision rules. According to Greco et al (1999) a \( D_{++} \) decision rule [\( D_{+} \) decision rule] \( a_{i} MMD_{q} a_{j} \rightarrow a_{i} P a_{j} \) [\( a_{i} not MMD_{q} a_{j} \rightarrow a_{i} N a_{j} \)] will be called minimal if there is not any other rule \( a_{i} MMD_{r} a_{j} \rightarrow a_{i} P a_{j} \) [\( a_{i} not MMD_{r} a_{j} \rightarrow a_{i} N a_{j} \)] such that \( R \subseteq Q \).

A \( D_{-}^{+} \) decision rule [\( D_{-}^{-} \) decision rule] \( a_{i} not MMD_{q} a_{j} \rightarrow a_{i} P a_{j} \) [\( a_{i} MMD_{q} a_{j} \rightarrow a_{i} N a_{j} \)] will be called minimal if there is not any other rule \( a_{i} not MMD_{r} a_{j} \rightarrow a_{i} P a_{j} \) [\( a_{i} MMD_{r} a_{j} \rightarrow a_{i} N a_{j} \)] such that \( R \subseteq Q \).

4. Application

To illustrate the application of the Rough Set approach modelling the preferences relation using mixed-data multi-attribute dominance for a reduced number of attributes, let us consider the example of the project selection problem where ten projects are evaluated in relation to three attributes. One attribute \( X_{1} \): discounted
cost is deterministic, the second $X_2$ : chances of success is probabilistic in relation to which seven experts evaluate the probability of the success on a scale of ten and $X_3$ : the technological level is a rather fuzzy attribute where experts make their evaluation using “about” to indicate the technological level on scale of ten from low-tech to high tech. The example of these evaluations in relation to the attribute $X_2$: chances of success and $X_1 :$ discounted cost (in hundred of thousands dollars) are presented in Table 2.

Table 2. Distributional and Deterministic Evaluations

<table>
<thead>
<tr>
<th>Scale of $X_2$ /Proj.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>$X_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2/7</td>
<td>1/7</td>
<td>2/7</td>
<td>1/7</td>
<td>1/7</td>
<td>-6.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td></td>
<td>1/7</td>
<td>1/7</td>
<td>2/7</td>
<td>3/7</td>
<td>-3.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td></td>
<td></td>
<td>4/7</td>
<td>3/7</td>
<td></td>
<td>-3.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P4</td>
<td></td>
<td></td>
<td></td>
<td>1/7</td>
<td>2/7</td>
<td>-4.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P5</td>
<td></td>
<td></td>
<td></td>
<td>2/7</td>
<td>3/7</td>
<td>1/7</td>
<td>1/7</td>
<td>-4.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P6</td>
<td></td>
<td></td>
<td>1/7</td>
<td>1/7</td>
<td>2/7</td>
<td>1/7</td>
<td>2/7</td>
<td>-4.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P7</td>
<td>3/7</td>
<td>1/7</td>
<td>2/7</td>
<td></td>
<td></td>
<td>-6.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P8</td>
<td>2/7</td>
<td>1/7</td>
<td>4/7</td>
<td></td>
<td></td>
<td>-7.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P9</td>
<td>1/7</td>
<td>2/7</td>
<td>2/7</td>
<td></td>
<td></td>
<td>-5.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P10</td>
<td>1/6</td>
<td>2/6</td>
<td></td>
<td>1/6</td>
<td>2/6</td>
<td>-5.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The evaluation “about” is expressed by a fuzzy number whose membership function can be defined in the following manner:

$$\eta_{ik}(x_k) = 0 \text{ if } x_k \leq a-c \text{ or } x_k \geq a-c;$$

$$\eta_{ik}(x_k) = \frac{x_k}{c} + (1 - a/c) \text{ if } a-c < x_k < a; \quad (19)$$
\[ \eta_{ik}(x_k) = 1 \text{ if } x_k = a; \]

\[ \eta_{ik}(x_k) = -\frac{x_k}{c} + (1 + \frac{a}{c}) \text{ if } a < x_k < a+c; \]

The estimation depends on two parameters where a -design estimated technological level is introduced by a project where the membership function reaches the maximum i.e. the value equal to one and the parameter \( c \), which is linked to the possible domain of the definition of the membership function. The values of these parameters for ten projects are given in Table 3.

<table>
<thead>
<tr>
<th>P</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>5</td>
<td>8</td>
<td>7</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>c</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>b</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>1</td>
<td>1/2</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>1/2</td>
</tr>
</tbody>
</table>

The parameter \( b \) necessary to calculate the function \( f_{ik}(x_k) \) for considered membership function according to condition (2) is as follow:

\[ b = \frac{1}{c} \quad (20) \]

To apply our approach, it is first necessary to establish the types of pairwise dominance relations MD for each pair of projects using each attribute.
Table 4 shows us the MD identified in relation to the attribute $X_2$ and table 5 shows us the MD identified in relation to the attribute $X_3$.

Table 4. Dominance relations MD identified in respect to the attribute : $X_2$-Chances of success

<table>
<thead>
<tr>
<th>Projects</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<td>-</td>
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</tr>
<tr>
<td>P2</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
<td>SMD</td>
<td>FMD</td>
<td>FMD</td>
<td>FMD</td>
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</tr>
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<td>P3</td>
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<td>FMD</td>
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</tr>
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<td>FMD</td>
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</tr>
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<td>P10</td>
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<td>-</td>
<td>-</td>
<td>FMD</td>
<td>FMD</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5. Dominance relations MD identified in respect to the attribute : $X_3$- Technological level

<table>
<thead>
<tr>
<th>Projects</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
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</thead>
<tbody>
<tr>
<td>P1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>SMD</td>
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</tr>
<tr>
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<td>-</td>
<td>FMD</td>
<td>-</td>
<td>-</td>
<td>FMD</td>
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<td>FMD</td>
<td>FMD</td>
<td>FMD</td>
</tr>
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<td>SMD</td>
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<td>FMD</td>
<td>FMD</td>
</tr>
<tr>
<td>P4</td>
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<td>FMD</td>
<td>FMD</td>
<td>FMD</td>
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</tr>
<tr>
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<td>FMD</td>
<td>FMD</td>
<td>FMD</td>
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<tr>
<td>P9</td>
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<td>FMD</td>
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<tr>
<td>P10</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>SMD</td>
<td>SMD</td>
<td>FMD</td>
<td>FMD</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Four sorting projects given by the DM create a training set to build the decision table as a pairwise comparison table presented in Table 6.
Table 6. Decision Table

<table>
<thead>
<tr>
<th></th>
<th>X₁</th>
<th>X₂</th>
<th>X₃</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P₃,P₄)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>P</td>
</tr>
<tr>
<td>(P₃,P₂)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>P</td>
</tr>
<tr>
<td>(P₃,P₁)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>P</td>
</tr>
<tr>
<td>(P₄,P₂)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>P</td>
</tr>
<tr>
<td>(P₄,P₁)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>P</td>
</tr>
<tr>
<td>(P₂,P₁)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>P</td>
</tr>
<tr>
<td>(P₄,P₃)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>(P₂,P₃)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>(P₁,P₃)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>(P₂,P₄)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>(P₁,P₄)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
<tr>
<td>(P₁,P₂)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>N</td>
</tr>
</tbody>
</table>

In the decision table, we have the result of identified dominance MD in relation to each condition attribute, and the decision attribute D makes a dichotomic partition of the set of pair projects: $V_D = P$ means preference, $V_D = N$ means non-preference. According to the definition (15) we can obtain two dominances which intersect with three pairs of projects and which verify the condition of the lower approximation. In table 7, we can then see that these two dominances have the same largest intersection.
Table 7.

<table>
<thead>
<tr>
<th>MMDᵣ /H</th>
<th>(P3,P1)</th>
<th>(P4,P1)</th>
<th>(P2,P1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R={X₁, X₂, X₃}</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>R={ X₂, X₃}</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Among two MMDᵣ which verify the condition of the largest intersection with the pairs of alternative we chose the minimal decision rule which can be formulated as follows:

**Rule 1**
If $Pᵣ MMD₂ Pⱼ$ in relation to two attributes $\{X₂, X₃\}$ then $Pᵣ Pⱼ$

In the same way we can do the approximation of non-preference $N$ by the Multi-Attribute non-dominance not MMDᵣ. According to the definition (16) in table 6 we obtain two non-dominances which intersect with three pairs of projects and which verify the condition of lower approximation.

Table 8.

<table>
<thead>
<tr>
<th>Not MMDᵣ /H</th>
<th>(P1,P3)</th>
<th>(P1,P4)</th>
<th>(P1,P2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R={X₁, X₂, X₃}</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>R={ X₂, X₃}</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Based on the same principle as in the case of preferences choosing the minimal decision rule we suggest the following:

**Rule 2**
If $Pᵣ not MMD₂ Pⱼ$ in relation to two attributes $\{X₂, X₃\}$ then $Pᵣ N Pⱼ$

The last step of the suggested methodology is to apply the rules to order the whole set of sites. The overall binary preference relation noted ($P$) is identified if, rule 1
is fulfilled between projects. If the second rule is fulfilled, the overall non-preference is identified which is noted (N). In general we can have one of the four following situations of the preference defined in (18):

For each project \( P_i \), let

\[
SC^{++}(P_i) = \text{card}\{ P_j \in A: \text{there is at least one } D_{++} \text{-decision rule and/or at least one } D_{\leftrightarrow} \text{ decision rule stating that } P_i \mathbin{P} P_j \},
\]

\[
SC^{+-}(P_i) = \text{card}\{ P_j \in A: \text{there is at least one } D_{\rightarrow} \text{-decision rule stating that } P_j \mathbin{P} P_i \},
\]

\[
SC^{-+}(P_i) = \text{card}\{ P_j \in A: \text{there is at least one } D_{\leftarrow} \text{-decision rule stating that } P_j \mathbin{N} P_i \},
\]

\[
SC^{--}(P_i) = \text{card}\{ P_j \in A: \text{there is at least one } D_{\nleftrightarrow} \text{-decision rule and/or at least one } D_{\rightarrow} \text{-decision rule stating that } P_i \mathbin{NS} P_j \}.
\]

For each \( P_i \in A \) we assign a score \( SNF(P_i) \), called *Net Flow Score*,

(see Greco et al., 1999), \( SNF(P_i) = SC^{++}(P_i) - SC^{+-}(P_i) + SC^{-+}(P_i) - SC^{--}(P_i) \).

In ranking problems, the final recommendation is the total preorder established by \( SNF(P_i) \) on the set of projects. The list of pairs of projects supporting decision rules are presented in Table 9. Exploitation of the preference relations \( P_i \mathbin{P} P_j \) and \( P_i \mathbin{N} P_j \) obtained from the application of decision rules assigns a score to the projects and leads to the ranking shown in Table 10.
Table 9. Pairs of projects supporting decision rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>Pairs of projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(P2,P1), (P3,P1), (P4,P1), (P5,P1), (P6,P1), (P9,P1), (P2,P6), (P3,P6), (P5,P6), (P2,P7), (P4,P7), (P3,P7), (P5,P7), (P1,P8), (P2,P8), (P3,P8), (P4,P8), (P5,P8), (P6,P8), (P7,P8), (P9,P8), (P10,P8), (P2,P9), (P3,P9), (P4,P9), (P5,P9), (P6,P9), (P10,P9), (P2,P10), (P4,P10), (P5,P10)</td>
</tr>
<tr>
<td>2</td>
<td>(P1,P1), (P8,P1), (P1,P2), (P2,P2), (P6,P2), (P7,P2), (P8,P2), (P9,P2), (P10,P2), (P1,P3), (P3,P3), (P6,P3), (P7,P3), (P8,P3), (P9,P3), (P10,P3), (P1,P4), (P4,P4), (P7,P4), (P8,P4), (P9,P4), (P10,P4), (P1,P5), (P2,P5), (P5,P5), (P7,P5), (P8,P5), (P9,P5), (P10,P5), (P6,P6), (P1,P6), (P7,P6), (P8,P6), (P9,P6), (P7,P7), (P8,P7), (P8,P8), (P1,P9), (P8,P9), (P9,P9), (P7,P10), (P8,P10), (P10,P10)</td>
</tr>
</tbody>
</table>

Table 10. Ranking according to the Rough Set approach

<table>
<thead>
<tr>
<th>Projects</th>
<th>S++</th>
<th>S+</th>
<th>S++</th>
<th>S-</th>
<th>SNF</th>
<th>Rank</th>
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</thead>
<tbody>
<tr>
<td>P5</td>
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<td>7</td>
<td>1</td>
<td>12</td>
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<tr>
<td>P2</td>
<td>6</td>
<td>0</td>
<td>7</td>
<td>2</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>P3</td>
<td>5</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>P4</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>P6</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>P10</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>-3</td>
<td>5</td>
</tr>
</tbody>
</table>
5. Conclusions

Three kinds of evaluations were considered in this paper: deterministic, stochastic or fuzzy in relation to each attribute. The mixed-data dominances (MD) were defined and suggested to model the preferences in relation to each attribute. The global preferences on the set of alternatives are approximated by means of mixed-data multi-attribute dominance rules (MMD_r) for a reduced number of attributes. The rules represent the preference model of the DM, which can be applied to a new set of potential alternatives. This approximation is based on the Rough Sets methodology.

References


