A MULTICRITERION APPROACH FOR SELECTING ATTRACTIVE PORTFOLIO

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Abstract

According to the conventional theory of finance, maximizing return with minimum risk should be a hallmark of every successful investor. However, contrary to the theoretical expectations of the conventional theory, the tests achieved on most financial markets have revealed the existence of other variables. Moreover, behavioral aspects, like the investor's attitude to solvency and liquidity, are not taken into consideration. Then the problem of selecting an attractive portfolio is a multicriteria issue which should be tackled by using the appropriate techniques. A Multicriterion approach is proposed and applied to the Tunisian Stock Market to select an attractive portfolio.

Keywords: Portfolio Management, Multicriteria Methods, Finance.

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1. Introduction

In financial theory, models allowing the selection of an optimal portfolio are all inspired from the conventional theory (Markowitz one) which is exclusively based on both the criteria of expected return and risk measured by the variance. The Markowitz's theory of the portfolio selection, based upon the mean-variance (M-V) criterion describes how we calculate a portfolio which exhibits the highest expected return for a given level of risk, or the lowest risk for a given level of expected return (efficient portfolio). The problem of portfolio selection is, according to this theory, a simple problem of quadratic programming which consists in minimizing risk while keeping in mind an expected return which should be guaranteed.
The (M-V) formulation developed by Markowitz (1959) can be represented as:

\[
\min_{X_p} \sigma_p^2
\]

subject to \( \text{E}(R_p) = \mu_p \)

\[
X_p'1 = 1
\]

where:

- \( \sigma_p^2 = X_p'VX_p = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \)

- \( \text{E}(R_p) \): Expected portfolio return. It is just a weighted average of the expected returns on the individual stocks; \( \text{E}(R_p) = X_p'R = \sum_{i=1}^{N} x_i r_i \)

- \( X_p \): the vector of proportions of the different assets.

- \( V \) is the matrix of the variances-covariances.

- \( x_i \): proportion invested in stock \( i \). (\( i = 1, \ldots, N \))

- \( \sigma_i^2 \): variance of return in stock \( i \).

- \( \sigma_{ij} \): covariance of returns on stock \( i \) and \( j \).

- \( R \): the vector of expected returns of the individual stocks.

- \( \mu_p \): the expected rate of return on a portfolio \( X_p \) required by an investor.

The fragility of the base hypotheses of Markowitz model was the starting point for some criticisms aimed at this model. In this respect, to obtain the set of the efficient portfolios within the framework of the "M-V" model, it is important to point out that at least one of the two following hypotheses must be verified: (i) that of a quadratic utility function to represent the investor's preferences, and (ii) that of the normal distribution of stock's returns.

The former supposes that the utility function of an investor whose wealth \( W \) is of the type: \( U(W) = a_0 + a_1W + a_2W^2 \). This function can't be consistent with economic theory unless \( a_1 \) is positive and \( a_2 \) is negative. These two restrictions are indispensable in order to find that the amount invested (absolute or relative) in the ventured portfolio increases with the increase in wealth.\(^1\)

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\(^1\) For a certain level of wealth (\( W \)) and a utility function (\( U \)), we define the measure of absolute risk aversion (ARA) as: \( \text{ARA} = -[U''(W) / U(W)] \), and the measure of relative risk aversion (RRA) as \( \text{RRA} = W \times \text{ARA} \). A utility functions must have decreasing ARA (\( \delta \text{ARA}/\delta W < 0 \)) and constant RRA (\( \delta \text{RRA}/\delta W = 0 \)). There are
The latter supposes that the distribution of the stock returns is perfectly symmetrical, which implicitly means that the negative returns are very unlikely! Besides this normality has almost never been verified. Most of the empirical tests have resulted in an asymmetrical and (or) leptokurtic distribution (Choquette et al., 1995).

We must note that the Markowitz model was a target for other criticisms inherent to the difficulty of its implementation arising from the very high number of parameters it requires. For a set of 100 stocks, we hopefully need to have 100 returns, 100 variances and 4950 covariances ($C^2_{100}$). At this level, the simplification suggested by Sharpe (1963) is obvious.\footnote{2}{The model developed by Sharpe (1963) assumes that there exists a linear relationship between the return on individual stock and the return on the market. This model is expressed as: $R_{it} = \gamma + \beta R_{mt} + U_{it}$, where $U_{i} \sim N(0,\sigma^2)$ and $Cov(U_{i},U_{j}) = 0 \forall i \neq j$.}

However it generates more serious problems. On the one hand, the failure to respect the normality of residuals, supposed by Sharpe model, incidentally like any econometric model, can only generate non efficient estimators. The loss of effectiveness is emphasized when the distribution tends to have thick tails (leptokurtic distribution). On the other hand, contrary to Markowitz model which allows the accurate calculation of the minimal variance (measure of the risk) of the efficient portfolio, that of Sharpe gives just an approximate value of this variance. The hypothesis of the residual independence generates an under-evaluation or an over-evaluation, of the variance, depending respectively on whether the covariances dealt with are positives or negatives.\footnote{3}{An estimator is supposed efficient when its variance is equal to the Fréchet's limit. This latter is the inverse of the information of the estimator of $\theta$ (In ($\theta$)), where $In(\theta) = - E[\delta^{2}L(X_{1},X_{n}, \theta)/\delta\theta^{2}]$ (L is the likelihood of the sample).}

In other respects, we must note that the models underlying this theory (Capital Assets Pricing Model and Arbitrage Pricing Theory) were not themselves screened from criticisms. The estimation of these models and the interpretation of the results related to efficiency are so doomed to failure that some authors are even talking about a statistical dragging (Jaffe et al., 1989). Contrary to the theoretical expectations of the conventional theory, the tests achieved on most financial markets (AMEX, NYSE, TSE, Paris'Stock Exchange etc.) have revealed the existence of other variables (baptized as anomalies) which could be used in portfolio selection. The most important anomalies discovered to date may be gathered in two categories: some anomalies are related to the characteristics of the company, such as the size

many utility functions. Those exhibiting linear risk tolerance include the quadratic, logarithmic, power and exponential functions.
measured by stock market capitalization, the price earning ratio (PER), the book value of shares, etc.; others on cyclic phenomena that can be noticed on the stock prices (January effect, week-end effect, etc.).

Together with all of these criticisms, it is important to note that the methodology suggested by the conventional theory to solve the problem of portfolio selection amounts broadly and simply to an optimization of an economic function. The merit of this approach is that it leads to mathematical problems well set, but which are not always representative of reality for: (i) the comparison of several possible actions is rarely made according to a single criterion, (ii) in many cases, the preferences over a criterion can hardly be modeled by a function, and (iii) when there are several objectives, it is impossible to reach them all at once.

Taking into account not only the limits related to Markowitz conventional theory, to the results from the estimation of the models, and to the philosophy of the optimization approach, but also the behavior of investors, who, in addition to the above mentioned anomalies, could have some preferences to solvency and liquidity, we strongly believe in the interest and even in the necessity of the multicriterion approach to solve the problem of portfolio selection.

The aim of this paper is to apply to the Tunisian Stock Market (BVMT)\(^4\) a multicriterion approach to select an attractive portfolio. This paper is organized as follows: following this introduction, section 2 provides the identification of criteria. In section 3, we present the multicriterion procedure. Section 4 presents the application to the Tunisian context. Section 5 summarizes the paper and indicates the possible directions for further research.

2. Multicriterion approach : Construction of criteria

Every decision maker who is faced with a decision problem seeks to find the solution that allows him to get as close as possible to his set objectives. Even if this solution depends on the adopted set of questions, the way to operate is generally the same. It consists in: (i) defining the set of actions, (ii) constructing the set of evaluation criteria, (iii) evaluating the actions according to the retained criteria, and (iv) applying a multicriterion aggregation procedure to these evaluations which represent the decision maker's preferences.

\(^4\) Bourse de Valeurs Mobilières de Tunis
To avoid any redundancy that may affect the clearness of the paper, we confine ourselves in this section to presenting the retained criteria for the portfolio evaluation.

In general, the existing literature makes a distinction between two categories of criteria: the accounting criteria and those based on market value. The set of actions is the set of portfolio that can be made up from of pre-selected stocks.

The accounting criteria are just the ratios used by the analysts (or managers) to give a synthesized and clear idea about the firm's financial situation. There is a large number of them and the use of one criterion or the other depends on the manager's attitude and objectives. However, these ratios are conveniently gathered in three categories the frontiers of which are rather badly defined in that some ratios are overlapping. In general, we distinguish (i) the profitability ratios, (ii) the liquidity and solvency ones, and (iii) those of the financial structure.

- The profitability ratios are expected to reflect the efficiency of the firm's management and exploitation. They aim, through various criteria, at measuring the firm's return under several facets (profit, return on investments, equity's return, earning per share, etc.).
- The solvency and liquidity ratios measure the firm's capacity to cope with its short term commitments. They appraise the firm's aptitude to change its circulating assets into liquidity to pay off its short term debts.
- The ratios of financial structure aim at measuring the importance of debts in relation to the other sources for financing the firm (equity, leasing, etc.). They make it possible to appreciate the risk of bankruptcy and the constraints that are likely to hinder the leaders' freedom of action.

The market criteria are the ones which contain all the information used by the stock analysts to appreciate a stock's performance. The criteria used at this level are the mean return, total risk (variance), systematic risk (bêta), the size measured by the stock capitalization, the PER (price - earning ratio), and the stock liquidity.

Taking into account the strong interaction between some accounting and market criteria, and to avoid any redundancy at the level of the the criteria selected, we have confined ourselves in our paper to the following criteria.
1. Return criterion (C₁): the rate of return is the main criterion recognized by all financial and micro theories to explain the investor behavior. Under some hypothesis, the traditional finance theory states: "the firm should adjust its capital stock until the marginal rate of return on further investment is equal to the cost of capital" (Modigliani and Miller, 1963). Return on the assets are defined as total return, i.e., dividends (Dₜ) plus capital gains (Pₜ - Pₜ₋₁). The equation for a monthly return is:

\[ R_{it} = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}. \]

The mean return is a criterion to maximize

2. Risk criterion (C₂): the major important improvement of the portfolio approach upon the traditional economic theory of the investment decision was the incorporation of the riskiness of an asset. Risk in investment decisions is due to uncertainty about the outcomes (payoffs) which is represented by the variance of the future return around its expected value. Risk can be categorized in two main categories: systematic risk which is associated with market factors that are beyond the control of investor, and unsystematic one which refers to the risk associated with an individual company and its operations and methods of doing business. Unlike systematic risk, the unsystematic one is within the control of the investor, so the financial equilibrium models consider only systematic or market risk measured by the beta. This latter is defined as the covariance of the stock's return with market index return divided by the variance of the market index return. We want to have a very small gap between the

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5 These hypothesis are: complete certainty, perfect capital market and rational wealth maximizing behavior.
6 Pᵣ is actual price of security at period t.
7 The central and normative proposition in the Markowitz theory of investment can be stated as: "If investors are rational, the, other things being equal, for given variance of future returns, they prefer more average return to less average one. If they are risk averters, then other things being equal, for given expected value of future return, they prefer less variance to more one of future return. According to portfolio approach, maximizing return with minimum risk should be hall mark of every successful investor".
8 The systematic risk is due to interest rate risk, reinvestment rate risk, purchasing power risk, exchange rate risk and political one. The subcategories of unsystematic risk are: business risk, financial risk, default risk, credit risk and liquidity one.
9 The well known capital assets pricing model (CAPM) states that the only variable which determines differential expected returns among securities is the systematic risk coefficient (βᵢ) ; the model asserts also that there is a linear relationship between (βᵢ) and expected return (E(Rᵢ)).
systematic risk of stock $i$ and the market one which is equal to one, so we have to, following Hurson and Zapounidis (1997), minimize $|\beta - 1|$ where $
abla\beta = \frac{\sigma_m}{\sigma_i}$.

3. Liquidity criterion (C3): Liquidity refer to the ability to buy or sell a particular security with minimal market impact. Even they think that there is no precise definition of adequate and inadequate liquidity, Stephen (1994) note that liquidity has a profound effect on the behavior of stock prices. The liquidity measure of a stock is the turnover rate. For each year, the turnover is calculated as:

$$\Omega = \frac{\text{total number of shares traded}}{\text{total number of shares outstanding}}$$

It is because liquid assets or current ones have a less volatile return than non current assets, that it could be argued that liquidity criterion depends on risk one. In our opinion, the differential riskness in their non current assets they hold, thus we can accept that the liquidity will not have high association with the market determined risk measure. On the other hand, it is true that higher asset turnover must necessarily generate higher transaction costs which agents should expect to be compensated for in the form of higher expected returns, however we can accept the independence hypothesis between liquidity and return because the pricing models do not depend upon measure of turnover reflecting a separation of expected returns and asset trade at all points in time. The liquidity criterion is to maximize.

4. Size and PER criteria (C4 and C5): Few believe that asset returns are well described by their first two moments. The small firm and the earning price ratio (E/P) effects have recently received wide attention in both the financial and the academic literature. Banz (1981) shows a significant negative relation between abnormal returns and market value of common equity for samples of NYSE. He shows that return and small firms exceed returns on large firms,

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10 A reliable estimate of the bid ask spread should contain three components: (i) the market by/sell quotations, (ii) broker commissions plus taxes, and (iii) the costs of gathering market impact and/or management fees.
even after adjusting for market risk using the CAPM. The American National Bank went a step further, it set up «Market Expansion Fund» of small firm stocks. On other hand, the findings reported in Basu (1977) indicate that the common stock of higher EPR earn, on average, higher risk-adjusted returns, than the common stock of low EPR firms. This EPR effect is clearly significant even after experimental control was exercised over differences in firm size. Thus the EPR and the size must be considered as different criteria which not reflect the same phenomenon. These two criteria are to be maximized.

3. A Multicriterion Aggregation Procedure
Several approaches can be considered for resolving the problem of aggregation of the evaluation according to several conflictual criteria.

- Pure ordinal methods (POM): these models are developed to handle multicriteria problems where large number of alternatives need to be processed and minimal data on preferences is available.
- Analytical hierarchy process (AHP) which are used when a small number of alternatives are to be evaluated, and pairwise comparison data on a ratio scale are available.
- Rating scale method (RSM) which include ELECTRE family proposed by Roy and al. (Roy and Bouyoussou, 1993), and PROMETHEE one proposed by Brans and al. (1984). These procedures are based on thresholds to modelize decision maker's preferences and when the number of criteria retained is relatively small.

In this paper, since the evaluations according to each of the criteria are on a cardinal level scale and the number of relevant criteria is limited just to five, not only to avoid superfluity explained by the important interaction between different criteria, but also in order to obey the condition of readability, while being aware of the exhaustiveness, a procedure belonging to RSM is retained. PROMETHEE II seemed appropriate since it is easier to understand and to carry out than ELECTRE III for example.

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11 We have to adjust for the market risk because it is widely believed that larger firms are less risky than smaller ones.
12 ELECTRE: Élimination et Choix Traduisant la Réalité. The family of ELECTRE methods includes ELECTRE I, II, III, IV, ELECTRE IS and ELECTRE TRI
13 PROMETHEE: Preference Ranking Organization Methods for Enrichment Evaluation. This family includes PROMETHEE I, II, III, IV and V.
PROMETHEE II approach is based on an extension of the criterion notion through the introduction of a function $\Psi$ expressing the decision maker's preference for an action in relation to another. Such a function, which varies between 0 and 1, is defined separately for each criterion. For a criterion $C_j$ to maximize we will have:

$$\Psi_j(a, b) = \begin{cases} 0 & \text{si } C_j(a) \leq C_j(b) \\ H_j(d_j) & \text{si } C_j(a) > C_j(b) \end{cases}$$

where $d_j = C_j(a) - C_j(b)$

In PROMETHEE, we have six types of generalized criteria (see figures 1)

- **True criterion** $\Psi_j(a, b) = \begin{cases} 0 & \text{if } C_j(a) \leq C_j(b) \\ 1 & \text{if } C_j(a) > C_j(b) \end{cases}$  
  $\Rightarrow H_j(d_j) = \begin{cases} 0 & \text{if } d_j = 0 \\ 1 & \text{if } d_j > 0 \end{cases}$ (figure 1, (a))

- **Quasi – criterion** $H_j(d_j) = \begin{cases} 0 & \text{si } d_j \leq q_j \\ 1 & \text{si } d_j > q_j \end{cases}$ where $q_j$ is an indiff erence threshold (figure 1, (b))

- **Preference criterion** $H_j(d_j) = \begin{cases} \frac{d_j}{p_j} & \text{if } d_j \leq q_j \\ 1 & \text{if } d_j > q_j \end{cases}$ where $p_j$ is a strict preference threshold (figure 1, (c))

- **Level** $H_j(d_j) = \begin{cases} 0 & \text{if } d_j \leq q_j \\ 1/2 & \text{if } q_j < d_j \leq p_j \\ 1 & \text{if } d_j > p_j \end{cases}$ (figure 1, (d))

- **Linear Preference with indiff erence zone criterion**

  - $H_j(d_j) = \begin{cases} 0 & \text{if } d_j \leq q_j \\ \frac{d_j - q_j}{p_j - q_j} & \text{if } q_j < d_j \leq p_j \\ 1 & \text{if } d_j > p_j \end{cases}$ (figure 1, (e))

- **Gaussian inversed criterion** $H_j(d_j) = 1 - \exp\left(-\frac{d_j^2}{2\sigma_j^2}\right)$ (figure 1, (f))

where $\sigma_j$ corresponds to bend point of the curve
As we can see, except for the true criterion, for every criterion, one or two thresholds, are required to define these criterion functions.

Then an index of global preference is calculated for each pair of actions :

\[ \Psi(a,b) = \sum_{j=1}^{n} w_j \Psi_j(a,b) \]

where \( w_j \) refers to the weight assigned to the criterion \( C_j \).

Thus we can construct a graph or a matrix of the preference indexes of all actions. After that the out flows and in flows are calculated as follows :

\[ \phi^+(a,b) = \sum_{b\geq a} \Psi(a,b), \text{out flows} \]

\[ \phi^-(a,b) = \sum_{b\leq a} \Psi(b,a), \text{in flows} \]
With PROMETHEE II, we use the notion of net flow : \( \phi(a) = \phi^+(a) - \phi^-(a) \). We conclude that \( a \) is preferred to \( b \) if \( \phi(a) > \phi(b) \) and \( a \) is indifferent to \( b \) if \( \phi(a) = \phi(b) \).

With such procedure, each pair of actions is compared according to each criterion, which implies that the set of comparable actions must be small enough. In the paper of Martel, Khoury and Bergeron (1988), where they use the ELECTRE III procedure, they have proposed a finite set of portfolios to compare. Since the overall portfolios that can be made up from a set of pre-selected stocks is very big, and even infinite, it is impossible to compare all the pairs of portfolios. In a such context, Khoury and Martel (1990) and Zmitri, Martel and Dumas (1990) have proposed a procedure that allows an absolute evaluation of each action by comparing it with two fictitious profiles (portfolios) : one ideal (\( \overline{P} \)) and the other anti-ideal (\( \overline{P} \)). For a criterion \( C_j \) to maximize, we will have \( C_j (\overline{P}) = \max C_j (s_i) \) and for a criterion to minimize \( C_j (\overline{P}) = \min C_j (s_i) \). (where \( S = \{s_1, s_2, \ldots, s_N\} \) the set of \( N \) preselected stocks). In the same way \( C_j (\overline{P}) = \min C_j (s_i) \) or \( \max C_j (s_i) \) depending on whether \( C_j \) is to maximize or to minimize.

Any portfolio \( P \) that can be made up must be between \( P \) et \( \overline{P} \), e. g. \( \phi(P) \leq \phi(P) \leq \phi(\overline{P}) \) (where \( P = X_p \cdot S = \sum_{i=1}^{N} x_i s_i \), \( x_i \) proportion invested in \( s_i \) for the portfolio \( P \)). Since to each vector \( X_p \) corresponds a portfolio, then we will have an infinite number of portfolios.

- For a criterion \( C_j \) to maximize, we have (with \( \Psi \) preference function) :

\[
\Psi_j(P, \overline{P}) = \begin{cases} 
0 & \text{if } C_j(P) = C_j(\overline{P}) \\
H_j\left(C_j(P), C_j(\overline{P})\right) & \text{if } C_j(P) > C_j(\overline{P}) 
\end{cases}
\]

and

\[
\Psi_j(\overline{P}, P) = \begin{cases} 
0 & \text{if } C_j(P) = C_j(\overline{P}) \\
H_j\left(C_j(\overline{P}), C_j(P)\right) & \text{if } C_j(P) < C_j(\overline{P}) 
\end{cases}
\]

- For a criterion \( C_j \) to minimize, we have :
\[
\Psi_j (P, \overline{P}) = \begin{cases} 
0 & \text{if } C_j(P) = C_j(\overline{P}) \\
H_j(C_j(P), C_j(\overline{P})) & \text{if } C_j(P) < C_j(\overline{P}) 
\end{cases}
\]

and

\[
\Psi_j (\overline{P}, P) = \begin{cases} 
0 & \text{if } C_j(P) = C_j(\overline{P}) \\
H_j(C_j(\overline{P}), C_j(P)) & \text{if } C_j(P) > C_j(\overline{P}) 
\end{cases}
\]

where

\[
C_j(P) = \sum_{i=1}^{N} x_i C_j(s_i), \quad C_j(\overline{P}) = \sum_{i=1}^{N} x_i C_j(\overline{s}_i), \quad C_j(P) = \sum_{i=1}^{N} x_i C_j(P), \quad C_j(\overline{P}) = \sum_{i=1}^{N} x_i C_j(\overline{P}),
\]

\[
C_j(P) \text{ and } C_j(\overline{P}) = \min \text{ ou } \max C_j(s_i) \text{ and } \sum_{i=1}^{N} x_i = 1.
\]

According to PROMETHEE II, the absolute evaluation (net flow) of a portfolio P is :
\[
\phi(P) = \phi^+(P) - \phi^-(P).
\]

According to the approach proposed in this paper,
\[
\phi(P) = \sum_{j=1}^{n} w_j \left( \phi_j^+(P) - \phi_j^-(P) \right) = \sum_{j=1}^{n} w_j \left( \Psi_j(P, P) - \Psi_j(\overline{P}, P) \right).
\]

Let's recall to mind that the calculation of \( \Psi_j(P, P) \) and \( \Psi_j(\overline{P}, P) \) requires preference functions \( H_j \) and the thresholds \( q_j, p_j \) or \( \sigma_j \). These functions \( H_j \) on the criterion \( C_j \) and on the portfolio \( P \) used to make comparison. We design these functions by \( H_j^+ \) and \( H_j^- \) when the comparison is made respectively to \( (P) \) or \( (\overline{P}) \). \( q_j^+ \) et \( p_j^+ \) are thresholds connected with the ideal profile and \( q_j^- \) et \( p_j^- \) are thresholds connected with the anti-ideal.

The \( d_j \) argument of \( H_j^+ \) is :
\[
\max \text{ ou } \min C_j(s_i) - \sum_{i=1}^{N} x_i C_j(s_i),
\]

and the \( d_j \) argument of \( H_j^- \) is :
\[
\sum_{i=1}^{N} x_i C_j(s_i) - \min \text{ ou } \max C_j(s_i).
\]

(he minimum or maximum is depending on the optimization sense of the criterion \( C_j \)).

Therefore we are seeking to identify the portfolio \( P \) which maximizes \( \phi(P) \) while verifying the investor's constraints.
Max \( \phi(P) \)

subject to : \( \sum_{i=1}^{N} x_i = 1 \)

and \( 0 \leq x_i \leq x_M \)

where

\( \phi(P) = \sum_{j=1}^{n} w_j \phi_j(P) \)

\( P = X_p' S \)

\( X_p = \{ x_1, \ldots, x_i, \ldots, x_N \} \)

\( S = \{ s_1, \ldots, s_i, \ldots, s_N \} \)

\( x_i : \) proportion invested in stock \( i (i = 1, \ldots, N) \) in portfolio \( P \)

\( N : \) the number of preselected stocks

\( x_M : \) The maximum of proportion which could be invested in each stock

4. Application to Tunisian Context

In this section, we try to lead an application of the proposed methodology to the Tunisian Stock Market (BVMT). The presented table 1 resume the evaluation of 37 pre-selected stocks according to the five criteria mentioned.

- The first column is about the stocks listed on the BVMT.
- The second column is about mean returns. The data used to calculate mean returns consist of monthly prices, and the sample period extends from January 1998 to June 2000. In order to adjust returns calculations for the bouncing of transaction prices between a security's bid and ask prices, we use a bid-ask adjustment technique, first used by Keim (1987) to calculate average daily portfolio returns. Thus we calculate stock returns as follows:

\[
\hat{R}_{jt} = \frac{(1 + R_{jt})(1 + R_{j,t-1})}{1 + R_{j,t-1}} - 1
\]

- The third column is about the absolute value of bêta less one. Bêtas are calculated using monthly returns.
- The fourth column is about turnover (liquidity criterion)
- The fifth column gives the EPR of each stock. The formula used to calculate the EPR is:

\[
\text{EPR} = \frac{\text{Fully diluted earning per share at stock price of the end of June 2000}}{\text{book value per share at the end of the previous fiscal year}}
\]

- The last column is about the measure of size. It has been calculated using the ratio market value (M) to the book value (B); M/B = Stock price at the end of June 2000 to book value per share at the end of the previous fiscal year.
To illustrate our methodology, we use the linear preference criterion which includes only one threshold. Consequently, if we take: $H_j^+ = H_j^-$ and $p_j^+ \neq p_j^-$, then we will have:

Table 1: Stock’s values for the constructed criteria

<table>
<thead>
<tr>
<th>Stocks</th>
<th>Mean return(%) (1)</th>
<th></th>
<th>β - 1</th>
<th>Turnover (%)</th>
<th>EPR</th>
<th>M/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIAT</td>
<td>9,43</td>
<td>0,455</td>
<td>7,72</td>
<td>0,0877</td>
<td>2,2</td>
<td></td>
</tr>
<tr>
<td>BT</td>
<td>22</td>
<td>0,341</td>
<td>4,23</td>
<td>0,0641</td>
<td>7,4</td>
<td></td>
</tr>
<tr>
<td>SPCD</td>
<td>0,01</td>
<td>0,124</td>
<td>3,86</td>
<td>0,0769</td>
<td>8,4</td>
<td></td>
</tr>
<tr>
<td>Monoprix</td>
<td>0,71</td>
<td>0,078</td>
<td>6,34</td>
<td>0,0769</td>
<td>2,79</td>
<td></td>
</tr>
<tr>
<td>UBCI</td>
<td>13,86</td>
<td>0,854</td>
<td>2,16</td>
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\[
\phi_j(P) = \begin{cases} 
\frac{C_j(P) - C_j(P) - p_j^-}{p_j^-} & \text{when } C_j(P) \leq C_j(P) \leq C_j(P) + p_j^+ \\
1 - 1 = 0 & \text{when } C_j(P) + p_j^- \leq C_j(P) \leq C_j(P) - p_j^+ \\
\frac{p_j^+ + C_j(P) - C_j(\overline{P})}{p_j^+} & \text{when } C_j(\overline{P}) - p_j^+ \leq C_j(P) \leq C_j(\overline{P}) 
\end{cases}
\]

This function is illustrated in the figure 2.

![Figure 2](image)

**Figure 2**: Net flows of a linear preference criterion

In order to avoid the multiple solutions, we set \( p_j^+ = p_j^- = \frac{|C_j(\overline{P}) - C_j(P)|}{2} \). Then the net flow is located on a straight line going from -1 to +1 and there is no landing. There is no landing either if we set \( p_j^+ + p_j^- = \left|C_j(\overline{P}) + C_j(P)\right| \), but then the net flow consists of two segments of a straight line with different slopes.

We propose such values for our application:

\[
\begin{align*}
p_j^+ & \quad 0.61 \quad 0.54 \quad 0.70 \quad 0.08 \quad 6.4 \\
p_j^- & \quad 0.40 \quad 0.4594 \quad 0.2468 \quad 0.0497 \quad 3.73
\end{align*}
\]

Moreover, when we consider that the investor is always feel attached to the conventional theory which has been applied since 1959, we can suggest the following weights (table 2)
Table 2: Weight of each criterion

| Criterion | Mean return | |β-1| | Liquidity | EPR | Size |
|-----------|-------------|------------|--------|----------|--------|------|
| weight    | 0.3         | 0.3        | 0.15   | 0.15     | 0.10   |

The optimal solution with $x_M = 0.15$ gives $\phi(P) = 0.1405034$. The optimal proportion $(x_i)$ to invest in each stock that generate such maximum are the following (table 3):

Table 3: Optimal proportions to invest in stocks

<table>
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<tr>
<th>Stocks</th>
<th>$X_i$</th>
<th>$x^*$</th>
<th>$x^*/x_M = 0.25$</th>
<th>$x^*/x_M = 0.30$</th>
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We must notice that this optimal solution depends not only on the weights given to each criterion, but also on the number of criteria selected.

Table 3 clearly shows that the optimal solution also depends on constraints which are imposed to $x_M$: without any constraint, only five stocks among 37 absorb all the investments, two of the five, namely $x_{34}$ (ELMAZRAA) and $x_5$ (UBCI) absorb more than 70% of the investments. Results also show that the diminution of the threshold imposed on the maximal proportion constraint does not systematically increase the number of stocks constituting the attractive portfolio of the investor (8 for $x_M = 0.3$, but only 7 for $x_M = 0.25$). This conclusion is in perfect coherence with that of Markowitz who points out that is not enough to diversify in order to minimize risk, but one should well know how to diversify.

In other respects, and as it is expected, the model does not lead to the choice of stocks which have the highest yield or the minimum risk, but it leads to a result that reassures the investor not only at the level of yield and risk, but also at the level of liquidity and solvability.
5. Summary and Conclusions

In this paper, we have tried to illustrate how the multicriterion approach makes it possible to integrate, within the portfolio selection process, the conventional criteria (M-V) with other relevant criteria. The methodology proposed consists on comparing the performance of each stock (according to any selected criterion) to the ideal and anti-ideal portfolios. Results show that the optimal solution depends on (1) the weights given to each criterion, (2) the number of criteria selected, and (3) the value of maximum proportion to invest in each stock ($x_M$). Results show also that the number of stocks included in the "attractive portfolio" does not increase systematically with the decrease of the value of $x_M$. This result can be interpreted as one which is in coherence with that of Markowitz who remarks that the increase of the number of stocks does not decrease systematically the risk of portfolio. It is important also to point out that the methodology proposed is mathematically founded because when we apply the suggested model (multicriterion approach with ideal and anti-ideal portfolios) to the two conventional criteria, we meet exactly with the efficient set suggested by Markowitz (Veilleux, 1999). This result indicate the possibility to use the proposed methodology to test the efficiency hypothesis in comparing the proportions which result from the two models (multicriterion model and conventional one). The acceptance of nil hypothesis supposes that information about liquidity, solvability, EPR, etc., are included in the prices, and so the market could be considered as efficient.
References


Veilleux, M., (1999), "Une Approche Multicritère à la Composition de Portefeuilles", Thèse de Doctorat (not published), FSA, Université LAVAL.