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Trip packing in petrol stations replenishment¹

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Abstract

This paper considers a generalized version of the trip packing problem that we encountered as a sub-problem of the petrol stations replenishment problem. In this version we have to assign a number of trips to a fleet composed of a limited number of non-identical tank-trucks. Each trip has a specific duration, working time of vehicles is limited and the net revenue of each trip depends on the truck used. The paper provides a mathematical formulation of the problem and proposes some construction, improvement and neighbourhood search solution heuristics. A set of benchmark problem instances is created in a way that reflects real-life situations and used to analyse the performance of the proposed heuristics. A real-life case is also used to further assess the proposed heuristics.

Keywords: Vehicle scheduling, Trip packing, Mixed integer models, Heuristics.

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1. INTRODUCTION

In this paper we address a new trip packing problem which arises as a sub problem of the *petrol stations replenishment problem* (PSRP) that will be called hereafter the *generalized trip packing problem* (GTPP). To solve the PSRP (see Cornillier *et al.* [1]), we start by designing a set of trips to deliver the petrol products needed or ordered by the petrol stations and then pack these trips into subsets such that each subset can be performed within a working day by one of the available trucks. The GTPP is defined by a limited number of non identical tank-trucks and a limited number of overtime hours allowed. The heterogeneous fleet implies that the revenue of a given trip depends on the truck used for the trip, and that some trips may not be feasible for a given truck due to the characteristics and capacity of the vehicle. The objective considered is to maximize the overall net revenue of operations which is the sum over all trips of net revenue minus the marginal cost of overtime hours required. The net revenue of a trip is its revenue minus the variable operating cost (fuel consumption and cost of driver regular hours) of the used truck. The fixed cost of trucks (depreciation and regular maintenance cost) is not taken into consideration as it is paid whether the truck is used or not. Some other objectives can be overseen. Mainly, three alternative objectives are considered: to maximize the total amount of products to deliver, to maximize overall net revenue per litre delivered, and to minimize the maximum overtime hours. However, in business the main and most used objective is to maximize overall net revenue.

To the best of our knowledge, the GTPP has never been addressed in the literature. A more simplified version of this problem, called hereafter the *Simple Trip Packing Problem* (STPP), has been mentioned in the literature dealing with the *multi-trip vehicle routing problem* (MTVRP). While some contributions try to solve the MTVRP as a whole (see Brandão and Mercer [2], [3]), most others solve the problem in two steps: first generate the set of routes to be used and then pack these trips (see Fleischmann [4], Taillard *et al.* [5] Golden *et al.* [6], Zhao [7], Petch and Salhi [8] and Olivera and Viera [9]).

STPP usually assumes that we have an unlimited number of identical trucks (unlimited homogeneous fleet), that all working hours are regular hours and that trip cost and revenue are the same whichever trucks are used. Consequently, the problem is reduced to the standard *Bin Packing Problem* (BPP). This explains why the contributions addressing the homogeneous fleet

MTVRP solve the trip packing sub-problem as a bin packing problem. For a survey of the literature on bin packing problems see Coffman *et al.* [10].

Only Prins [11] considered the Multi-trip vehicle routing problem (MTVRP) with limited heterogeneous fleet. However he also solves the trip packing sub-problem as a bin packing problem using a straightforward adaptation of the first fit decreasing (FFD) heuristic. The objective considered is to minimize the number of used vehicles. Trips cost and revenue are not taken into consideration.

In contrast, in the GTPP, trips are packed into working days that have maximum regular and overtime working hours, trucks have different capacities and trip net revenues depend on the trucks used. This implies that solution methods designed to solve the bin packing problem (BPP) or the simple trip packing problem (STPP) can not be used to solve the GTPP.

Finally, it is worth noting that in practice we have about one hour to solve the petrol stations replenishment problem (PSRP). This should be done at the beginning of each day as petrol stations can issue orders at any time and ordered products should be delivered within the following 24 hours. The solution procedure proposed in [1] is an iterative procedure where each iteration consists in designing a set of trips and then solving the corresponding GTPP. Thus we need to design very fast trip packing methods.

2. LITERATURE REVIEW

As mentioned above, to the best of our knowledge, there is no publication that deals with the GTPP. Moreover, there are few publications that deal with the simple trip packing problem (STPP). Most of these publications use an adaptation of the BFD (best fit decreasing) heuristic to solve the packing problem. Most of these publications consider the case of homogeneous fleets.

Fleischmann [4] was the first to address the STPP as a sub-problem of the MTVRP. He proposed a modified version of the savings algorithm (Clarke and Wright [12]) to design a set of feasible trips and used a bin packing heuristic to assign trips to trucks. Taillard *et al.* [5] proposed a three-phase approach to solve the MTVRP. First they used the Tabu search heuristic of Rochat and Taillard [13] to generate a set of trips. Then they constructed a set of solutions as if each

vehicle is used for only one trip. Finally they used a bin packing heuristic to allocate trips to vehicles. Golden *et al.* [6] solved the MTVRP with the objective of minimizing the longest trip. They used a Tabu search procedure to produce several solutions for the VRP and then used an adaptation of the BFD and an interchange procedure to assign trips to trucks.

Zhao *et al.* [7] proposed a Tabu search heuristic with an embedded bin packing procedure. Thus, within the Tabu search, each visited neighbour solution is evaluated only after assigning trips to vehicles. Petch and Salhi [8] developed methods to generate several solutions where each vehicle is used for only one trip and, for each solution, used a modified version of the BFD to assign trips to vehicles. The best obtained MTVRP solution is then retained. In all these contributions, the fleet is composed of an unlimited number of identical vehicles (unlimited homogeneous fleet).

Olivera and Viera [9] also decompose the MTVRP and solve the trip packing sub-problem as bin packing problem. Although they consider a limited number of identical vehicles (limited homogeneous fleet), their objective is to minimize the number of vehicles to use. They indicate that they failed to solve some of their test instances.

Finally, as mentioned above, only Prins [11] considered the MTVRP with heterogeneous unlimited fleet. He assumed that all trucks have the same maximum working hours and decomposed the problem into two sub-problems: trip construction and trip packing. He developed some heuristics to construct individual trips and suggested using an adaptation of the FFD (first fit decreasing) algorithm to pack trips into a minimum number of trucks. Trip revenue and cost are not taken into consideration.

Unfortunately, none of the above mentioned trip packing methods can be used to solve the GTPP as they are not designed to maximize the overall net revenue and often unable to pack trips for a heterogeneous limited fleet. Furthermore, these methods are not designed to handle the possibility of working overtime hours in addition to regular hours.

3. PROBLEM FORMULATION

This paper considers the GTPP arose in solving the petrol station replenishment problem (see Taqa Allah *et al.* [14], Malépart *et al.* [15], Cornillier *et al.* [16] and Cornillier *et al.* [17]). This real-life problem is considered under the following assumptions:

1. There are a limited number of non identical tank-trucks. Trucks are owned by the transportation company and, consequently, the fixed costs of trucks (depreciation and regular maintenance cost) are paid whether they are used or not;
2. In addition to regular working hours, each driver can work a limited number of overtime hours;
3. When drivers work overtime, they are paid a number of hours equal to the smallest integer larger than or equal to real overtime worked;
4. For each truck, some trips are feasible and others are not depending on truck characteristics and capacity;
5. Trip revenue and variable cost depends on the truck to be used for the trip but trip duration is the same whichever the truck used.

The following notation will be used to formulate the generalized trip packing problem (GTPP) considered in this paper:

Parameters:

T set of trips to be packed; $|T|=n$

J set of available trucks; $|J|=m$

i trip index; $i \in T$

j truck index; $j \in J$

L maximum number of regular work hours

M maximum number of overtime hours

C marginal cost per overtime hour

d_i duration of trip i (in hours)

J_i set of trucks that can perform trip i , $|J_i|=m_i$

T_j set of trips that can be carried out by truck j

V_{ij} net revenue (revenue minus variable operating cost) of trip i if assigned to truck j

Decision variables:

x_{ij} binary equals 1 if trip i is assigned to truck j

O_j overtime hours for truck j

Using this notation, the generalized trip packing problem can be formulated as follows.

Find: O_j integer ≥ 0 and $x_{ij} \in \{0,1\}$, $\forall i \in T$ and $\forall j \in J$ which:

Maximize: $Z = \sum_{i \in T} \sum_{j \in J_i} V_{ij} x_{ij} - C \sum_{j \in J} O_j$

Subject to: $\sum_{j \in J_i} x_{ij} = 1 \quad ; \forall i \in T$
 $\sum_{i \in T_j} d_i x_{ij} - O_j \leq L \quad ; \forall j \in J$
 $O_j \leq M \quad ; \forall j \in J$

In this formulation, the objective is to maximize the overall net revenue of operations (sum over all trips of net revenue minus the marginal cost of overtime hours). Notice that O_j should take an integer value (see assumption 3). The first set of constraints assigns each trip to one and only one truck. The second set allows determining the overtime hours for each truck and the third set prevent overtime hours from exceeding the given upper limit.

Proposition The generalized trip packing problem (GTPP) is NP-hard.

Proof: To prove that GTTP is NP-hard we are going to use the restriction technique (see Garey and Johnson [18] page 63). So we are going to show that a special case of GTTP is NP-hard.

Let us consider the special case where $M = 0$ (no overtime hours and all working hours are regular hours). This implies that we no longer need neither the decision variable O_j nor the third constraint of the above given formulation. The model for this special case is:

Find: $x_{ij} \in \{0,1\}$, $\forall i \in T$ and $\forall j \in J$ which:

Maximize: $\sum_{i \in T} \sum_{j \in J_i} V_{ij} x_{ij}$

Subject to: $\sum_{j \in J_i} x_{ij} = 1 \quad ; \forall i \in T$
 $\sum_{i \in T_j} d_i x_{ij} \leq L \quad ; \forall j \in J.$

Thus this special case is a maximum sum generalized assignment problem. It can be easily transformed into a minimum sum problem by replacing V_{ij} by $V - V_{ij}$ where $V \geq \max_{i,j} V_{ij}$. Now, Sahni and Gonzalez [19] showed that the minimum sum generalized assignment problem is NP-hard. This completes the proof that GTTP is NP-hard.

Numerical illustration

Let us consider an example where we need to assign 10 trips ($n=10$) to 4 trucks ($m=4$) over a period including 12 regular working hours ($L=12$) and at most 3 overtime hours ($M=3$). The marginal cost of overtime hours is 1 ($C=1$). Trip durations and net revenues are given in Table 1 where NF indicates unfeasible assignments.

Table 1: Duration and revenues for the numerical example

i	1	2	3	4	5	6	7	8	9	10
d_i	3.5	5.5	3	4.5	7	7.5	2	9	4.5	3.5
V_{i1}	10	8	NF	10	9	NF	9	NF	8	9
V_{i2}	9	10	NF	9	10	9	NF	8	10	NF
V_{i3}	NF	NF	9	8	10	8	10	9	NF	8
V_{i4}	9	10	8	10	NF	10	NF	9	9	NF

For this numerical example, the optimal solution as obtained by the commercial MIP code CPLEX (version 11.0) is:

- Trips 1, 4 and 10 assigned to truck 1 (total duration 11.5 hours);
- Trips 5 and 9 assigned to truck 2 (total duration 11.5 hours);
- Trips 3, 7 and 8 assigned to truck 3 (total duration 14 hours); and
- Trips 2 and 6 assigned to truck 4 (total duration 13 hours).

This solution gives a sum of net revenues of 97 and 3 overtime hours. Thus $Z^* = 94$.

4. PROPOSED HEURISTICS

In this paper we propose several heuristic methods to solve the GTTP. First, we present six construction heuristics that can produce solutions in extremely short computation times. Second we introduce four improvement heuristics that can be used to improve solutions produced by the

construction heuristics. Third, we develop a multi-start local improvement heuristic that combines construction and improvement heuristics. Finally, we propose an adaptation of the simulated annealing algorithm with multiple reheating cycles introduced by Boctor [20]. It worth noting that, although we used these heuristics to maximize the overall net revenue, it is straightforward to modify them in order to optimize any of the other alternative objectives discussed in section 1.

4.1. CONSTRUCTION HEURISTICS

The proposed construction heuristics are greedy heuristics. They order trips based on one of their characteristics and assign them to trucks one by one according to this order. If we fail to assign one of the trips, we modify the order by moving this trip to the top of the list and try again to assign trips to trucks according to the new order. This is done at most Q_{max} times where Q_{max} is a parameter to be chosen by the user. If, in spite of these attempts, no feasible solution is obtained, we conclude that the heuristic is unable to solve the considered problem instance.

Heuristic C1

1. Number trips according to one of their characteristics (e.g., in descending order of their duration d_i or in ascending order of m_i , the number of trucks able to perform them, or randomly). Set $Q=0$.
2. Increment Q . If $Q > Q_{max}$ then the heuristic fails to find a feasible solution. Otherwise, set all trips free (unassigned) and $u_j = 0, \forall j \in J$ (u_j is the sum of the duration of trips assigned to truck j). Go to step 3.
3. For $i=1$ to n , do
 - 3.1 Assign trip i to the truck that can perform it within its regular hours (i.e., such that $u_j + d_i \leq L$,) while adding the largest trip net revenue V_{ij} . Let k be the selected truck, update u_k (i.e., set $u_k = u_k + d_i$.)
 - 3.2 If there is no such truck, assign trip i to the truck k that leads to the largest increase of the objective function (overall net revenue); i.e.:

$$k = \arg \max_j \{ V_{ij} - C (\lceil u_j + d_i - L \rceil - O_j) \mid u_j + d_i \leq L + M \},$$

where O_j is the overtime hours of truck j before assigning trip i to it. In case of tie choose the truck having the smallest number. Update u_k ($u_k = u_k + d_i$).

- 3.3 If trip i is still not assigned, find all other already-assigned trips ℓ that can be reassigned to another truck, say k (i.e., $u_k + d_\ell \leq L + M$), leaving enough time to allow adding trip i to the original truck of ℓ , say truck j (i.e., such that $u_j + d_i - d_\ell \leq L + M$). Let ℓ^* be the best trip to be reassigned (i.e., the one allowing to achieve the highest overall net revenue), j^* be its original truck and k^* be the truck to which it can be added. Move trip ℓ^* to k^* and add trip i to j^* . Update u_{k^*} and u_{j^*} .
- 3.4 If trip i is still free, move it to the top of the trip list and go back to step 2.

Step 3.3 requires examining at most n possible reassignments and is embedded in step 3 which is repeated n times. In addition, the main loop of the heuristic (steps 2 and 3) is repeated at most Q_{max} times. Thus the complexity of this construction heuristic is $O(Q_{max} n^2)$.

Heuristic C2

Heuristic C2 is similar to C1 except that we modify step 3.1 as follows:

- 3.1 If trip i is free, assign it to the truck that can do it within its regular hours (i.e., such that $u_j + d_i \leq L$, where u_j is the sum of the duration of trips assigned to truck j) while adding the largest value to the objective function. Update u_j and if there is a trip such that its duration equals $L - u_j$, then assign it to j . In case of tie assign the one adding the highest net revenue.

Obviously, the complexity of heuristic C2 is the same as that of C1; i.e.; $O(Q_{max} n^2)$.

Variants of the proposed construction heuristics

Several versions of the proposed construction heuristics will be tested. We call $C1d$, the version of C1 where trips are numbered in descending order of their duration, $C1m$, the version where trips are numbered in ascending order of the number of trucks able to perform them, and $C1r$, the version where trips are numbered randomly. Heuristics $C2d$, $C2m$ and $C2r$ can be defined similarly.

4.2. IMPROVEMENT HEURISTICS

Four local improvement heuristics were developed and are presented hereafter.

Heuristic I1

The idea of this improvement heuristic is to move trips to different trucks as long as this can improve the value of overall net revenue. Figure 1 gives the steps of the heuristic.

Until no trip can be moved to a different truck, do:

FOR $i=1$ to n

Let j be the truck to which trip i is assigned

Set $\delta=0$ (best possible improvement) and $j_0=0$ (the corresponding truck)

FOR $k=1$ to m

IF $k \neq j$ and truck k can carry out trip i in addition to its assigned trips (*i.e.*, if $u_k + d_i \leq L + M$), THEN

Calculate δ_{ijk} , the change of net revenue that results if trip i is moved from truck j to truck k :

$$\delta_{ijk} = V_{ik} - C\{\max(0, \lceil u_k + d_i - L \rceil) - O_k\} - V_{ij} + C\{O_j - \max(0, \lceil u_j - d_i - L \rceil)\}$$

IF $\delta_{ijk} > \delta$, set $\delta = \delta_{ijk}$ and $j_0 = k$

End IF

Next k

IF $j_0 \neq 0$, move trip i to truck j_0

Next i

Figure 1: Pseudo code of the heuristic I1

Within each iteration of this improvement heuristic we examine the possibility of moving each of the n trips to each of the $m-1$ alternative trucks. Thus the complexity of each iteration of this improvement heuristic is $O(mn)$. Obviously, we can not predict the number of iterations needed to reach a local optimum. This implies that we cannot determine the complexity of the heuristic as a whole.

Heuristic I2

Within this heuristic we consider every pair of trips, say i and l , assigned to two different trucks, say j and k respectively. We reassign trip i to truck k and trip l to truck j as long as this allows improving the value of the overall net revenue. It works as shown by the following pseudo code given by Figure 2.

In the worst case (where there are only 2 trucks with $n/2$ trips assigned to each of them), each iteration of this heuristic evaluates the impact of $\frac{n^2}{4}$ pair-exchanges; thus the complexity of one iteration is $O(n^2)$. Again, we can not predict the number of iterations needed to reach a local optimum.

Until no pair of trips exchange their trucks do:

```

FOR  $i=1$  to  $n-1$ 
  Let  $j$  be the truck to which trip  $i$  is assigned
  Set  $\delta=0$  (best possible improvement),  $l_0=0$  (the corresponding trip) and  $k_0=0$  (the corresponding truck)
  FOR  $l=i+1$  to  $n$ 
    Let  $k$  be the truck to which trip  $l$  is assigned
    IF  $k \neq j$  and trips  $i$  and  $l$  can be permuted (i.e., if  $u_k+d_i-d_l \leq L+M$  and  $u_j+d_l-d_i \leq L+M$ ), THEN
      Calculate  $\delta_{ijk}$ , the change in net revenue that results from this exchange:
      
$$\delta_{ijk} = V_{ik} - V_{ij} + C\{O_k - \max(0, \lceil u_k+d_i-d_l-L \rceil)\} + V_{lj} - V_{lk} + C\{O_j - \max(0, \lceil u_j+d_l-d_i-L \rceil)\}$$

      IF  $\delta_{ijk} > \delta$ , set  $\delta = \delta_{ijk}$ ,  $l_0=l$  and  $k_0=k$ 
    End IF
  Next  $l$ 
  IF  $l_0 \neq 0$ , move trip  $i$  to truck  $k_0$  and trip  $l_0$  to truck  $j$ 
Next  $i$ 

```

Figure 2: Steps of the heuristic *I2*

Heuristic I3:

Within this heuristic we consider every triplet of trips, say i , l and t , such that i and l are assigned to the same truck, say j , while trip t is assigned to different truck, say k . We reassign trips i and l to truck k and reassign t to truck j , as long as this allows improving the value of the overall net revenue. The steps of this heuristic are given in Figure 3.

Until no triplet of trips exchange their trucks do:

```

FOR  $i=1$  to  $n-2$ 
  Let  $j$  be the truck to which trip  $i$  is assigned
  Set  $\delta=0$  (best possible improvement),  $l_0=0$  and  $t_0=0$  (the corresponding trips) and  $k_0=0$  (the truck of  $t_0$ )
  FOR  $l=i+1$  to  $n-1$ 
    IF trip  $l$  is also assigned to truck  $j$  then
      FOR  $t=l+1$  to  $n$ 
        Let  $k$  be the truck to which trip  $t$  is assigned
        IF  $k \neq j$  and trips  $i$  and  $l$  can be assigned to truck  $k$  while moving  $t$  to truck  $j$ ,
        (i.e., if  $u_k+d_i+d_l-d_t \leq L+M$  and  $u_j+d_t-d_i-d_l \leq L+M$ ) THEN
          Calculate  $\delta_{iljtk}$ , the change in net revenue that results from this truck exchange:
          
$$\delta_{iljtk} = V_{ik} + V_{lk} - V_{ij} - V_{lj} + C\{O_k - \max(0, \lceil u_k+d_i+d_l-d_t-L \rceil)\} + V_{jt} - V_{tk} + C\{O_j - \max(0, \lceil u_j+d_t-d_i-d_l-L \rceil)\}$$

          IF  $\delta_{iljtk} > \delta$ , set  $\delta = \delta_{iljtk}$ ,  $l_0=l$ ,  $t_0=t$  and  $k_0=k$ 
        End IF
      Next  $t$ 
    End IF
  Next  $l$ 
  IF  $l_0 \neq 0$ , move trips  $i$  and  $l_0$  to truck  $k_0$  and trip  $t_0$  to truck  $j$ 
Next  $i$ 

```

Figure 3: Pseudo code of the improvement heuristic *I3*

In the worst case (where there are only 2 trucks with $n/2$ trips assigned to each of them), each iteration of this heuristic evaluates the impact of $\frac{n^2}{4}(\frac{n}{2} - 1)$ triplet-exchanges; thus the complexity of each iteration is $O(n^3)$. Again we cannot predict the number of iterations needed to reach a local optimum.

Heuristic I4:

Within this heuristic we consider every triplet of trips, say i , l and t , assigned to three different trucks, say j , k and v respectively, and reassign i to k , l to v and t to j , as long as this improves the value of the overall net revenue. The heuristic works as shown by the pseudo code given by Figure 4.

Until no triplet of trips exchange their trucks do:

```

FOR  $i=1$  to  $n-2$ 
  Let  $j$  be the truck to which trip  $i$  is assigned
  Set  $\delta=0$  (best improvement),  $l_0=0$  and  $t_0=0$  (the corresponding trips)  $k_0$  and  $v_0$  (the corresponding trucks)
  FOR  $l=i+1$  to  $n-1$ 
    Let  $k$  be the truck to which trip  $l$  is assigned
    IF  $k \neq j$  THEN
      FOR  $t=l+1$  to  $n$ 
        Let  $v$  be the truck to which trip  $t$  is assigned
        IF  $v \neq j$ ,  $v \neq k$  and trip  $i$ ,  $l$  and  $t$  can be moved to trucks  $k$ ,  $v$  and  $j$  respectively,
        (i.e., if  $u_k+d_i-d_l \leq L+M$ ,  $u_v+d_l-d_t \leq L+M$  and  $u_j+d_t-d_i \leq L+M$ ) THEN
          Calculate  $\delta_{ijkltv}$ , the change in net revenue that results from this truck exchange:
          
$$\delta_{ijkltv} = V_{ik} - V_{ij} + C\{O_k - \max(0, \lceil u_k + d_i - d_l - L \rceil)\} + V_{lv} - V_{lk} + C\{O_v - \max(0, \lceil u_v + d_l - d_t - L \rceil)\} + V_{jt} - V_{tv} + C\{O_j - \max(0, \lceil u_j + d_t - d_i - L \rceil)\}$$

          IF  $\delta_{ijkltv} > \delta$ , set  $\delta = \delta_{ijkltv}$ ,  $l_0=l$ ,  $t_0=t$ ,  $k_0=k$  and  $v_0=v$ 
        End IF
      Next  $t$ 
    End IF
  Next  $l$ 
  IF  $l_0 \neq 0$ , move trips  $i$ ,  $l_0$  and  $t_0$  respectively to trucks  $k_0$ ,  $v_0$  and  $j$ 
Next  $i$ 

```

Figure 4: Pseudo code of the improvement heuristic I4

In the worst case (where there are only 3 trucks with $n/3$ trips assigned to each of them), each iteration of this heuristic evaluates the impact of $\frac{n^3}{27}$ triplet-exchanges; thus the complexity of each iteration is $O(n^3)$.

Numerical illustration

In the following we apply some of the proposed heuristics to the numerical example presented in section 3.

Heuristic C1d

In step 1 we arrange trips in descending order of their duration. In step 2 we set all trips free and all trucks unused. The needed iterations of step 3 are summarized in Table 2. Applying the proposed improvement heuristics to the obtained solution leads to the following results. Heuristic **I1** does not lead to any improvement. **I2** brings us to move trip 1 to truck 1 and trip 2 to truck 4 leading to overall net revenue of 94. **I3** and **I4** produce no further improvement.

Heuristic C2d

The needed iterations of step 3 are summarized in Table 3. The second row of the table shows that trip 3 was selected (in application of step 3.1) as its duration equals the remaining time of truck 3 ($d_3=L-u_3$). Applying **I1** to the obtained solution does not lead to any improvement. **I2** brings us to move trip 2 to truck 4 and trip 4 to truck 1 leading to overall net revenue of 94. **I3** and **I4** bring no further improvement.

Table 2: Step 3 of heuristic C1d

Trip	Duration	Selected Truck	Truck net revenue minus overtime cost	Remaining regular (used overt time) hours of truck			
				1	2	3	4
8	9	3	9	12	12	3	12
6	7.5	4	10	12	12	3	4.5
5	7	2	10	12	5	3	4.5
2	5.5	1	8	6.5	5	3	4.5
4	4.5	1	10	2	5	3	4.5
9	4.5	2	10	2	0.5	3	4.5
1	3.5	4	9	2	0.5	3	1
10	3.5	1	9-2	(1.5)	0.5	3	1
3	3	3	9	(1.5)	0.5	0	1
7	2	3	10-2	(1.5)	0.5	(2)	1
Overall net revenue			90				

Table 3: Step 3 of heuristic *C2d*

Trip	Duration	Selected Truck	Truck revenue minus overtime cost	Remaining regular (used overt time) hours of truck			
				1	2	3	4
8	9	3	9	12	12	3	12
3	3	3	9	12	12	0	12
6	7.5	4	10	12	12	0	4.5
4	4.5	4	10	12	12	0	0
5	7	2	10	12	5	0	0
2	5.5	1	8	6.5	5	0	0
9	4.5	2	10	6.5	0.5	0	0
1	3.5	1	10	3	0.5	0	0
10	3.5	1	9-1	(0.5)	0.5	0	0
7	2	3	10-2	(0.5)	0.5	(2)	0
Overall net revenue			92				

4.3. MULTI-START LOCAL IMPROVEMENT HEURISTIC (MSLIH)

The general framework of the proposed *Multi-Start Local Improvement Heuristic* (MSLIH) is:

Repeat the following steps P times and retain the best solution obtained:

- 0- Set $p = 0$
- 1- Construct a new initial solution. Set $p = p + 1$
- 2- Apply the improvement heuristics $I1$.
- 3- Apply $I2$. If an improved solution is found go back to step 2.
- 4- Apply $I3$. If an improved solution is found go back to step 2.
- 5- Apply $I4$. If an improved solution is found go back to step 2.
- 6- IF $p < P$ Go to step 1.

To find a new initial solution we can apply a different construction heuristic each time. Also if we apply $C1r$ or $C2r$ several times with different random seeds we may obtain different solutions. The value of the parameter P is chosen by the user.

4.4. ADAPTATION OF THE SIMULATED ANNEALING ALGORITHM (SA)

The general framework of the proposed adaptation of the simulated annealing (SA) algorithm is given in Figure 5. Within this adaptation we execute H heating cycles and within each cycle we

start with an initial temperature $T_{\max}$ and end with a final temperature $T_{\min}$. The temperature is reduced (using a reduction coefficient α) once we reach $R_{\max}$ iterations without improving the best feasible solution found so far. It has been shown (Boctor [19]) that performing several heating cycles improves the final results. It was also shown in this same paper that a well designed adaptation of the SA algorithm can outperform other metaheuristics. That is why we decided to use SA to solve our problem.

At each iteration of our SA algorithm, we examine a neighbour of the current solution. Three neighbourhood structures are used and we randomly select one of them. Then we randomly select one neighbour in the randomly selected neighbourhood. These neighbourhood structures are:

- (1) Move a randomly selected trip to a randomly selected truck;
- (2) Permute the trucks of two randomly selected trips, and;
- (3) Randomly select three trips (say i, j and k) assigned to three different trucks and circularly permute the trucks of the selected trips (i.e.; move i to the truck of j , j to the truck of k and k the truck of i).

- Use a construction heuristic to construct an initial feasible solution
- Store the obtained solution as the current solution and as the best solution found so far
- Initialize the heating cycle counter $h:=0$
- Repeat until $h=H$
 - $h=h+1$
 - Initialize the cooling temperature $\tau=T_{\max}$
 - Repeat until $\tau < T_{\min}$
 - Initialize the repetition counter $r=0$
 - Repeat until $r=R_{\max}$
 - Randomly select a neighbourhood definition
 - Randomly select a neighbour of the current solution from the selected neighbourhood
 - Let δ be the difference between the value associated to the neighbour solution and that of the current solution
 - If $\delta > 0$ or a randomly drawn value is less than $e^{-\delta/\tau}$ then
 - Store the neighbour solution as the new current solution
 - If the neighbour solution is feasible and better than the best found so far, store it as the new best solution and set $r=0$
 - End If
 - End Repeat
 - Set $\tau=\alpha \tau$
 - End Repeat
- End Repeat

Figure 5: General Framework of the proposed adaptation of the simulated annealing algorithm

A selected neighbour solution could be either feasible or unfeasible. Unfeasible solutions are penalized as follows. A penalty p_1 is added for each trip assigned to a truck that cannot carry it out and another penalty p_2 is added for every working hour exceeding allowable hours. Penalizing unfeasible solutions instead of always rejecting them allows investigating larger number of different neighbours and reduces the chance that the search procedure is trapped in a local optimum.

5. A DYNAMIC PROGRAMMING FORMULATION

The Generalized Trip Packing Problem (GTPP) can be seen as a clustering or partitioning problem. There are two types of partitioning problems: restricted and unrestricted partitioning. Unrestricted partitioning implies that an element can be assigned to any cluster without restrictions. In the case of the GTPP we have to partition the n trips into at most m subsets in a way that maximizes overall net revenue. It is a restricted partitioning problem as some trips can not be assigned to a given truck and the sum of the duration of trips assigned to any truck should be less than or equal to the allowable working time.

There are very few papers that present a dynamic programming solution approaches to solve unrestricted partitioning problems and, to the best of our knowledge, there is no publication that proposes such approach for any restricted partitioning problem. Jensen [21] was the first to suggest using dynamic programming to deal with unrestricted partitioning problems. Hubert, Arabie and Meulman [22] were the first to provide a complete FORTRAN program to solve them. Van Os and Meulman [23] proposed to improve this method by removing redundant solutions. However, as they noted, removing redundancy slow down the process and consequently a balance should be sought between fast implementations with (some) redundancy and slower implementations that is completely non-redundant.

The GTPP can be formulated as a dynamic program as follows. Let us define E_j as a subset of trips that can be carried out by truck j within the maximal allowable (regular and overtime) work hours, $V_j(E_j)$ as the total net revenue that truck j collect from carrying out the trips of E_j , $O_j(E_j)$ as the corresponding overtime cost and $W_j(E_j) = V_j(E_j) - O_j(E_j)$ as the truck net revenue. The problem is to find a collection of m mutually exclusive and exhaustive subsets of I , say $E = \{E_1, \dots, E_m\}$,

which maximizes $W_m(I)$, the sum over all the m trucks of $W_j(E_j)$. Let E^j be all trip subsets that can be carried out by truck j , I_j all trip subsets that can be feasibly assigned to the first j trucks, and $W_j^*(I')$ denote the optimal value corresponding to the optimal assignment of $I' \subseteq I_j$ to the j first trucks. Assuming that there is a feasible assignment of I' to the j first trucks, then, dynamic programming provides an optimal solution by using the recursion:

$$W_j^*(I') = \max_{E_j \in E^j, I' - E_j \in I_{j-1} \text{ and } E_j \subseteq I'} \{W_{j-1}^*(I' - E_j) + W_j(E_j)\}; \forall I' \in I_j$$

The overall optimal solution is then given by $W_m^*(T)$. Recall that T is the whole set of trips.

To solve this dynamic program we need to calculate $W_j(E_j)$ for every truck j and for every trip subset E_j that can be carried out by j within allowable working hours. The complexity of this step is $O(m2^n - 1)$. We also need to enumerate all the subsets I_j ; $j=1, \dots, m$. This requires a huge storage capacity and makes it impossible to solve real-life problems.

Alternatively, we may transform the GTPP into an unrestricted problem by assigning a large negative revenue to trip subsets that are not feasible for the considered truck. In this case we can use the FORTRAN Code provided by Van Os and Meulman [22]. But still we need to calculate $W_j(E_j)$ for every truck j and for every trip subset E_j that can be carried out within allowable working hours. However, as noted by Van Os and Meulman, the number of recursive evaluations is of order $O(3^n)$ and the storage capacity needed is of order $O(m2^n)$. They stated that as a result of these requirements, the maximum problem size to be handled by modern desktop computers is in the range of 25 to 29. They also concluded that it is not expected that this limit will be extended in the near future given the order $O(3^n)$ of the algorithm. They also indicated that on a PC with a Pentium IV 3.1 GHz processor it took 14 hours and 1.4 GB memory to solve an unrestricted portioning problem with 26 elements and 7 clusters. In addition they indicated that the computation time is multiplied by a factor between 2.7 and 3.7 each time we add a new element the set to be partitioned. Thus if we increase the size of their problem to 50 elements, partitioning the set should require 3^{24} times 14 hours; which is about 451 million years. Unfortunately, this implies that we are not able to solve our 50 trips-15 trucks GTPP test instances by dynamic programming.

6. COMPUTATIONAL RESULTS

There are no benchmark instances for the GTPP. Thus, one hundred instances of the problem were randomly-generated and used to assess the quality of the solutions produced by the proposed heuristics. These instances were generated in a way that reflects real-life situation. We have also solved a real life instance using data provided by a petrol products distributor in Eastern Quebec, Canada.

6.1. RESULTS FOR THE RANDOMLY GENERATED TEST INSTANCES

Each of the generated instances includes 50 trips and 5 truck types. The number of available trucks of each type is 4, 2, 4, 3 and 2 for a total of 15 trucks. This is the size of real-life instances that medium sized transportation companies usually encounter. It is also the size of the real-life instance we solved (see section 6.2). Durations and trip revenues are drawn from the empirical distribution driven from the data of the real-life instance. Regular working hours are 10 and there is a maximum of 5 overtime hours. Overtime hourly cost is 60. These test instances can be obtained from the authors on a simple request.

The commercial MIP code CPLEX 11.0 was used and ran on a personal computer with a Xeon 3.6 GHz processor and 1.00 GB of RAM. CPLEX failed to solve any of the generated instances to optimality and produced an out of memory error for all instances. However CPLEX always produced an integer solution. In average, best obtained integer solutions are 0.208% worse than the best solution found heuristically. The best heuristic solution was better than the integer solution found by CPLEX for 88 instances and CPLEX produced a slightly better solution for 11 instances. The average run time of CPLEX (time to stoppage) was 1658 seconds with a standard deviation of 145 seconds. The average optimality gap was 0.7768%.

We also tried to solve these instances using CPLEX with the strong branching variable-selection option. This option requires less memory but often requires more computational times. Again CPLEX failed and reached an out of memory error after about 20 000 seconds of computation time.

Results obtained by construction heuristics alone

Table 4 presents the results obtained with each of the proposed construction heuristics alone (not followed by any improvement heuristic). Average computational time (clock time) for all these heuristics is less than 0.01 seconds. Table 4 shows that best results were obtained by *C1d* followed by *C2d*. As CPLEX failed to reach the optimal solutions, the average deviation is calculated with respect to the best solution found with either CPLEX or any of the tested heuristics. The average deviation from the best obtained solution varies between 2.90% and 3.59%. None of these construction heuristics produced the best solution for any of the test instances. *C2r* produced the worst solution for 40 test instances.

Table 4: Results obtained by construction heuristics alone

Heuristic	<i>C1d</i>	<i>C2d</i>	<i>C1m</i>	<i>C2m</i>	<i>C1r</i>	<i>C2r</i>
Average deviation from the best obtained solution	2.91%	2.92%	3.31%	3.27%	3.58%	3.60%
Standard deviation	0.45%	0.47%	0.48%	0.56%	0.53%	0.54%
Minimum deviation	1.90%	1.70%	2.35%	2.17%	2.23%	2.36%
Maximum deviation	4.04%	4.04%	4.56%	5.15%	5.05%	5.25%
Number of times it produced the best solution	0	0	0	0	0	0
Number of times it produced the worst solution	3	5	19	14	35	40

Results obtained by construction heuristics followed by improvement heuristics

Table 5 presents the results obtained by each construction heuristic followed the improvement heuristics where each one was applied once. Again deviations are with respect to the best obtained solution among those given either by CPLEX or any of the tested heuristics. Table 5 shows that slightly better results can be obtained if the initial solution is produced by *C1d* or *C2d*. However, using the means-comparison statistical test, we can conclude that there is no significant difference between the obtained average deviations. The average computing time (clock time) varies from 1.19 and 1.24 seconds. Although the average percentage deviation is less than 0.52%, none of these heuristics produced the best found solution.

Table 5: Results obtained by construction heuristics followed by each improvement heuristic once

Initial solution by	<i>C1d</i>	<i>C2d</i>	<i>C1m</i>	<i>C2m</i>	<i>C1r</i>	<i>C2r</i>
Average deviation from the best obtained solution	0.46%	0.46%	0.50%	0.48%	0.50%	0.52%
Standard deviation	0.21%	0.20%	0.19%	0.21%	0.23%	0.23%
Minimum deviation	0.04%	0.01%	0.15%	0.08%	0.06%	0.01%
Maximum deviation	0.99%	1.05%	1.05%	1.13%	1.13%	1.24%
Number of times it produced the best solution	0	0	0	0	0	0
Number of times it produced the worst solution	0	0	0	0	0	0
Average computing (clock) time in seconds	1.19	1.24	1.19	1.21	1.19	1.21

Multi-Start local improvement heuristic and best parameters

Table 6 presents the results obtained for different values of P while the initial solution was always constructed using *C1r*. As shown in the table very good results were obtained with $P = 15$ or 20. With $P=20$, the multi-start local improvement heuristic gave a better solution than the best obtained by CPLEX for 65 of the 100 test instances while requiring in average less than 24 seconds. The table shows that average deviation from the best obtained solution improves as P increases. However, computation time also increases as P increases. Thus the value of P should be chosen in function of the user's preferences.

Table 6: Results obtained using different values of P (initial solution by *C1r*)

Number of restarts P	2	3	5	9	15	20
Average deviation from the best solution found	0.40%	0.35%	0.30%	0.23%	0.20%	0.17%
Standard deviation	0.16%	0.14%	0.14%	0.12%	0.10%	0.11%
Minimum deviation	0.01%	0	0	0	0	0
Maximum deviation	0.91%	0.70%	0.68%	0.57%	0.45%	0.45%
Number of times it produced the best solution	0	2	1	3	2	4
Number of times it gave a better solution than CPLEX	15	21	35	50	54	65
Average computation time (clock time) in seconds	2.37	2.95	5.99	10.82	17.80	23.89

Simulated annealing and best heating and cooling schedules

The proposed simulated annealing algorithm requires selecting the value of several parameters: H the number of heating cycles, $T_{\max}$ the initial temperature, $T_{\min}$ the final temperature, α the temperature reduction coefficient and $R_{\max}$ the number of repetitions. We also need to fix the infeasibility penalties p_1 and p_2 . A large number of parameter combinations were tested and Table 7 presents the results obtained by some of the best combinations of H and $R_{\max}$ in the ascending order of their average computation time. For all the reported results we used:

$T_{\max} = 16$, $T_{\min} = 1$, $\alpha = 0.5$, $p_1 = 10000$ and $p_2 = 600$. Other values for these 5 parameters were also tested but it appeared that they have small effect on the quality of the obtained solutions.

From Table 7 we can see that average deviation decreases as computation time increases. It also shows that with $H=6$ and $R_{\max}=25000$, the heuristic produced an average deviation of 0.11% in 16.37 second of average time. Furthermore, comparing the results of Tables 6 and 7 we can conclude that, using the same computation time, our adaptation of the simulated annealing algorithm produces slightly better results than the multi-start local improvement heuristic (see Figure 6). For example, with approximately 11 seconds of computing time, the multi-start local improvement heuristic produced an average deviation of 0.22% while the simulated annealing adaptation produced an average deviation of 0.20%.

Table 7: Results obtained the simulated annealing adaptation in function of its parameters

H	6	6	10	6	10	6	10	10
$R_{\max}$	10 000	15 000	10 000	20 000	15 000	25 000	20 000	25 000
Average deviation from the best obtained solution	0.26%	0.19%	0.21%	0.16%	0.17%	0.11%	0.13%	0.11%
Standard deviation	0.13%	0.13%	0.11%	0.13%	0.12%	0.10%	0.11%	0.10%
Minimum deviation	0	0	0	0	0	0	0	0
Maximum deviation	0.62%	0.59%	0.51%	0.58%	0.59%	0.35%	0.63%	0.53%
Number of times it produced the best solution	2	4	2	12	8	18	16	18
Number of times it gave a better solution than CPLEX	41	49	56	61	64	74	71	75
Average computation (clock) time in seconds	6.51	9.79	10.86	13.10	16.33	16.37	21.81	27.08

Figure 6: Average deviation from best obtained solution in function of computing time

6.2. RESULTS FOR THE REAL LIFE INSTANCE

We used the data provided by a petrol products distributor in Eastern-Quebec for a single working day where 87 petrol stations are replenished using 51 routes and 14 trucks of 5 different configurations (see Table 8). The stations location is given on Figure 7 and the data for the 51 routes is given in the appendix. This real life instance was solved using 4 different scenarios:

Scenario A: 10 regular and 5 overtime hours with C (marginal overtime cost / hour) = 30\$

Scenario B: 10 regular and 5 overtime hours with $C = 60$ \$

Scenario C: 12 regular and 3 overtime hours with $C = 30$ \$

Scenario D: 12 regular and 3 overtime hours with $C = 60$ \$

Table 8: Truck types and numbers

Truck type	Number of Tanks	Total capacity (1000 litres)	Capacity of tank (1000 litres)						Number of available trucks
			1	2	3	4	5	6	
1	5	58	15	10	7.5	9.5	16	-	4
2	6	60	17	6	10	10	7	10	2
3	6	60	16	5	12	6	5	16	4
4	6	54	15	7	5	5	8	14	2
5	5	54	16	6	6	10	16	-	2

We tried to solve these instances using CPLEX version 11.0. Unfortunately, it failed to find the optimal solution for any of the attempted scenarios and always produced an out of memory error. So we tried again while using its strong branching variable-selection option. Again, CPLEX failed to solve any of the scenarios and produced an out of memory error. However, in all these trials an integer solution was found before the program failure. The obtained solution was worse than the best solution obtained heuristically for 3 of the 4 tested scenarios. In the fourth one (scenario C) a very close solution was found heuristically (overall net revenue of 12978 instead of 12981). Heuristic solutions are obtained in much shorter computation time (between 10.5 and 16.75 seconds). Solutions obtained by CPLEX 11 are given in table 9 and those obtained by the proposed heuristics are presented in Table 10.

delivered, the net revenue per 1000 litres delivered and the maximum overtime hours, for each of the solutions given in Table 10. Table 11 gives the results for scenario A. It shows that solutions with less net revenue have slightly higher values for the quantity of products to deliver. However the difference is quite small and represents 0.003%. In the same time, the solution with the largest net revenue has also the largest net revenue per 1000 litres to deliver. All these solutions have the same value for the maximum, among all trucks, of overtime hours.

Tables 12, 13 and 14 compare these same 4 criteria for scenarios B, C and D respectively. Comparing the results for all 4 scenarios, it seems that solutions with largest or the second largest net revenue have also the largest revenue per 1000 litres. For 3 of these scenarios all solutions have the same maximum overtime hours. The results indicate that in only one case (scenario B), the solution having the largest net revenue also have the largest quantity to deliver.

7. HOW TO DEAL WITH DISRUPTIONS AND DELAYS

In real life, disruptions and delays occur and we have to be able to deal with them. Mainly, there are two events that may cause disruptions: vehicles breakdown and drivers absence. Usually, transportation companies are able to replace absent drivers quite quickly and at most at the next day. It is also possible to rent a truck to replace the broken one the next day. Small delays due to small traffic problems are not very important as trip durations include a margin of about 10%. Larger delays may make it impossible to finish the remaining trips as originally planned.

Thus, in case of a sudden disruption like a truck breakdown, we need to update our operations plan by repacking all the remaining (not yet started) trips and assigning them to the remaining vehicles. This can be done very easily using the same solution methods developed in this paper. However, in presenting our methods we implicitly assumed that all trucks are available at the beginning of the planning period and have the same regular and overtime working hours. In updating our plan due to an important disruption, we have to apply our packing heuristics while taking into consideration only the remaining working time for every truck.

For example, if the working day is composed of 10 regular and 5 overtime hours, starts at 7 a.m., and a truck breaks down at 2 p.m. The remaining regular working hours for the other trucks

is the time from their return to the charging terminal after finishing their current trip and 5 p.m. If a truck returns after 5 p.m., it has no remaining regular hours and the time between its return time and 10 p.m. is its remaining overtime hours.

8. CONCLUSIONS

This paper introduced a generalized version of the trip packing problem, gave a mathematical formulation of the problem, developed heuristic solution procedures to solve it and presented a computational experiment to assess the quality of the solutions obtained by these heuristics. The considered problem involves a heterogeneous fleet of tank trucks that should deliver different petrol products to a number of petrol stations using a set of pre-designed trips. But as we have trucks of different configurations and capacities, some trips can not be carried out by some trucks. Furthermore, the cost and revenue of each trip depends on the truck to be used.

The problem is shown to be NP-hard and commercial MIP commercial codes are not able to solve problem instances of even medium size to optimality. In addition this may require long computing hours while we need very quick solutions. The heuristics presented in this paper were able to produce very good solutions with little computational time and effort.

To the best of our knowledge, this is the first research work that addressed this new problem. Further research is needed to produce more efficient heuristics and to try to develop optimal solution methods. We are designing some nature-inspired heuristics (see Doerner *et al.* [24]) to produce heuristic solutions and we are planning to use a Greedy Randomized Adaptive Memory Programming Search like the one used in [25]. Two optimal approaches could be investigated: the Lagrangean decomposition approach inspired by the one used in [26] and the branch-and-cut algorithms.

Table 11: results corresponding to scenario A (10 regular and 5 overtime hours; C= 30)

Evaluation criterion	Multi-start local improvement heuristic		Simulated annealing	Best CPLEX solution
	$P=6$	$P=10$	$H=6$	
	Initial solutions by the 6 construction heuristics	Initial solutions by $C1r$	$R_{max} = 25\ 000$	
Net revenue	12 179	12 162	12 173	12162
Quantity to deliver (litres)	2755.5	2763	2764.5	2763
Net revenue per 1000 litres	4.420	4.400	4.409	4.400
Maximum overtime (hrs)	5	5	5	5

*Boldface figures indicate the best found solution for each scenario

Table 12: results corresponding to scenario B (10 regular and 5 overtime hours; C= 60)

Evaluation criterion	Multi-start local improvement heuristic		Simulated annealing	Best CPLEX solution
	$P=6$	$P=10$	$H=6$	
	Initial solutions by the 6 construction heuristics	Initial solutions by $C1r$	$R_{max} = 25\ 000$	
Net revenue	11 309	11 255	11 314	11262
Quantity to deliver (litres)	2755.5	2763	2764	2761
Net revenue per 1000 litres	4.104	4.073	4.093	4.079
Maximum overtime (hrs)	5	5	5	5

Table 13: results corresponding to scenario C (12 regular and 3 overtime hours; C= 30)

Evaluation criterion	Multi-start local improvement heuristic		Simulated annealing	Best CPLEX solution
	$P=6$	$P=10$	$H=6$	
	Initial solutions by the 6 construction heuristics	Initial solutions by $C1r$	$R_{max} = 25\ 000$	
Net revenue	12 944	12 978	12 948	12981
Quantity to deliver (litres)	2754.5	2753.5	2759.5	2753
Net revenue per 1000 litres	4.699	4.713	4.692	4.715
Maximum overtime (hrs)	3	2	1	3

Table 14: results corresponding to scenario D (12 regular and 3 overtime hours; C= 60)

Evaluation criterion	Multi-start local improvement heuristic		Simulated annealing	Best CPLEX solution
	$P=6$	$P=10$	$H=6$	
	Initial solutions by the 6 construction heuristics	Initial solutions by $C1r$	$R_{max} = 25\ 000$	
Net revenue	12 889	12 918	12 884	12863
Quantity to deliver (litres)	2745	2753.5	2756.5	2760
Net revenue per 1000 litres	4.695	4.691	4.674	4.660
Maximum overtime (hrs)	2	2	2	2

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APPENDIX: Data for the 51 routes of the real life instance

Route	Length (km)	Duration (min)	Quantity to deliver by truck type					Net revenue for truck type				
			1	2	3	4	5	1	2	3	4	5
1	296,6	312	54	59	57	54	54	400	430	418	400	400
2	292,2	308	58	59	58	54	54	419	425	419	396	396
3	416,3	408	58	60	NF	NF	NF	563	580	NF	NF	NF
4	396,6	392	42	42	42	42	42	413	413	413	413	413
5	128	177	31	31	31	31	31	159	159	159	159	159
6	53,4	117	57.5	60	60	NF	NF	141	144	144	NF	NF
7	33,8	102	46	46	46	46	46	111	111	111	111	111
8	32,1	100	57	60	58	54	54	117	119	117	115	115
9	51,2	115	NF	57	55	53	NF	NF	138	136	134	NF
10	237,8	265	NF	57	57	54	NF	NF	351	351	337	NF
11	136,8	169	57	57	57	54	54	236	236	236	228	228
12	149,6	194	15	15	15	15	15	125	125	125	125	125
13	61,4	109	58	59	59	54	54	151	152	152	146	146
14	41	107	58	58	58	54	NF	128	128	128	124	NF
15	376,9	376	56.5	59	59	54	54	506	525	525	487	487
16	453,2	422	58	60	60	54	54	606	624	624	569	569
17	136,8	169	52	52	52	52	52	222	222	222	222	222
18	136,8	169	52	52	52	52	52	222	222	222	222	222
19	19,7	90	57.5	60	60	NF	NF	103	104	104	NF	NF
20	99,3	154	57.5	60	60	NF	NF	194	199	199	NF	NF
21	188	225	55	55	55	54	52	287	287	287	283	276
22	68,8	130	57	57	57	54	54	158	158	158	154	154
23	99	139	53	53	53	53	52	185	185	185	185	183
24	149,6	194	30	30	30	30	30	170	170	170	170	170
25	33,60	86	55.5	56	56	54	53	117	118	118	116	116
26	48,80	114	53	59	59	53	54	132	138	138	132	133
27	39,50	106	32	32	32	32	32	105	105	105	105	105
28	39,40	106	55	55	55	53	53	123	123	123	122	122
29	35,50	103	58	60	60	54	54	121	123	123	118	118
30	22,40	77	58	60	60	54	54	106	107	107	104	104
31	29,80	98	57.5	58	58	54	54	114	115	115	112	112
32	38,20	105	58	60	60	54	54	124	126	126	121	121
33	31,60	100	56	56	57	53	54	115	115	116	113	114
34	9,50	82	55	55	56	54	54	90	90	91	90	90
35	9,60	67	55	55	55	54	54	91	91	91	90	90
36	306,00	304	58	58	59	54	54	435	435	441	410	410
37	288,70	305	58	60	60	54	NF	415	426	426	392	NF
38	323,10	333	58	59	60	54	NF	455	461	468	429	NF
39	298,60	298	58	59	60	54	54	426	432	438	402	402
40	291,20	307	51	51	51	51	51	377	377	377	377	377
41	307,30	320	56.5	59	60	53	54	427	443	449	406	412
42	831,80	740	58	60	58	54	NF	1045	1078	1045	978	NF
43	132,00	165	56.5	59	59	54	54	229	236	236	223	223
44	151,40	196	58	58	59	54	54	256	256	259	244	244
45	176,00	215	54	54	54	53	53	270	270	270	267	267
46	165,60	192	53	53	53	53	53	256	256	256	256	256
47	174,00	199	54	54	54	53	54	268	268	268	264	268
48	174,00	214	54	54	55	52	53	268	268	271	261	264
49	42,80	109	57	57	57	54	53	129	129	129	126	125
50	53,30	117	58	60	60	54	NF	142	144	144	138	NF
51	27,40	96	54	55	55	53	54	110	110	110	109	110